SOLUTIONS TO THE PROBLEMS OF THE BOOK

"An Introduction to Atmospheric Thermodynamics"

by

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CHAPTER 3

(3.1)

$$m = \frac{pV}{RT} = \frac{1013 \times 10^2 \times 60}{287 \times 293} = 72.3\,\mathrm{kg}$$

(3.2)

$$V_0 = \frac{mRT_0}{p} \Longrightarrow V_0 = AT_0 \Longrightarrow A = 3.663\,\mathrm{cm^3\,K^{-1}} \quad \text{(first sample)}$$
$$A = 7.326\,\mathrm{cm^3\,K^{-1}} \quad \text{(second sample)}$$

Curves are:

$$\text{first sample } V = 3.663\,\mathrm{T}$$
$$\text{second sample } V = 7.326\,\mathrm{T}$$

(3.3)

$$\text{(a)} \quad V = \frac{nR^*T}{p} = 0.0000821\,\mathrm{T}\ (\mathrm{m^3\,K^{-1}})$$
$$= 82.1\,\mathrm{T}\,(\mathrm{cm^3\,K^{-1}})$$

$$\text{so } V = 82.1\,\mathrm{T}\,(V\,\text{in}\,\mathrm{cm^3})$$

$$\text{(b)} \quad V = 41.05\,\mathrm{T}$$

(3.4)

$$\text{per unit V:} \begin{cases} 0.95 \times M_{\mathrm{CO_2}} = 41.8 \\ 0.05 \times M_{N_2} = \underline{\ 1.4\ } \\ \qquad\qquad\quad 43.2 \end{cases}$$

So by mass:

$$\mathrm{CO_2}: \frac{41.8}{43.2} = 96.76\%$$
$$N_2: \frac{1.4}{43.2} = 3.24\%$$

Then it follows

$$\overline{M} = \frac{\sum m_i}{\sum \frac{m_i}{M_i}} = 43.2\,\mathrm{g\,mol^{-1}}$$
$$R = \frac{R^*}{\overline{M}} = 192.5\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

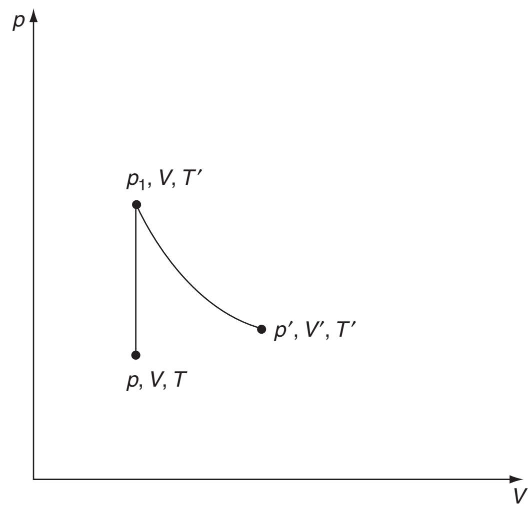
(3.5)

$$x = \frac{6.022 \times 10^{23}}{22\,400} = 2.6884 \times 10^{19}$$

(3.6)

$$\begin{aligned} &(1) \quad p, V, T \longrightarrow p_1, V, T' \, (T' > T \Longrightarrow p_1 > p) \quad &\text{[isochoric]} \\ &(2) \quad p_1, V, T' \longrightarrow p', V', T' \, (V' > V \Longrightarrow p' < p) \quad &\text{[isothermal]} \end{aligned}$$

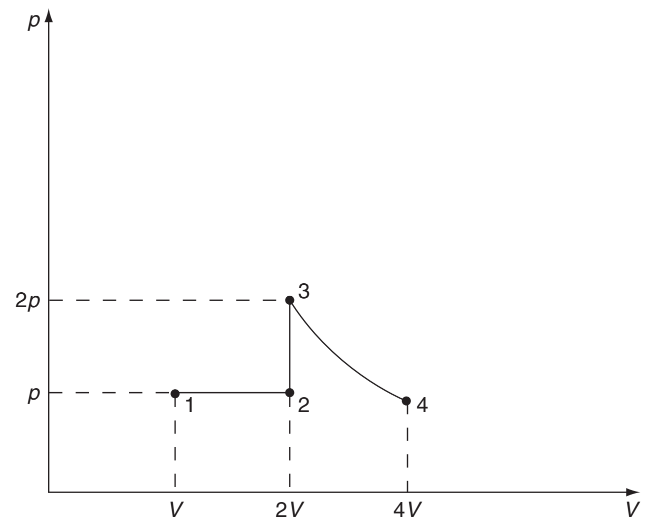
$$(pV = \text{constant is a hyperbola})$$



(3.7)

$$\begin{aligned} &(1) \quad p, V, T \\ &(2) \quad p, 2V, 2T \\ &(3) \quad 2p, 2V, 4T \\ &(4) \quad p, 4V, 4T \end{aligned}$$

$$pV = \text{constant is a hyperbola}$$


p
2p
p
3
1
2
4
V
2V
4V
V

CHAPTER 4

(4.1)

(a) $\theta = T\left(\frac{1000}{p}\right)^{\frac{R}{c_p}}$

$\theta = 213(5)^{0.286} = 337.5\,\mathrm{K}$

(b) $q = c_p\,dT = 1005(298 - 337.5) = 39\,697.5\,\mathrm{J\,kg^{-1}} = -9.5\,\mathrm{cal\,g^{-1}}$

Remove $9.5\,\mathrm{cal\,g^{-1}}$

(4.2)

$$V_f = 1.2V_i = 1.2\frac{mRT_i}{p} = 0.10332\,\mathrm{m^3}$$

$$T_f = \frac{pV_f}{mR} = 324\,\mathrm{K}$$

$$Q = C_p\Delta T = 5427\,\mathrm{J}$$

$$W = p\Delta V = 1550\,\mathrm{J}$$

(4.3)

