3

Pinch technique to all orders

In this chapter, we present the generalization of the pinch technique (PT) beyond one loop. The key observation is that the one-loop PT rearrangements described in Chapter 1 constitute the lowest-order manifestation of a fundamental cancellation taking place between graphs of distinct kinematic nature. This cancellation is encoded in the Slavnov–Taylor identity satisfied by a special Green's function, which serves as a common kernel to all higher-order self-energy and vertex diagrams. This allows for the collective treatment of entire sets of diagrams, providing a compact way of extending the PT construction to higher orders. In addition, we will show that, quite remarkably, the correspondence between the pinch technique and the background Feynman gauge established in Chapter 1 is not accidental but persists to all orders.

3.1 The s-t cancellation to all orders

The generalization of the pinch technique to all orders relies on the following basic observations. The vast PT cancellations between one-loop Feynman diagrams, studied in Chapter 1, are in fact encoded in the Slavnov–Taylor identity obeyed by the kernel $A_\mu A_\nu q\bar{q}$ (with the gluons off shell and the quarks on shell). In the Feynman gauge, this Slavnov–Taylor identity is triggered by the longitudinal momenta k_1^μ and k_2^ν contained in $\Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q, k_1, k_2)$. The tree-level version of this Slavnov-Taylor identity gives rise precisely to the s-t cancellation discussed in Section 1.7.2 (but with the gluons on shell) for the tree-level process $gg \to q\bar{q}$, namely, the lowest-order contribution to the aforementioned amplitude $A_\mu A_\nu q\bar{q}$.

Indeed, as explained in Section 1.7.2, at tree-level, the preceding amplitude, denoted by $\mathcal{T}^{mn}_{\mu\nu}$ is the sum of two distinct parts: an s-channel subamplitude, $\mathcal{T}^{mn}_{s,\,\mu\nu}$, given in Figure 1.16($c$) and t- and u-channel subamplitudes containing an internal quark propagator, $\mathcal{T}^{mn}_{t,\,\mu\nu}$, shown in diagrams ($a$) and ($b$) of the same figure.

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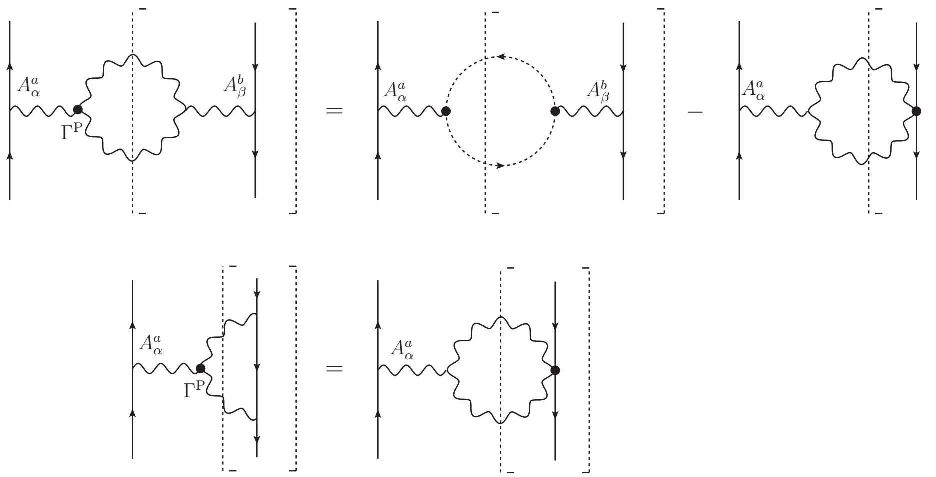

Figure 3.1. The one-loop pinch technique seen in terms of the fundamental s-t cancellation. The self-energy-like contribution coming from the vertex cancels exactly against the contribution coming from the propagator. Notice that none of the effective vertices induced after the cancellation is contained in the original Lagrangian of the theory; their field-theoretic interpretation will be presented in Chapter 4.

