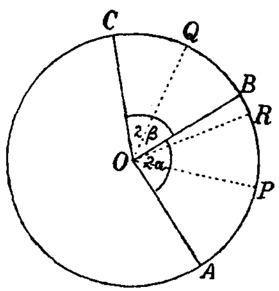
Elementary Method of investigating the Centroid of a Uniform Circular Arc.

Let AB and BC be two circular arcs subtending angles 2α and 2β at the common centre O. From symmetry the centroids G_1, G_2 and G of AB, BC and AC lie on the bisectors OP, OQ and



OR of the angles which they subtend at the centre. Also, G is the centroid of two particles placed at G_1 and G_2, and with masses proportional to the arcs AB and BC. Hence G_1, G, and G_2 are collinear, and

$$\frac{G_1G}{GG_2} = \frac{\beta}{\alpha}. \quad \text{..............................(1)}$$

Now $\angle AOR = \angle ROC = \alpha + \beta$, therefore

$$\angle POR = \angle AOR - \angle AOP = \beta,$$

and $\angle ROQ = \angle ROC - \angle QOC = \alpha.$

Hence $$\frac{G_1G}{GG_2} = \frac{OG_1}{OG_2} \cdot \frac{\sin G_1OG}{\sin GOG_2} = \frac{OG_1 \sin\beta}{OG_2 \sin\alpha}. \quad \text{...............(2)}$$

Equating (1) and (2) we have

$$OG_1 : OG_2 = \frac{\sin\alpha}{\alpha} : \frac{\sin\beta}{\beta}.$$

Hence the ratio $OG_1 : \frac{\sin\alpha}{\alpha}$ is independent of α, and therefore

$$OG_1 = k\frac{\sin\alpha}{\alpha} = k\frac{\text{chord}}{\text{arc}},$$

the angle α being in circular measure.

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To find k, let the arc diminish and tend to zero, then the ratio chord : arc $\rightarrow$ 1, and $OG_1 \rightarrow$ the radius, hence finally

$$OG = \text{radius} \times \frac{\text{chord}}{\text{arc}}.$$

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✕ Nomogram for the Solution of the Equation $z^3 + pz + q = 0$.

The curve $y = x^3$ is constructed with a scale of 1″ horizontally and $\frac{1}{10}$″ vertically. It is graduated with the values of x. Then draw the two lines $x = \pm 1$ and graduate them on the scale of the y-axis, $x = +1$ positive downwards, $x = -1$ positive upwards.

To solve the equation $z^3 + pz + q = 0$, set $p + q$ on the line $x = +1$ and $p - q$ on the line $x = -1$. Join the two points and get the intersections with the curve. The accompanying diagram has only a range between $\pm 3{\cdot}5$, but it is easy to divide the roots by 2, 3, or 10. If accurately drawn, it should give two figures of the root with ease, as a rule, and may give three. Sometimes, of course, it may be difficult to separate the roots, *e.g.* in the equation $z^3 - 13z + 18 = 0$. The line seems to touch about $+2{\cdot}2$, and the 3rd root is off the paper. Dividing the roots by 2, we get $z^3 - 3{\cdot}25z + 2{\cdot}25 = 0$. We see that 1 is a root, another root is $-2{\cdot}08$, and therefore the third root is $+1{\cdot}08$. Hence the roots of the given equation are 2, $2{\cdot}16$, $-4{\cdot}16$.

I find it convenient to use, instead of a ruler, which hides part of the diagram, a strip of celluloid with a fine line ruled on the lower surface.

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