(a) $T_i p_i^{\frac{1-\gamma}{\gamma}} = T_f p_f^{\frac{1-\gamma}{\gamma}} \Longrightarrow T_f = T_i\left(\frac{p_i}{p_f}\right)^{\frac{1-\gamma}{\gamma}}$

$$\delta w = -du = -c_V dT \Longrightarrow w = -c_V T_i\left[\left(\frac{p_i}{p_f}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]$$

(b) $\delta w = p da \Longrightarrow w = RT_i \ln\frac{a_f}{a_i} = RT_i \ln\frac{p_i}{p_f}$

(c) in (a)

$$\left.\begin{aligned} d\theta &= 0 \Longrightarrow \theta_1 = \theta_2 \\ \theta_1 &= T_i\left(\frac{1000}{p_i}\right)^{0.286} \\ \theta_2 &= T_f\left(\frac{1000}{p_f}\right)^{0.286} \end{aligned}\right\} \Longrightarrow \frac{\theta_1}{\theta_2} = \left(\frac{p_f}{p_i}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{p_f}{p_i}\right)^{0.286} = 1$$

in (b)

$$\theta_2 - \theta_1 = T_i\, 1000^{0.286}\left[p_f^{-0.286} - p_i^{-0.286}\right]$$

(4.4)

(a) $p_1 V_1^\eta = p_2 V_2^\eta \Longrightarrow \eta = \dfrac{\ln \frac{p_1}{p_2}}{\ln \frac{V_2}{V_1}} = 0.67 \quad \eta = \dfrac{0.929207}{1.38029}$

(b) $p_f V_f = mRT_f \Longrightarrow T_f = \dfrac{p_f V_f}{mR} = \dfrac{400 \times 10^2 \times 4 \times 0.0224}{0.02897 \times 287}$

$$= 431\,\mathrm{K}$$

(c)

(1) $\Delta U = C_V \Delta T = 0.02897 \times 718 \times 158 = 3286\,\mathrm{J}$

(2) $pV^{0.67} = \text{constant} \Longrightarrow \text{constant} = 7948$

$$W = \int_{V_i}^{V_f} \frac{7948}{V^{0.67}}\, dV = \frac{7948}{0.33}\left[V^{0.33}\right]_{V_i}^{V_f}$$

$$= 24\,085\,[0.0896^{0.33} - 0.0224^{0.33}]$$

$$= 24\,085[0.45109 - 0.28548] = 3989\,\mathrm{J}$$

(3) $Q = 3989 + 3286 = 7275\,\mathrm{J}$

(4.5)

$$1\,\mathrm{ft} = 0.305\,\mathrm{m}$$

$$\left.\begin{aligned} 15000\,\mathrm{ft} &= 4575\,\mathrm{m} \\ 3500\,\mathrm{ft} &= 1067.5\,\mathrm{m} \end{aligned}\right\} \quad 3507.5\,\mathrm{m} = 3.5075\,\mathrm{km}$$

$$3.5075 \times 9.8 = 34.3735 \approx 34.4$$

$$\Longrightarrow T_{3500} = 34.4 - 12 = 22.4\,^\circ\mathrm{C}$$

(4.6)

$$p_1 = 1\,\mathrm{atm}, \quad V_1 = 10\,l, \quad T_1 = 27\,^\circ\mathrm{C}$$

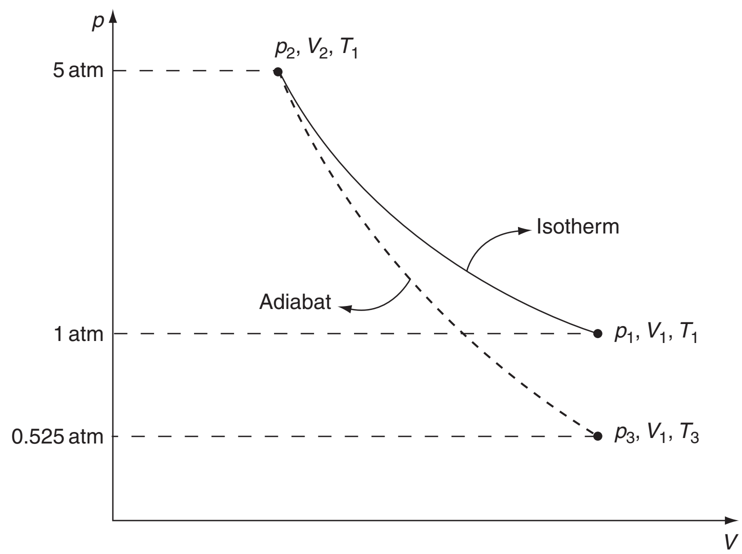
$$p_2 = 5\,\mathrm{atm}, \quad V_2 = 2\,l, \quad T_2 = 27\,^\circ\mathrm{C}$$

$$p_2 V_2^\gamma = p_3 V_3^\gamma \Longrightarrow p_3 = p_2\left(\frac{V_2}{V_3}\right)^\gamma = 5(5)^{-\gamma} = 5^{1-\gamma} = 0.525\,\mathrm{atm}$$

$$T_3 V_3^{\gamma-1} = T_2 V_2^{\gamma-1} \Longrightarrow T_3 = T_2\left(\frac{V_2}{V_3}\right)^{0.4} = 300(0.2)^{0.4} = 158\,\mathrm{K}.$$