When $\mathcal{T}^{mn}_{\mu\nu}$ is contracted by k_1^μ or k_2^ν, a characteristic cancellation takes place between $\mathcal{T}^{mn}_{s\,\mu\nu}$ and $\mathcal{T}^{mn}_{t\,\mu\nu}$. To see this, use the elementary Ward identity satisfied by $\Gamma^\alpha_{\mu\nu}(q,k_1,k_2)$ and note that the term proportional to q^2 cancels the $d(q)$, thus allowing communication with the t-channel graphs (in Section 1.7.2, we used $\Gamma^{\rm F}$ instead, but this makes no difference; see the comment following Eq. (1.130)). Using, in addition, current conservation, $q^\alpha\mathcal{V}^c_\alpha = 0$, and keeping the gluons off shell (i.e., not setting $k_1^2 = k_2^2 = 0$, as we did in Section 1.7.2), we have

$$\begin{aligned} k_1^\mu \mathcal{T}^{mn}_{s\,\mu\nu} &= g^2 f^{mnc}\mathcal{V}^c_\alpha(k_2^2 g^\alpha_\nu - k_{2\nu}k_2^\alpha)d(q^2) - g^2 f^{mnc}\mathcal{V}^c_\nu \\ k_1^\mu \mathcal{T}^{mn}_{t\,\mu\nu} &= g^2 f^{mnc}\mathcal{V}^c_\nu, \end{aligned} \tag{3.1}$$

so that

$$k_1^\mu \mathcal{T}^{mn}_{\mu\nu} = g^2 f^{mnc}\mathcal{V}^c_\alpha(k_2^2 g^\alpha_\nu - k_{2\nu}k_2^\alpha)d(q^2). \tag{3.2}$$

The important point to realize is that one can recast the entire one-loop PT construction in terms of the s-t cancellation. The precise way in which the preceding cancellation is realized inside the one-loop self-energy and vertex graphs, giving rise to the PT rearrangements described in Chapter 1, is shown schematically in Figure 3.1.

It turns out that the pinch technique may be extended to higher orders simply by pursuing the preceding cancellations beyond tree level [1, 2]. Specifically, the

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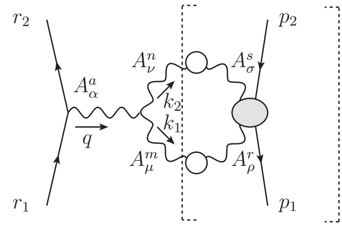

Figure 3.2. The fundamental amplitude receiving the action of the longitudinal momenta stemming from Γ^{P}. The shaded blob represents the (connected) kernel corresponding to the process $AA \to q\bar{q}$.

all-order version of the Slavnov–Taylor identity satisfied by $\mathcal{T}_{\mu\nu}^{mn}$, appropriately interpreted, allows the generalization of the PT construction to all orders.

The subset of all graphs that receive the action of the longitudinal momenta contained in $\Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q, k_1, k_2)$ is shown in Figure 3.2: it comprises precisely the kernel $A_\mu^m(k_1)A_\nu^n(k_2) \to q(p_1)\bar{q}(p_2)$, i.e., the all-order version of $\mathcal{T}_{\mu\nu}^{mn}$. In terms of Green's functions,

$$\mathcal{T}_{\mu\nu}^{mn} = \bar{u}(p_1)\big[\mathcal{C}_{\rho\sigma}^{mn}(k_1, k_2, p_1, p_2)\Delta_\mu^\rho(k_1)\Delta_\nu^\sigma(k_2)\big]u(p_2). \tag{3.3}$$

Clearly, the two internal gluons are off shell, whereas the two external quarks are on-shell, satisfying $\bar{u}(p_1)S^{-1}(p_1)\big|_{\not{p}_1=m} = S^{-1}(p_2)u(p_2)\big|_{\not{p}_2=m} = 0$, where $S(p)$ is the full-quark propagator.

Let us focus on the Slavnov–Taylor identity satisfied by the amplitude $\mathcal{T}_{\mu\nu}^{mn}$. Following standard techniques [3], one exploits ghost charge conservation to write the trivial position space identity:

$$\big\langle T[\bar{c}^m(x)A_\nu^n(y)q(z)\bar{q}(w)]\big\rangle = 0, \tag{3.4}$$

with T denoting the time-ordered product of fields. Rewriting the fields in terms of their BRST-transformed counterparts, using their equations of motion and equal-time commutation relations, and Fourier transforming the final result to momentum space, we find

$$k_1^\mu C_{\mu\nu}^{mn} - k_{2\nu}G_1^{mn} + \mathrm{i}gf^{nrs}Q_{1\nu}^{mrs} - gX_{1\nu}^{mn} - g\bar{X}_{1\nu}^{mn} = 0, \tag{3.5}$$

where the various Green's functions appearing on the right-hand side (rhs) are defined in Figure 3.3. Note that the terms $X_{1\nu}$ and $\bar{X}_{1\nu}$ vanish on shell because they are missing one fermion propagator; at lowest order, they are simply the terms proportional to the inverse tree-level propagators $(\not{p}_1 - m)$ and $(\not{p}_2 - m)$ first encountered in the one-loop PT calculations of Chapter 1. After multiplying Eq. (3.5) by the two inverse propagators $S^{-1}(p_1)S^{-1}(p_2)$, we thus arrive at the

