Foreword

For about twenty years after its invention, quantum electrodynamics remained an isolated success in the sense that the underlying ideas seemed to apply only to the electromagnetic force. In particular, its techniques did not seem to be useful in dealing with weak and strong interactions. These interactions seemed to lie outside the scope of the framework of local quantum field theory and there was a wide-spread belief that the best way to handle them would be via a more general, abstract S-matrix theory. All this changed dramatically with the discovery that non-Abelian gauge theories were renormalizable. Once the power of the gauge principle was fully recognized, local quantum field theory returned to the scene and, by now, dominates our thinking. Quantum gauge theories provide not only the most natural but also the only viable candidates we have for the description of electroweak and strong forces.

The basic dynamical variables in these theories are represented by non-Abelian connections. Since all the gauge invariant information in a connection is contained in the Wilson loops variables (i.e., traces of holonomies), it is natural to try to bring them to the forefront. This is precisely what is done in the lattice approaches which are the most successful tools we have to probe the non-perturbative features of quantum gauge theories. In the continuum, there have also been several attempts to formulate the theory in terms of Wilson loops. In the perturbative approach, it is known that Wilson loop "Schwinger functions" are finite to all orders *after renormalization. This is a strong indication that they may be also mathematically meaningful in a non-perturbative treatment of the continuum theory. Since these are functions on an appropriate space of loops, one can derive differential equations they satisfy on that loop space. The hope is that once a complete set of equations is obtained, physical "boundary conditions" will lead to unique solutions which in turn will determine the theory. Thus, the space of loops offers a natural arena for the quantum theories of connections.*

https://doi.org/10.1017/9781009290203.001 Published online by Cambridge University Press

In the last few years, this relation between connections and loops has acquired another dimension. In these developments, the emphasis is on the Hamiltonian formulation. It turns out that there is a remarkable mathematical interplay between measures on the spaces of connections and functions on the loop space, which gives rise to a generalization of the Fourier transform, called the loop transform. This transform can be defined rigorously. As a result, quantum states can be regarded either as gauge invariant functions of connections or as suitable functions of loops. The loop picture suggests new strategies for defining operators and provides a number of new insights.

Quite surprisingly, it turns out that these insights are especially useful while dealing with a force that one does not, normally, associate with theories of connections: gravity. General relativity is usually thought of as a theory of metrics and, therefore, quite removed from theories of other interactions. One can, however, think of it also as a dynamical theory of connections. This idea is not new. Indeed, such a reformulation was obtained already by Einstein and Schrödinger. In their new version, the Levi Civita connection is regarded as the basic variable; metric is a secondary, derived object. The problem was that the equations of the theory became more complicated. It turns out, however, that if one uses chiral connections in place of Levi Civita, the equations actually simplify. With this observation, general relativity moves closer to theories governing other fundamental forces. As in other theories, one can now represent states of quantum gravity as functions of (chiral) connections or, via loop transform, of loops. Thus, the loop representation offers a unified arena for the quantum description of all four fundamental interactions. In the case of general relativity, further structures arise because physical states are required to be invariant under the action of diffeomorphisms. In the loop representation, they depend not on individual loops but also on the (generalized) knot to which the loop belongs. There is thus an unexpected interplay between loops, knots, gauge fields and gravity.

This monograph is devoted to this interplay. The authors are eminently qualified to unfold this saga as they are among the leaders in the field. Indeed, many of the developments that I have alluded to are due to them and their close colleagues. They provide not only a comprehensive summary of the entire subject, but, in the last few chapters, also a glimpse of two frontier areas of active research. Graduate students would find this unified treatment of a large subject extremely useful. More advanced researchers would be able to appreciate the fascinating confluence of ideas from particle physics general relativity and contemporary mathematics.

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