Thus

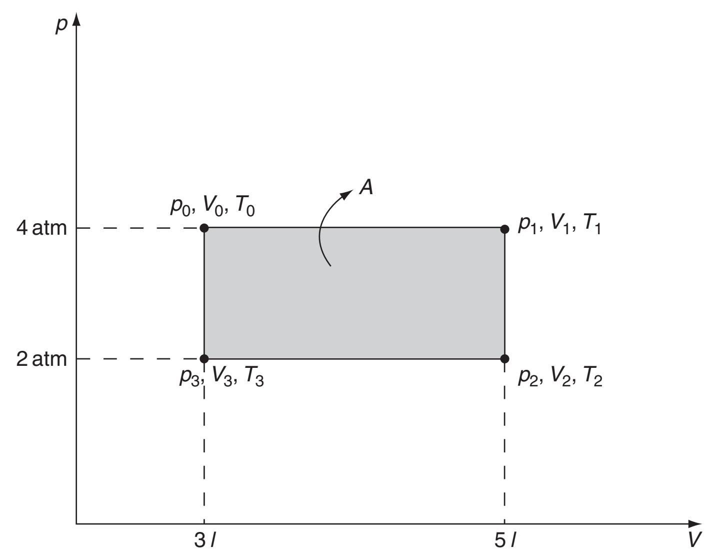
$$p_3 = 0.525,\ V_1 = V_3 = 10\,l,\ T_3 = -115\,^\circ\mathrm{C}$$



(4.7)

$$\begin{aligned} p_0 &= 4\,\text{atm}, & V_0 &= 3\,l & T_0 &= 300\,\text{K} \\ p_1 &= p_0 = 4\,\text{atm}, & V_1 &= 5\,l & T_1 &= 500\,\text{K} \\ p_2 &= 2\,\text{atm}, & V_2 &= 5\,l & T_2 &= 250\,\text{K} \\ p_3 &= 2\,\text{atm}, & V_3 &= 3\,l & T_3 &= 150\,\text{K} \\ p_0 &= 4\,\text{atm}, & V_0 &= 3\,l & T_0 &= 300\,\text{K} \end{aligned}$$

$$W = A = 2 \times 1013 \times 10^2 \times 2 \times 10^{-3} = 405.2\,\text{J}$$



(4.8)

$$U \approx C_V T = \text{constant} \Longrightarrow \frac{pVC_V}{mR} = \frac{pVc_V}{R} \Longrightarrow \boxed{pV = R/c_V}$$

(4.9) The change in U is the same ($dU = 0$).

Since $dT = 0$, the heat absorbed is equal to work done.

Thus the heat absorbed is greater in the case of the step-like curve (which gives a greater area in the (p, V) diagram).

(4.10)

$$\begin{aligned} Q &= C_p \Delta T = 1005 \times 50 = 50\,250\,\text{J} \\ \Delta U &= C_V \Delta T = 718 \times 50 = 35\,900\,\text{J} \\ W &= Q - \Delta U = 14\,350\,\text{J} \end{aligned}$$

(4.11)

$$p_1 V_1 = nR^* T_1 \Longrightarrow n = 0.208\,\text{moles}$$

$$\Longrightarrow m_{\text{hydrogen}} = 0.208 \times 0.002 = 0.416 \times 10^{-3}\,\text{kg}$$

$$\begin{aligned} T_2 &= \frac{p_2 V_2}{nR^*} \approx 586\,\text{K} \\ \Delta T &= 293 \\ C_p &= \frac{7}{2} nR^* = 6.0526\,\text{J}\,\text{K}^{-1} \\ Q &= C_p \Delta T = 1773.4\,\text{J} \end{aligned}$$

(4.12)

$$\begin{aligned} R &= \frac{R^*}{M} \\ R &= \frac{2c_V}{5} = \frac{R^*}{M} \Longrightarrow \frac{2C_V}{5m} = \frac{R^*}{M} \Longrightarrow \frac{2\Delta U}{5m\Delta T} = \frac{R^*}{M} \\ &\Longrightarrow M = \frac{5m\Delta T R^*}{2\Delta U} \end{aligned}$$

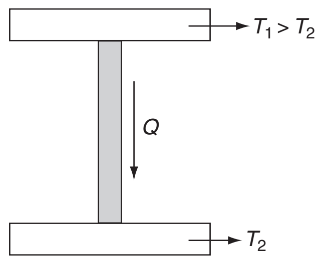
$$T_1 V_1^{0.4} = T_2 V_2^{0.4} \Longrightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{0.4} = 685.75\,\text{K}$$

$$\Longrightarrow \Delta T = 412.75\,\text{K}$$

$$\Longrightarrow M = \frac{5 \times 0.088 \times 412.75 \times 8.314}{2 \times 17\,158} = 0.044\,\text{kg}\,\text{mol}^{-1}$$

(i.e. CO_2)

(4.13) Heat transfer due to conduction



in this case $W = 0$

Going the other way we must do work (which means that the environment suffers a change, which means irreversible process).

(4.14) Initial volume:

$$V_1 = \frac{mRT_1}{P_1} = 0.0007835\,\mathrm{m}^3$$

Final temperature:

$$T_2 = \frac{V_1 P_2}{mR} = 286.6\,\mathrm{K}$$

Change in temperature:

$$13.6\,^\circ\mathrm{C}$$

Amount of heat absorbed:

$$Q = C_V \Delta T = m c_V \Delta T = 2.33\,\mathrm{cal}$$

(4.15)

$$Q = 0 \Longrightarrow$$

$$\Delta u = -w = \frac{-\Delta K}{m} = -\left[\frac{1}{2}V_f^2 - \frac{1}{2}V_1^2\right] = -262.5\,\mathrm{J\,Kg^{-1}}$$

CHAPTER 5

(5.1)

$$\delta Q = C_p dT - V dp$$

$$\frac{\delta Q}{T} = C_p \frac{dT}{T} - nR^* \frac{dp}{p}$$

$$\oint \frac{\delta Q}{T} = C_p \oint d\ \ln\ T - nR^* \oint d \ln p$$

$$\oint \frac{\delta Q}{T} = 0$$

(5.2)

$$dz = M dx + N dy$$

for an exact differential

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

for reversible processes

$$dU = TdS - pdV \quad (1)$$

$$dH = TdS - V dp \quad (2)$$

$$dF = -SdT - pdV \quad (3)$$

$$dG = -SdT + V dp \quad (4)$$

U, H, F, G all are exact differentials. Thus:

$$(1): \quad \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

$$(2): \quad \left(\frac{\partial T}{\partial p}\right)_S = -\left(\frac{\partial V}{\partial S}\right)_p$$

$$(3): \quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$(4): \quad \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p .$$