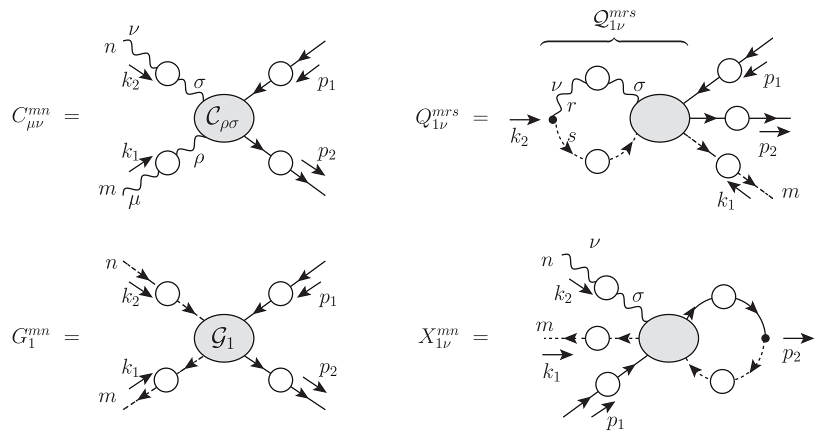

Figure 3.3. Diagrammatic representation of the Green's functions appearing in the Slavnov–Taylor identity (Eq. (3.5)). Ghost Green's functions receive a contribution from similar terms with the ghost arrows reversed (not shown).

on-shell Slavnov–Taylor identity

$$k_1^{\mu} \mathcal{T}^{mn}_{\mu\nu} = \mathcal{S}^{mn}_{1\nu}, \tag{3.6}$$

with[1]

$$\mathcal{S}^{mn}_{1\nu} = \bar{u}(p_1)\left[g f^{nrs}\mathcal{Q}^{mrs}_{1\nu}(k_1,k_2) - k_{2\nu}\mathcal{G}^{mn}_1(k_1,k_2)D(k_2)\right]D(k_1)u(p_2), \tag{3.7}$$

with $\mathcal{G}^{mn}_1$ and $\mathcal{Q}^{ars}_{1\nu}$ defined in Figure 3.3.

In perturbation theory, both $\mathcal{T}^{mn}_{\mu\nu}$ and $\mathcal{S}^{mn}_{1\nu}$ are given by Feynman diagrams, which can be separated into distinct classes according to their kinematic dependence and topological properties (Figure 3.4). Graphs that do not contain information about the external test quarks are self-energy graphs, whereas those depending on the quantum numbers of the test quarks are vertex graphs. The former depend only on the variable s, the latter on both s and the mass m of the test quarks; equivalently, we will refer to them as s- or t-channel graphs, respectively. In addition to the s-t classification, Feynman diagrams can be separated into 1PI and 1PR graphs. The crucial point is that the action of the momentum k_1^{μ} or k_2^{ν} on $\mathcal{T}^{mn}_{\mu\nu}$ does not respect, in general, the original s-t and 1PI-1PR separations furnished by the Feynman diagrams. In other words, even though Eq. (3.6) holds for the entire amplitude, it is not true for the individual subamplitudes, i.e.,

$$k_1^{\mu}\left[\mathcal{T}^{mn}_{\mu\nu}\right]_{x,\mathrm{Y}} \neq \left[\mathcal{S}^{mn}_{1\nu}\right]_{x,\mathrm{Y}} \qquad x = s, t;\ \mathrm{Y} = \mathrm{I}, \mathrm{R}, \tag{3.8}$$

[1] In what follows, the only momenta we indicate in the Green's functions are the ones corresponding to the gluons (k_i); the quark momenta (p_i) will instead be omitted.

