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CORRIGENDUM

On the stability of relativistic nonlinear cold plasma waves

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In this note we investigate the stability of large-amplitude longitudinal relativistic plasma waves. We find that they are secularly stable, that is, small perturbations grow in time proportional to time but not exponentially with time. Similar results have recently been obtained for transverse waves by Romeiras (1978).

The present stability problem was considered by Infeld & Rowlands (1973) who erroneously concluded the waves to be marginally stable. In the notation of that paper it was found that the perturbations were of the form

$$\delta\gamma = A(t)\frac{\partial\gamma}{\partial\xi} + B(t)\frac{\partial\gamma}{\partial\lambda_1}$$

where $\gamma(\xi)$ represents the spatial variation of the nonlinear wave and λ_1 a parameter which distinguishes these various waves. It was then stated that both $\partial\gamma/\partial\xi$ and $\partial\gamma/\partial\lambda_1$ were periodic functions of ξ and the condition that $\delta\gamma$ must itself be a periodic function implied that A and B must be periodic functions of time leading to marginal stability.

However, since γ represents a periodic nonlinear wave we may write it in the form

$$\gamma(\xi) = \gamma(\xi/L(\lambda_1), \lambda_1)$$

where L is the period of the nonlinear wave. With this form we see that

$$\frac{\partial\gamma}{\partial\lambda_1} = -\frac{1}{L}\frac{dL}{\partial\lambda_1}\xi\frac{\partial\gamma}{\partial\xi} + \frac{\partial\gamma}{\partial\lambda_1}$$

and that if $dL/d\lambda_1 \neq 0$ then $\partial\gamma/\partial\lambda_1$ is spatially secular. It would then appear that we must demand $B(t) \equiv 0$ in order for $\delta\gamma$ to remain bounded in space and this was, in fact, the condition we demanded in our 1973 paper. However, the quantity t is itself of the form

$$t = \tau - \int^{\xi} \frac{\gamma}{(\gamma^2 - 1)^{\frac{1}{2}}} d\xi$$

where τ is the true time variable. Thus if we take B to be constant, B_0, and

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A proportional to t, $A = \alpha t$, then we may choose α to make $\delta\gamma$ spatially periodic, namely

$$\alpha = \frac{B_0}{L^2}\frac{dL}{d\lambda_1}\oint\frac{\gamma}{(\gamma^2-1)^{\frac{1}{2}}}\,d\xi.$$

The perturbation may now be written in the form

$$\delta\gamma = \alpha\tau\frac{\partial\gamma}{\partial\xi} + \text{spatial periodic function}$$

showing that $\delta\gamma$ is linearly unstable.

The integral involved in α is over a complete period of the nonlinear wave and using the corrected form of equation (4.8) of Infeld & Rowlands, namely

$$\gamma_0 - \gamma\left(\frac{\gamma_0^2-1}{\gamma^2-1}\right)^{\frac{1}{2}} = -\frac{\partial^2\gamma}{d\xi^2},$$

we see that

$$\oint\frac{\gamma}{(\gamma^2-1)^{\frac{1}{2}}}\,d\xi = \frac{\gamma_0 L}{(\gamma^2-1)^{\frac{1}{2}}}.$$

In the non-relativistic limit, the period L is independent of λ_1 and hence we must demand $B \equiv 0$, implying marginal stability as found by Rowlands (1969).

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