(5.3) From equation (5.8), we have

$$\Delta s \geq c_V \ln \left(\frac{a_f}{a_i}\right)^{\gamma-1} \qquad (T = \text{constant})$$

$$\Delta s \geq 718 \ln (2^{0.4}) \approx 200\,\mathrm{J\,K^{-1}\,kg^{-1}}$$

(5.4) H = constant $\Longrightarrow (C_V + nR^*)T$ = constant $\Longrightarrow T$ = constant $\Longrightarrow pV$ = constant $\Longrightarrow$ hyperbolas.

(5.5)

$$dS = C_p \frac{dT}{T} - \frac{V\,dp}{T}$$

Since

$$dS = 0$$

it follows that

$$C_p \frac{dT}{T} = mR \frac{dp}{p}$$

or

$$c_p \frac{dT}{T} = R \frac{dp}{p}$$

or

$$\ln \frac{T_2}{T_1} = \frac{R}{c_p} \ln \frac{p_2}{p_1} \Longrightarrow T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{R}{c_p}} = 273(10)^{0.286}$$

$$= 527.5\,\mathrm{K}$$

(5.6)

$$Q = W_{12} + W_{34} = 31\,165 - 24\,282 = 6883\,\mathrm{J}$$

$$\eta = 1 + \frac{Q_2}{Q_1} = 1 - \frac{24\,282}{31\,165} = 0.22$$

(5.7)

$$\Delta S \geq \frac{Q}{T} = \frac{W}{T} = \frac{nR^* T \ln \frac{V_2}{V_1}}{T} = 8314 \ln 4 = 11.5\,\mathrm{J\,K^{-1}}$$

(expansion in a vacuum is isothermal)

$$dG \leq -S dT + V dp = nR^* T \frac{dp}{p} \Longrightarrow \Delta G$$

$$\leq nR^* T \ln \frac{p_2}{p_1} = nR^* T \ln \frac{V_1}{V_2} = -3146\,\mathrm{J}$$

(5.8)

$$S_i = 2k \ln \frac{N!}{\left(\frac{N}{2}!\right)^2}$$

(a)

$$S_f = k \ln \frac{2N!}{(N!)^2}$$

$$\begin{aligned}\Delta S &= K(2N \ln 2N - 2N - 2N \ln N + 2N) \\ &\quad - 2K(N \ln N - N - N \ln N/2 + N) \\ &= 2KN \ln 2 + 2KN \ln N - 2KN \ln N \\ &\quad - 2KN \ln N + 2KN \ln N - 2KN \ln 2 \\ &= 0\end{aligned}$$

(b) as in solved example (5.1)

(5.9)

$$ds = c_p \frac{dT}{T} - \frac{a}{T} dp = -R \frac{dp}{p} \Longrightarrow \Delta s = -287 \ln \frac{8}{10} = 64\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

(5.10)

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$-30 = 1005 \ln \frac{T_2}{273} - 287 \ln \frac{950}{900} \Longrightarrow T_2 = 269\,\mathrm{K}$$

$$\theta_2 = 269 \left(\frac{1000}{950}\right)^{0.286} = 273\,\mathrm{K}$$

(5.11)

$$ds = c_p \frac{dT}{T} - \frac{adp}{T}$$

$$c_p d \ln \theta = c_p d \ln T \Longrightarrow$$

$$\frac{d\theta}{\theta} = \frac{dT}{T} \Longrightarrow d\theta = dT \left(\frac{\theta}{T}\right)$$

or since $\left(\frac{\theta}{T}\right) = \text{constant}$

$$\Delta\theta = \Delta T \left(\frac{\theta}{T}\right) \quad \text{or}$$

$$\frac{\Delta\theta}{\theta} = \frac{\Delta T}{T}$$

(5.12) In this case

$$c_p d\ln\theta = c_p d\ln T - Rd\ln p$$

or

$$\frac{d\theta}{\theta} = \frac{dT}{T} - \frac{R}{c_p}\frac{dp}{p}$$

The air's T will increase due to conduction. So $\frac{dT}{T} > 0$. If $\frac{d\theta}{\theta} > 0$ but $\frac{dT}{T} < \frac{d\theta}{\theta}$, then for the air moving over warmer surface $\frac{dp}{p} < 0$ which means the surface pressure will decrease. If $\frac{d\theta}{\theta} > 0$ but $\frac{dT}{T} > \frac{d\theta}{\theta}$ the air moving over warmer surface $\frac{dp}{p} > 0$ which will result in an increase of the surface pressure and so on!

(5.13)

$$\Delta S_T = C_{V_1}\ln\frac{T}{T_1} + C_{V_2}\ln\frac{T}{T_2}$$

For

$$T_1 = T_2 \Longrightarrow T = T_1 = T_2 \Longrightarrow \Delta S_T = 0$$

For

$$T_1 \neq T_2 \Longrightarrow \Delta S_T = C_{V_1}\left[\ln\left(\frac{b+x}{1+b}\right) + \frac{1}{b}\ln\left(\frac{1+b/x}{1+b}\right)\right]$$

where

$$b = \frac{C_{V_1}}{C_{V_2}}, x = \frac{T_2}{T_1} \text{ and } T = \frac{C_{V_1}}{C_{V_1}+C_{V_2}}T_1 + \frac{C_{V_2}}{C_{V_1}+C_{V_2}}T_2.$$

Define

$$f(x) = \frac{\Delta S_T}{C_{V_1}}.$$

Then:

$$\frac{df}{dx} = \frac{1}{b+x} - \frac{1}{x(b+x)}.$$

Now we have that

$$\frac{df}{dx} = 0 \quad \text{at} \quad x = 1.$$

For

$$x = 1, \quad f(1) = 0 \quad (f(x)\,\text{has an extremum at}\,x = 1).$$

Now

$$\frac{d^2 f}{dx^2} = \frac{1}{(x+b)^2}\left[\frac{2x-b}{x^2} - 1\right].$$

The second derivative is positive for $x = 1$. Thus the extremum is minimum. It follows that $f(x)$ is zero when $x = 1$ (i.e. $T_1 = T_2$) and positive otherwise.