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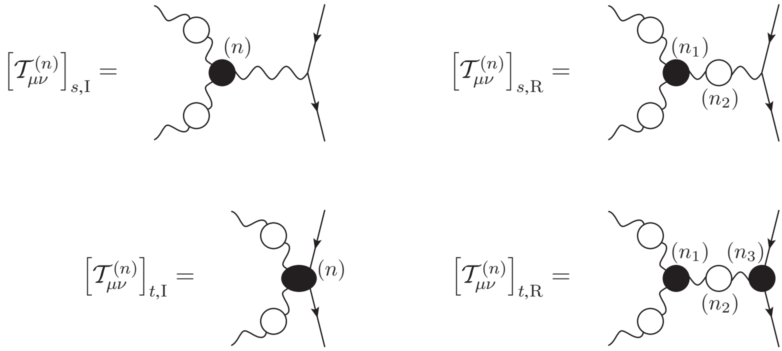

Figure 3.4. Decomposition at an arbitrary perturbative level n of the fundamental amplitude $\mathcal{T}_{\mu\nu}^{mn}$ in terms of s and t channels and 1PI and 1PR components.

where I (R) indicates the one-particle irreducible (reducible) parts of the amplitude involved. Evidently, whereas the characterization of graphs as propagator- and vertex-like is unambiguous in the absence of longitudinal momenta (e.g., in a scalar theory or in QED), their presence in non-Abelian gauge theories tends to mix propagator- and vertex-like graphs. Similarly, 1PR graphs can be effectively converted into 1PI graphs (the opposite cannot happen). The inequality between the two sides of Eq. (3.8) is precisely due to propagator-like terms, such as those encountered in the one-loop PT calculations; they have the characteristic feature that, when depicted by means of Feynman diagrams, contains unphysical vertices, i.e., vertices that do not exist in the original Lagrangian (Figure 3.5). All such terms cancel *exactly* against each other. Thus, after the PT cancellations have been enforced, we have

$$\left[k_1^{\mu}\mathcal{T}_{\mu\nu}^{mn}\right]_{t,\mathrm{I}}^{\mathrm{PT}} \equiv \left[\mathcal{S}_{1\nu}^{mn}\right]_{t,\mathrm{I}}. \tag{3.9}$$

The nontrivial step for generalizing the pinch technique to all orders is then the following: instead of going through the arduous task of manipulating the left-hand side of Eq. (3.9) to determine the pinching parts and explicitly enforce their cancellation, use directly the rhs, which already contains the answer! Indeed, the rhs involves only conventional (ghost) Green's functions expressed in terms of standard Feynman rules with no reference to unphysical vertices. Thus, its separation into propagator- and vertexlike graphs can be carried out unambiguously because all possibility for mixing has been eliminated.

3.2 Quark-gluon vertex and gluon propagator to all orders

The considerations just presented can be used to generalize the PT construction to all orders. In what follows, we will denote with a caret superscript the PT boxes,

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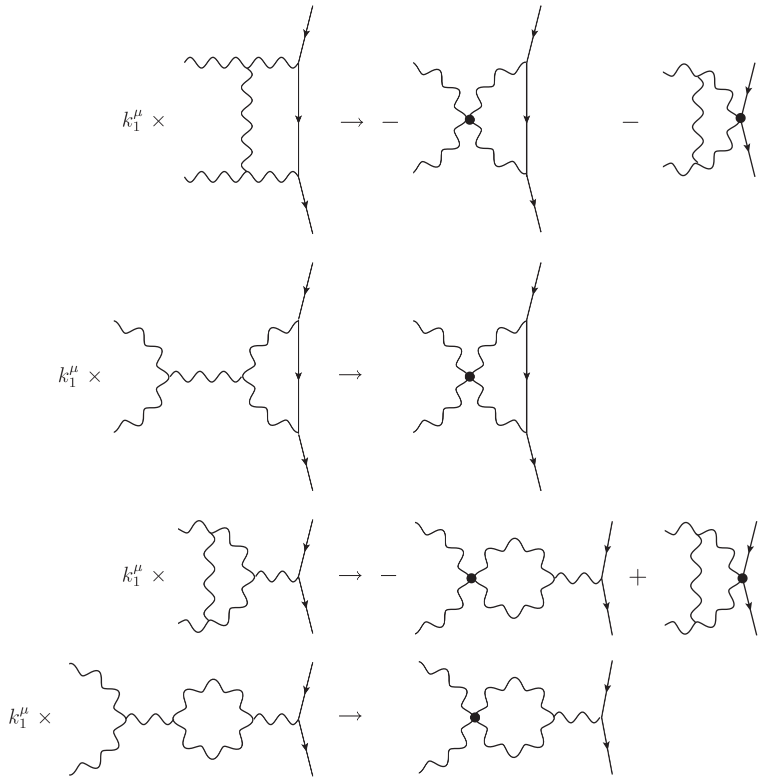

Figure 3.5. Some schematic two-loop examples of PT terms containing unphysical vertices, together with the Feynman diagrams from which they originate. Notice that the sum of all these terms is zero.

self-energies, and vertices and with a tilde the corresponding background Feynman gauge objects; the conventional renormalizable Feynman gauge terms will not carry any superscript.

To begin, it is immediate to recognize that in the renormalizable Feynman gauge, box diagrams of arbitrary order n, $B^{(n)}$, coincide with the PT boxes $\widehat{B}^{(n)}$ because all three-gluon vertices are internal; that is, they do not provide longitudinal momenta because inside the loops there is no preferred direction. Thus, they coincide with the background Feynman gauge boxes, $\widetilde{B}^{(n)}$, i.e.,

$$\widehat{B}^{(n)} = B^{(n)} = \widetilde{B}^{(n)} \tag{3.10}$$

for every n. The same is true for the PT quark self-energies; for exactly the same reason, they coincide with their renormalizable Feynman gauge (and background Feynman gauge) counterparts, i.e.,

$$\widehat{\Sigma}^{ij\,(n)} = \Sigma^{ij\,(n)} = \widetilde{\Sigma}^{ij\,(n)}. \tag{3.11}$$

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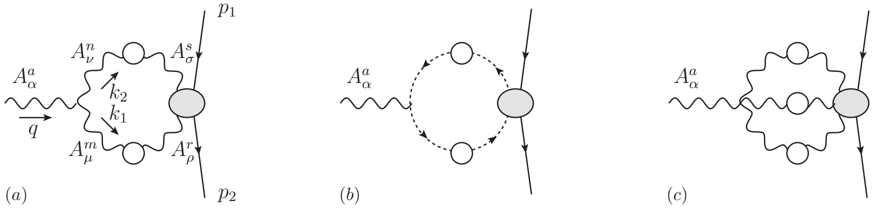

Figure 3.6. The Feynman diagrams contributing to the quark-gluon vertex Γ^a_α in the R_ξ gauge. Diagram (b) has a similar contribution (b') with the ghost arrow reversed. Kernels appearing in these diagrams are t-channel and 1PI with respect to s-channel cuts.

For the construction of the quark-gluon 1PI vertex $\widehat{\Gamma}^a_\alpha$, start by noting that of all diagrams contributing to this vertex in the renormalizable Feynman gauge (shown in Figure 3.6), the only one receiving the action of the pinching momenta is diagram (a). Thus, we carry out the PT vertex decomposition of Eq. (1.41) in diagram (a) and concentrate on the Γ^{P} part only; specifically,

$$(a)^{\mathrm{P}} = gf^{amn}\int_{k_1}(g^\nu_\alpha k^\mu_1 - g^\mu_\alpha k^\nu_2)\left[\mathcal{T}^{mn}_{\mu\nu}(k_1,k_2)\right]_{t,\mathrm{I}}. \tag{3.12}$$

Following the discussion presented in the previous subsection, the pinching action amounts to the replacements

$$k^\mu_1\left[\mathcal{T}^{mn}_{\mu\nu}\right]_{t,\mathrm{I}} \to \left[k^\mu_1\mathcal{T}^{mn}_{\mu\nu}\right]^{\mathrm{PT}}_{t,\mathrm{I}} = \left[\mathcal{S}^{mn}_{1\nu}\right]_{t,\mathrm{I}} \tag{3.13}$$

$$k^\nu_2\left[\mathcal{T}^{mn}_{\mu\nu}\right]_{t,\mathrm{I}} \to \left[k^\nu_2\mathcal{T}^{mn}_{\mu\nu}\right]^{\mathrm{PT}}_{t,\mathrm{I}} = \left[\mathcal{S}^{mn}_{2\mu}\right]_{t,\mathrm{I}}, \tag{3.14}$$

or equivalently,

$$(a)^{\mathrm{P}} \to gf^{amn}\int_{k_1}\left\{\left[\mathcal{S}^{mn}_{1\alpha}(k_1,k_2)\right]_{t,\mathrm{I}} - \left[\mathcal{S}^{mn}_{2\alpha}(k_1,k_2)\right]_{t,\mathrm{I}}\right\}. \tag{3.15}$$