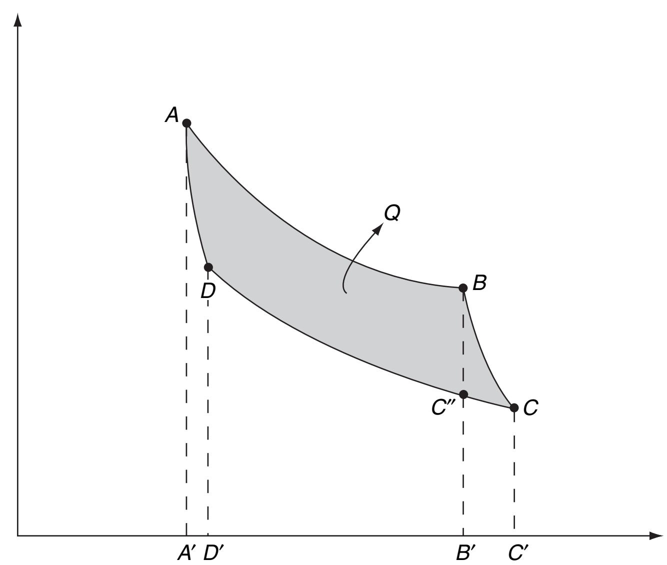
(5.16)

$$ds = c_V \frac{dT}{T} + R\frac{da}{a} = 0 \Longrightarrow c_V \ln\frac{T_2}{T_1} = -R\ln\frac{a_2}{a_1} \Longrightarrow$$

$$\ln\frac{T_2}{T_1} = \ln\left[\left(\frac{a_1}{a_2}\right)^{R/c_V}\right] \Longrightarrow T_2 = T_1(0.6)^{0.4} \approx 244.5\,\mathrm{K}$$

$$p_2 = \frac{RT_2}{a_2} \approx 1404\,\mathrm{mb}$$

(5.17)



$$\begin{aligned} ABCD &= AA'B'B - A'D'DA - D'DC''B' + BB'C'C - CC'B'C'' \\ &= AA'B'B - A'D'DA - D'D'CC' + BB'C'C \\ Q &= W_{12} - W_{41} - W_{34} + W_{23} \end{aligned}$$

since

$$W_{23} = -W_{41} = -C_V(T_2 - T_1)$$

it follows

$$Q = W_{12} + W_{34}\,(W_3 < 0).$$

CHAPTER 6

(6.1) & (6.2)

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\delta Q = dU + pdV = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$$

$$dS = \frac{\delta Q}{T} = \underbrace{\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V}_{M} dT + \underbrace{\frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]}_{N} dV$$

Since dS is an exact differential we can write

$$\left(\frac{\partial M}{\partial V}\right)_T = \left(\frac{\partial N}{\partial T}\right)_V$$

or

$$\frac{\partial}{\partial V}\left[\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V\right]_T = \frac{\partial}{\partial T}\left[\frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\right]_V \Longrightarrow$$

$$\frac{\partial}{\partial V}\left[\frac{1}{T}\right]_T \left(\frac{\partial U}{\partial T}\right)_V + \frac{1}{T}\left[\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right]_T$$

$$= \frac{\partial}{\partial T}\left(\frac{1}{T}\right)_V \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] + \frac{1}{T}\frac{\partial}{\partial T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]_V$$

$$= \frac{\partial}{\partial T}\left(\frac{1}{T}\right)_V \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] + \frac{1}{T}\left[\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right]_V + \frac{1}{T}\left(\frac{\partial p}{\partial T}\right)_V$$

Now,

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V = 0,$$

$$\frac{\partial}{\partial V}\left[\frac{1}{T}\right]_T = 0,$$

and

$$\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T = 0.$$

It follows that

$$-\frac{1}{T^2}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] + \frac{1}{T}\left(\frac{\partial p}{\partial T}\right)_V = 0$$

or

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

for ideal gases

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial}{\partial T}\left(\frac{RT}{V}\right)\right)_V - p = \frac{TR}{V} - p = 0 \Longrightarrow$$

$$U \neq f(V) \Longrightarrow U = f(T).$$

(6.4) From ice at $-20\,^{\circ}\mathrm{C}$ to ice at $0\,^{\circ}\mathrm{C}$ the amount of heat required is

$$Q_1 = C_{V_i}\Delta T = 5030\,\mathrm{cal}.$$

Thus, heat at a rate of 100 cal/min must be supplied for 50.3 min.
From ice at $0\,^{\circ}\mathrm{C}$ to water at $0\,^{\circ}\mathrm{C}$ the amount of heat required is

$$Q_2 = ml_f = 39\,850\,\mathrm{cal}.$$

In this case, heat at a rate of 100 cal/min must be supplied for 398.5 min. Since heat is supplied for a total of 700 min. It follows that an additional amount of heat of 70 000 − 44 880 = 25 120 cal will be supplied to water to raise its temperature from $0\,^{\circ}\mathrm{C}$ to T_{final} according to

$$25\,120 = C_{Vw}\Delta T$$

which gives

$$T_{\mathrm{final}} = 50\,^{\circ}\mathrm{C}.$$

Since $dQ/dt = 100\,\mathrm{cal/min}$ we have that

$$Q = 100t$$

and the three graphs should look like those that follow.