At this point, the construction of the effective PT quark-gluon vertex has been completed, and we have

$$\begin{aligned}\widehat{\Gamma}^a_\alpha(q,p_2,-p_1) &= (a)^{\mathrm{F}} + (b) + (b') + (c)\\ &\quad + gf^{amn}\int_{k_1}\left\{\left[\mathcal{S}^{mn}_{1\alpha}(k_1,k_2)\right]_{t,\mathrm{I}} - \left[\mathcal{S}^{mn}_{2\alpha}(k_1,k_2)\right]_{t,\mathrm{I}}\right\}.\end{aligned} \tag{3.16}$$

We emphasize that in the construction presented thus far, we have never resorted to the BFM formalism but have only used the BRST identity of Eq. (3.6) and the replacements (3.13) and (3.14).

The next important question is whether the one-loop correspondence between the pinch technique and the background Feynman gauge persists to all orders. This is

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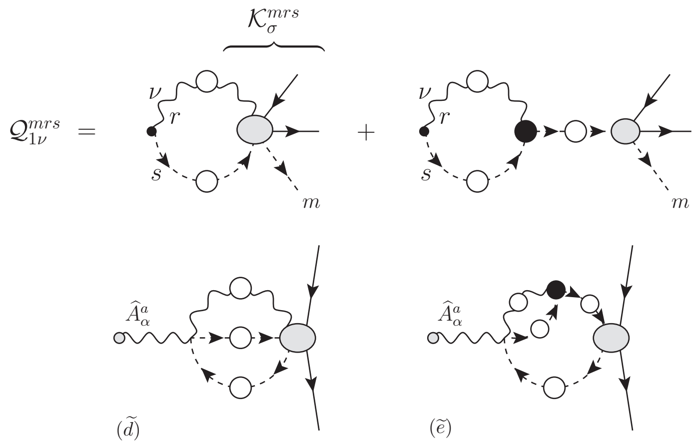

Figure 3.7. (*Top*) The decomposition of the auxiliary function $\mathcal{Q}_{1\nu}^{mrs}$ in terms of its 1PI and 1PR components. Notice that the kernel $\mathcal{K}_\sigma^{mrs}$ is 1PI with respect to s-channel cuts. (*Bottom*) Additional topologies present in the BFM quark-gluon vertex and dynamically generated in the PT procedure. Both diagrams have similar contributions (d') and (e') with the ghost arrows reversed.

indeed so, as can be seen by comparing directly the PT vertex $\widehat{\Gamma}_\alpha^a$ just constructed and the quark-gluon vertex $\widetilde{\Gamma}_\alpha^a$ written in the background Feynman gauge. We start by observing that all inert terms contained in the original renormalizable Feynman gauge Γ_α^a vertex carry over to the same subgroups of background Feynman gauge graphs. To facilitate this identification, we recall (see also the Feynman rules reported in the appendix) that to lowest order, one has the identities $\Gamma^{\mathrm{F}} = \Gamma_{\widehat{A}AA}$ and $\Gamma_{AAAA} = \Gamma_{\widehat{A}AAA}$, so that

$$(a)^{\mathrm{F}} = (\widetilde{a}) \qquad (c) = (\widetilde{c}), \tag{3.17}$$

where a tilde means that the (external) gluon A_α^a has been effectively converted into a background gluon $\widehat{A}_\alpha^a$.

As should be familiar by now, the only exception to this rule are the ghost diagrams (d) and (d'): they must be combined with the remaining terms from the PT construction to arrive at the characteristic ghost sector of the background Feynman gauge (see Figure 3.7), namely, the symmetric ghost-gluon vertex $\Gamma_{\widehat{A}c\bar{c}}$ and the four-particle ghost vertex $\Gamma_{\widehat{A}Ac\bar{c}}$, absent in the conventional R_ξ gauge fixing. Indeed, using Eq. (3.7), we find (omitting the spinors)

$$\begin{aligned} gf^{amn}\int_{k_1}\big[\mathcal{S}_{1\alpha}^{mn}(k_1,k_2)\big]_{t,\mathrm{I}} &= -gf^{amn}\int_{k_1} k_{2\alpha}D(k_1)D(k_2)\big[\mathcal{G}_1^{mn}(k_1,k_2)\big]_{t,\mathrm{I}} \\ &\quad + g^2 f^{amn} f^{nrs}\int_{k_1} D(k_1)\big[\mathcal{Q}_{1\alpha}^{mrs}(k_1,k_2)\big]_{t,\mathrm{I}}, \end{aligned} \tag{3.18}$$