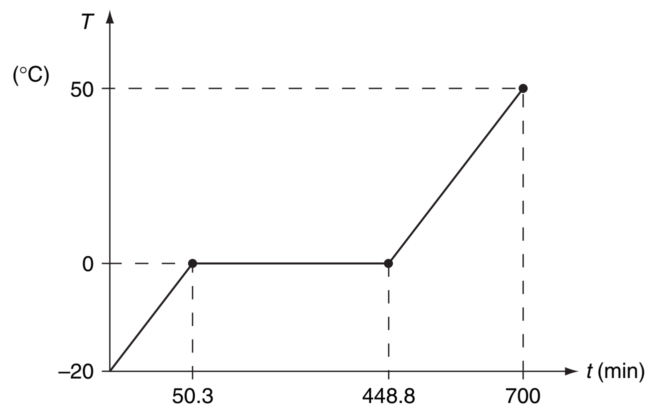
(6.5)

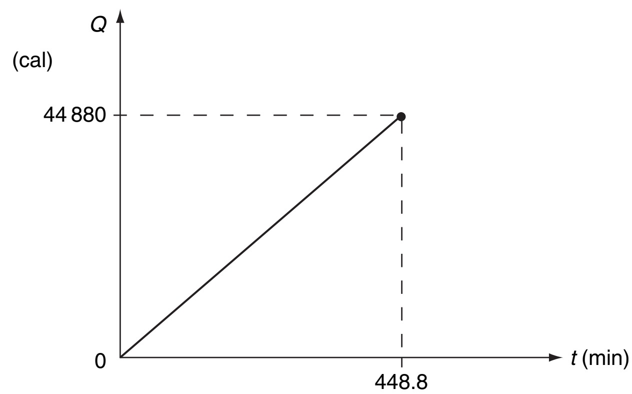
$$\ln\frac{e_{sw}}{6.11} = 19.83 - \frac{5417}{T}$$

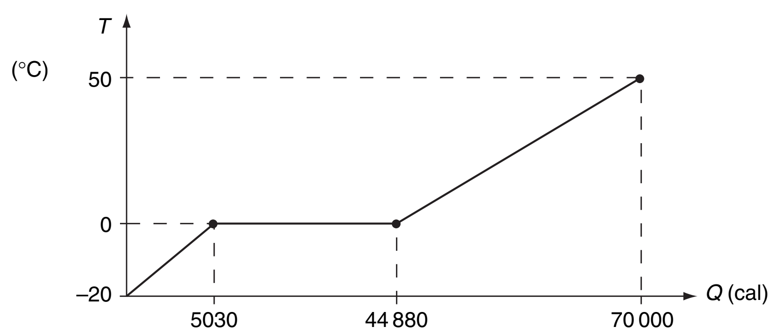
$$\Longrightarrow e_{sw}(393) = 2591\,\mathrm{mb}$$

$$e_{sw}(373) = 1237\,\mathrm{mb}$$

$$m_v(\mathrm{initial}) = \frac{e_{sw}(393)\cdot V}{R_v T} = 1.43\,\mathrm{kg}.$$







Similarly,

$$m_v \,(\text{final}) = 0.72\,\text{kg}.$$

So,

$$1.43 - 0.72 = 0.71$$

must escape.

(6.6) The least amount of water needed is the amount of water vapor corresponding to $e_{sw}(100\,^\circ\mathrm{C})$. At $100\,^\circ\mathrm{C}$:

$$\begin{aligned} e_{sw} &= 1000\,\mathrm{mb}\ \text{(use equation 6.16)} \\ \Longrightarrow m_v(100\,^\circ\mathrm{C}) &= \frac{1000 \times 10^2 \times 2 \times 10^{-3}}{461.5 \times 373} \\ &= 0.00116\,\mathrm{kg} \\ &= 1.16\,\mathrm{g} \end{aligned}$$

At standard conditions

$$\begin{aligned} \rho_d &= 1.293\,\mathrm{kg\ m^{-3}} \\ \Longrightarrow m_d &= \rho_d V = 0.002586\,\mathrm{kg}. \end{aligned}$$

It follows that

$$p_2 = \frac{m_d R_d T_2}{V} = 1384\,\mathrm{mb}.$$

Thus

$$p_{\text{total}} = 1384 + 1000 = 2384\,\mathrm{mb} = 2.35\,\mathrm{atm}$$

(6.7)

$$\begin{aligned} \left(\frac{\partial U}{\partial V}\right)_T &= T\left(\frac{\partial p}{\partial T}\right)_V - p \\ &= T\left(\frac{\partial\left(\frac{10^9 T^{1.1}}{V^{1.2}}\right)}{\partial T}\right)_V - p \\ &= 1.1T\frac{10^9 T^{0.1}}{V^{1.2}} - p \\ &= 1.1p - p = 0.1p \neq 0\ \text{(i.e. it is not an ideal gas)} \end{aligned}$$

Thus,

$$\begin{aligned} dU &\neq C_V dT \\ \Delta U &= \Delta Q - \int p dV \\ \Delta S &\geq \frac{\Delta Q}{T} \end{aligned}$$

(6.8) Increase the temperature to about $1000\,^\circ\mathrm{C}$.

(6.9)

$$\Delta S_1 \geq \int \frac{\delta Q_1}{T} = \int C_V \frac{dT}{T} =$$
$$= 5c_w \ln \frac{T_2}{T_1} = 5771\,\mathrm{J\,K^{-1}}$$
$$\Delta S_2 \geq \int \frac{\delta Q_2}{T} = \frac{1}{T} m l_v = 30\,201\,\mathrm{J\,K^{-1}}$$

Thus,

$$\Delta S \geq 30\,201 + 5771 = 35\,972\,\mathrm{J\,K^{-1}}$$

(6.11) Heat absorbed from warm source

$$(1)\ Q_1 = l_f \times 1000 = 333.7 \times 10^6\,\mathrm{J}$$

$$(2)\ -\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Longrightarrow Q_2 = -244.2 \times 10^6\,\mathrm{J}$$

CHAPTER 7

(7.1) Neglecting the water vapor terms and using

$$T = \frac{m_1 T_2 + m_2 T_2}{2}$$

and

$$T = \theta \left(\frac{1000}{p} \right)^{-k}$$

we arrive at

$$\theta \left(\frac{1000}{p} \right)^{-k} = \frac{m_1 \theta_1 (\frac{1000}{p})^{-k} + m_2 \theta_2 (\frac{1000}{p})^{-k}}{2}$$

or

$$\theta = \frac{m_1 \theta_1 + m_2 \theta_2}{2}.$$

Similarly using equation (7.74) and $q \approx w \approx \frac{\epsilon e}{p}$ we arrive at

$$e \approx \frac{m_1 e_1 + m_2 e_2}{m}$$

(7.2) We know that $w = w_s(T_{dew}, p) = w_s(T_{LCL}, p_{LCL})$. Then

$$\frac{\epsilon e_{sw}(T_{dew})}{p - e_{sw}(T_{dew})} = \frac{\epsilon e_{sw}(T_{LCL})}{p_{LCL} - e_{sw}(T_{LCL})} \Longrightarrow$$

$$\epsilon e_{sw}(T_{dew}) p_{LCL} = \epsilon e_{sw}(T_{LCL}) p \Longrightarrow$$

$$\frac{p_{LCL}}{e_{sw}(T_{LCL})} = \frac{p}{e_{sw}(T_{dew})} \qquad (1).$$

From equation (6.17)

$$\left. \begin{array}{l} e_{sw}(T_{dew}) = 6.11 \exp(19.83 - \left(\frac{5417}{T_{dew}}\right) \\ \text{and} \\ e_{sw}(T_{LCL}) = 6.11 \exp\left(19.83 - \frac{5417}{T_{LCL}}\right) \end{array} \right\} (2).$$

Combining (2) and (1) yields

$$\frac{p}{p_{LCL}} = \frac{\exp\left(19.83 - \frac{5417}{T_{dew}}\right)}{\exp\left(19.83 - \frac{5417}{T_{LCL}}\right)}$$

or

$$\frac{p}{p_{LCL}} = \exp\left(\frac{5417}{T_{LCL}} - \frac{5417}{T_{dew}}\right)$$

(7.3)

$$e = \frac{wp}{w + \epsilon}$$

$$\Longrightarrow e_i = 7.74\,\text{mb and}\, e_f = 5.98\,\text{mb}$$

From equation (6.17)

$$e_{sw}(T) = 6.11 \exp\left(19.83 - \frac{5417}{T}\right)$$

which gives

$$e_{swi} = 23.4, \quad e_{swf} = 12.16.$$

It follows that

$$r_i = 0.33, \quad r_f = 0.49.$$

Then using equation (7.16) we have that

$$T_{dew\,i} = 276.5\,\text{K}, \quad T_{dewf} = 272.8\,\text{K}$$

(7.4)

$$\delta Q = l_v dm = l_v dm_v$$
$$\delta Q = l_v m_d d\left(\frac{m_v}{m_d}\right) \Longrightarrow$$
$$\delta Q = l_v m_d dw$$

(7.5) Working as in example 7.4 we find that it is the same as the air at 1000 mb.

(7.6) Recall equation (7.80). Following the discussion in section 7.3.2, we have that

$$T_1 = T_0 = \theta_1 \left(\frac{p_1}{1000}\right)^k$$
$$T_2 = T_0 = \theta_2 \left(\frac{p_2}{1000}\right)^k$$

in general

$$\theta = T_0 \left(\frac{1000}{p}\right)^k.$$

Now

$$\overline{\theta} = \frac{-\int_{p_1}^{p_2} \theta dp}{p_1 - p_2} = -\frac{\int T_0 \frac{1000^k}{p^k} dp}{p_1 - p_2}$$
$$= -1000^k T_0 (p_2^{1-k} - p_1^{1-k})/(1-k)(p_1 - p_2).$$

Thus from equation (7.80)

$$T = p^k \frac{T_0}{1-k}(p_1^{1-k} - p_2^{1-k})/(p_1 - p_2)$$

(7.7) The lowest possible temperature will be T_w. Then from equation (7.22) we have

$$T_w + \frac{l_v}{c_{pd}} w_{sw} = T + \frac{l_v}{c_{pd}} w.$$

Since the air is initially very dry we can assume that $w = 0$. As such

$$T_w = T - \frac{l_v}{c_{pd}} w_{sw}$$

or

$$T_w \approx T - \frac{l_v}{c_{pd}} \frac{\epsilon}{p} 6.11 \exp\left(19.83 - \frac{5417}{T_w}\right).$$

The above equation can be solved numerically to obtain $T_w \approx 288\,\mathrm{K}$.

(7.8) At high temperatures the curve is fairly linear. Thus, for the mixture to become slightly supersaturated, both breath and the outside air must be very close to being saturated or be supersaturated. In any case the answer is yes but very high relative humidities are needed.

(7.9) From equation (7.16) we estimate that $r = 0.66$. From equation (6.17) we find that $e_{sw}(308) = 57.6\,\mathrm{mb}$. Then it follows that $e = 38.0\,\mathrm{mb}$ and that (at $p = 1000\,\mathrm{mb}$) $w = 24.6\,\mathrm{g\,kg^{-1}}$. Then from equation (7.22) we find (by numerical method) that $T_w \approx 303\,\mathrm{K}$.

This makes sense because $T_w > T_{dew}$. The difference between this answer and the answer in problem 7.7 is largely due to the fact that here $w \neq 0$.