with a similar relation holding for the $\mathcal{S}_2$ term. Then we find

$$\begin{aligned}
&(b) - g f^{amn} \int_{k_1} k_{2\alpha} D(k_1) D(k_2) \left[\mathcal{G}_1^{mn}(k_1, k_2)\right]_{t,\mathrm{I}} \\
&\quad = g f^{amn} \int_{k_1} (k_1 - k_2)_\alpha D(k_1) D(k_2) \left[\mathcal{G}_1^{mn}(k_1, k_2)\right]_{t,\mathrm{I}} = (\widehat{b}),
\end{aligned} \tag{3.19}$$

and using the decomposition for the $\mathcal{Q}_{1\nu}^{mrs}$ shown in Figure 3.2,

$$\begin{aligned}
&g^2 f^{amn} f^{nrs} \int_{k_1} D(k_1) \left[\mathcal{Q}_{1\alpha}^{mrs}(k_1, k_2)\right]_{t,\mathrm{I}} \\
&\quad = \mathrm{i} g^2 f^{amn} f^{nrs} \int_{k_1} \int_{k_3} D(k_1) D(k_3) \Delta_\alpha^\sigma(k_4) \left\{ \left[\mathcal{K}_\sigma^{mrs}(k_1, k_3, k_4)\right]_{t,\mathrm{I}} \right. \\
&\qquad \left. - \mathrm{i}\Gamma_\sigma^{grs}(-k_2, k_3, k_4) D(k_2) \left[\mathcal{G}_1^{mg}(k_1, k_2)\right]_{t,\mathrm{I}} \right\} = (\widetilde{d}) + (\widetilde{e}),
\end{aligned} \tag{3.20}$$

with $\mathcal{K}_\sigma^{mrs}$ representing the 1PI five-particle kernel shown in Figure 3.2, whereas Γ_σ^{grs} is the usual ghost-gluon vertex.

In exactly the same way, the remaining $\mathcal{S}_2$ will generate in $(\widetilde{b'})$ (when added to the R_ξ ghost diagram (b')) as well as $(\widetilde{d'})$ and $(\widetilde{e'})$ so that finally we get the relation

$$\widehat{\Gamma}_\alpha^a(q, p_2, -p_1) = \widetilde{\Gamma}_\alpha^a(q, p_2, -p_1). \tag{3.21}$$

The final step is to construct the all-order PT gluon self-energy $\widehat{\Pi}_{\alpha\beta}^{ab}(q)$. Notice that at this point, one would expect that it, too, coincides with the background Feynman gauge gluon self-energy $\widetilde{\Pi}_{\alpha\beta}^{ab}(q)$ because the boxes $\widehat{B}$ and the vertices $\widehat{\Gamma}_\alpha^a$ do coincide with the corresponding background Feynman gauge quantities, and the S-matrix is unique.

In what follows, we outline an indirect inductive proof of this result; the gluon self-energy will not be constructed explicitly here but rather in Chapter 6, in the more general context of the Schwinger-Dyson equations. We will use the *strong induction principle*, which states that a given predicate $P(n)$ on $\mathbb{N}$ is true $\forall\, n \in \mathbb{N}$, if $P(k)$ is true whenever $P(j)$ is true $\forall\, j \in \mathbb{N}$ with $j < k$.[2]

To avoid notational clutter, we suppress all color, Lorentz, and momentum labels. At one [4] and two loops (i.e., $n = 1, 2$) [5,6], we know from explicit calculations that the PT and background Feynman gauge Green's functions coincide. Let us then assume that the PT-BFM correspondence

$$\widehat{\Pi}^{(\ell)} = \widetilde{\Pi}^{(\ell)}, \qquad \widehat{\Gamma}^{(\ell)} = \widetilde{\Gamma}^{(\ell)}, \qquad \widehat{B}^{(\ell)} = \widetilde{B}^{(\ell)} \tag{3.22}$$

[2] In simple terms, whereas in the normal induction, one assumes the validity of a property at order $n - 1$ (only) and then demonstrates that it is true also at order n, the strong induction requires the property to be valid at all previous orders $1, 2, \ldots n - 1$.

holds for every $\ell = 1, \ldots, n-1$ (*strong induction hypothesis*). We will then show that the PT gluon self-energy is equal to the background Feynman gauge gluon self-energy at order n, i.e., $\widehat{\Pi}^{(n)} \equiv \widetilde{\Pi}^{(n)}$.

The S-matrix element of order n assumes the form

$$S^{(n)} = \{\Gamma\Delta\Gamma\}^{(n)} + B^{(n)}. \tag{3.23}$$

Moreover, because it is unique, whether written in the renormalizable Feynman gauge or the background Feynman gauge, as well as before and after the PT rearrangement, we have that $S^{(n)} \equiv \widehat{S}^{(n)} \equiv \widetilde{S}^{(n)}$. Using then Eq. (3.10) (which is valid to all orders, implying that Eq. (3.22) holds also when $\ell = n$), we find that

$$\{\Gamma\Delta\Gamma\}^{(n)} = \{\widehat{\Gamma}\widehat{\Delta}\widehat{\Gamma}\}^{(n)} = \{\widetilde{\Gamma}\widetilde{\Delta}\widetilde{\Gamma}\}^{(n)}. \tag{3.24}$$

The preceding amplitudes can then be split into 1PR and 1PI parts; in particular, because of the strong inductive hypothesis (3.22), the 1PR part after the PT rearrangement coincides with the 1PR part written in the background Feynman gauge because

$$\{\Gamma\Delta\Gamma\}_{\mathrm{R}}^{(n)} = \Gamma^{(n_1)}\Delta^{(n_2)}\Gamma^{(n_3)} \qquad \begin{cases} n_1, n_2, n_3 < n \\ n_1 + n_2 + n_3 = n. \end{cases} \tag{3.25}$$

Then Eq. (3.24) states the equivalence of the 1PI parts, i.e.,

$$\{\widehat{\Gamma}\widehat{\Delta}\widehat{\Gamma}\}_{\mathrm{I}}^{(n)} = \{\widetilde{\Gamma}\widetilde{\Delta}\widetilde{\Gamma}\}_{\mathrm{I}}^{(n)}, \tag{3.26}$$

which implies

$$\begin{aligned} 0 = {} & \left[\widehat{\Gamma}^{(n)} - \widetilde{\Gamma}^{(n)}\right]\Delta^{(0)}\Gamma^{(0)} + \Gamma^{(0)}\Delta^{(0)}\left[\widehat{\Gamma}^{(n)} - \widetilde{\Gamma}^{(n)}\right] \\ & + \Gamma^{(0)}\Delta^{(0)}\left[\widehat{\Pi}^{(n)} - \widetilde{\Pi}^{(n)}\right]\Delta^{(0)}\Gamma^{(0)}. \end{aligned} \tag{3.27}$$

At this point, we do not have the equality we want yet but have only that

$$\widehat{\Gamma}^{(n)} = \widetilde{\Gamma}^{(n)} + f^{(n)}\Gamma^{(0)} \tag{3.28}$$

$$\widehat{\Pi}^{(n)} = \widetilde{\Pi}^{(n)} - 2iq^2 f^{(n)}, \tag{3.29}$$

with $f^{(n)}$ being an arbitrary function of q^2. However, from the *explicit* construction of the PT quark-gluon vertex of the previous section, we have the all-order identity (3.21) so that the second of Eqs. (3.22) actually holds true even when $\ell = n$, i.e., $\widehat{\Gamma}^{(n)} \equiv \widetilde{\Gamma}^{(n)}$. Therefore $f = 0$, and one immediately concludes that

$$\widehat{\Pi}^{(n)} = \widetilde{\Pi}^{(n)}. \tag{3.30}$$

Hence, by strong induction, the preceding relation is true for any given order n.

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Reinstating the Lorentz and gauge group structures, we arrive at the announced result[3]:

$$\widehat{\Pi}^{ab}_{\alpha\beta}(q) \equiv \widetilde{\Pi}^{ab}_{\alpha\beta}(q). \tag{3.31}$$

Similar techniques have been used in [7] to generalize to all orders the PT algorithm in the electroweak sector of the standard model (we will briefly touch on this in Chapter 9).

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3 Owing to the validity to all orders of the PT background Feynman gauge correspondence, from now on, we will not make any distinction between PT and background Feynman gauge Green's functions and will indicate both with a caret.

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