(7.10) That will be at T_{dew}. From equation (7.16) we estimate that

$$T_{dew} = \frac{T}{-\frac{TR_v \ln r}{l_v} + 1} \approx 287.4\,\mathrm{K}.$$

(7.11) Since the decompression is adiabatic the final temperature inside the cabin is

$$T' = 298 \left(\frac{500}{900}\right)^{0.286} = 252\,\mathrm{K}$$

Now $e_{sw}(T') = 1.15\,\mathrm{mb}$ (from equation 6.17). Then $w_{sw}(T') = 0.001434$. At $T = 298\,\mathrm{K}$, $e_{sw}(T) = 31.9$ and $w_{sw}(T) = 0.0228$. Since the cabin is a closed system $w(T) = w_{sw}(T')$. Thus, $r = 6.29\%$.

(7.12) The dew point temperature of outside air with $T = 258\,\mathrm{K}$ is (from 7.16)

$$T_{dew} = \frac{T}{-\frac{TR_v \ln r}{l_v} + 1} = 252\,\mathrm{K}.$$

This is also the dew point for the air inside the room where $T = 298\,\mathrm{K}$. Thus, inside the room

$$\ln r = \frac{T - T_{dew}}{-\frac{R_v T T_{dew}}{l_v}} = -3.32 \quad \text{or} \quad r = 3.6\%.$$

At $T = 298\,\mathrm{K}$, $e_{sw} = 31.9\,\mathrm{mb}$ (from equation 6.17)
When the room reaches 50% relative humidity then the water vapor has a vapor pressure equal to

$$e = \frac{m_v R_v T}{V} \Longrightarrow re_{sw} = \frac{m_v R_v T}{V}$$

or

$$m_v = \frac{re_{sw} V}{R_v T} \qquad (1)$$

for $r = 0.5, e_{sw} = 31.9 \times 10^2\,\mathrm{Pa}$, $V = 100\,\mathrm{m}^3$, $R_v = 461.5\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and $T = 298\,\mathrm{K}$ it follows that

$$m_v = 1.16\,\mathrm{kg}.$$

Now this is *not* the amount it should be added because to start with there is some vapor. The amount of this vapor is found from equation (1) for $r = 0.036$ and it is equal to $0.0835\,\mathrm{kg}$. Thus, $1.16 - 0.0835 \simeq 1.08\,\mathrm{kg}$ must be evaporated for the relative humidity to become 50%.
The mass of dry air is

$$m_d = \rho_d V = 1.293 \times 100 = 129.3\,\mathrm{kg}.$$

Thus

$$w_t = \frac{1.16}{129.3} = 8.97\,\mathrm{g\,kg^{-1}}.$$

Finally

$$\Delta Q = l_v \Delta m = 2.5 \times 10^6 \times 1.08 = 2.7 \times 10^6\,\mathrm{J}.$$

(7.13) a. $w, q, p_{LCL}, T_{LCL}, \theta, \theta_v, s\,(w_t, q_t$ do not apply)

b. $w_t, q_t, r, p_{LCL}, T, \theta_e, \theta_w, s$

c. $w, q, e, \theta, \theta_v, h$

d. $r, h\,(\text{note}\,\theta_e \neq \theta_{ep}, \theta_w \neq \theta_{wp})$

(7.14) At 303 K, $e_{sw} = 43.08$ mb. Since initially $r = 0.5 \implies e(303\,\mathrm{K}) = 21.54$ mb. It follows that the amount of water vapor in the air inside the refrigerator is

$$m_v = \frac{eV}{R_v T} = 0.0308\,\mathrm{kg}.$$

In 2 m^3 there is $m_d = \rho_d V = 2.586$ kg of dry air. So there is $m = 2.6168$ kg of *moist* air inside the refrigerator.

To find the temperature at which condensation begins, T', we use equation (6.17) for $e_{sw}(T') = e$ i.e.

$$21.54 = 6.11\,\exp\left(19.83 - \frac{5417}{T'}\right).$$

Numerical solution of this equation gives $T' \cong 19\,^\circ\mathrm{C}$ (292 K).

Since the system is a closed system it follows that

$$m_t = m_v = \text{ constant } = 0.0308.$$

Then, $w = w_t = m_t/m_d = 0.012\,(m_t$ is the total mass of water). At 275 K, $e_{sw} = 6.97$ mb. This vapor pressure for $V = 2\,\mathrm{m^3}$ corresponds to a mass of water vapor

$$m_v' = 0.011\,\mathrm{kg}.$$

Thus $m_v - m_v' = 0.0198$ kg of vapor must condense to achieve a temperature of 275 K. In order to estimate the amount of heat given to the surroundings we proceed as follows. From $T = 303$ K to $T' = 292$ K an amount of heat $\Delta Q_1 = C_V \Delta T$ is given away. Here $C_V = 2.6168 \times c_{Vd} \times (1 + 0.97w)$. It follows that $\Delta Q_1 \approx 20\,908$ J. From $T' = 292$ K to $T = 275$ K the amount of heat given away is

$$\begin{aligned}\Delta Q_2 &= C_p \Delta T + l_v \Delta m \\ &= (c_{pd} + w_t c_w) m \Delta T + l_v \Delta m \\ &= (1005 + 0.012 \times 4218) \times 2.6168 \times 17 + 2.5 \times 10^6 \times 0.0198 \\ &\approx 46\,960 + 49\,500 \\ &= 96\,460\,\mathrm{J}\end{aligned}$$

Thus in total 117 368 J must be given to the surroundings.

(7.15) (a) We will assume an average temperature for the glass of $T = (T_1 + T_2)/2$ where $T_1 = 25\,^\circ\mathrm{C}$ and $T_2 = -10\,^\circ\mathrm{C}$. Thus, $T = 7.5\,^\circ\mathrm{C}$, which is assumed to be the dew point temperature. Then from equation 7.16, we find that $r \approx 0.32$. Thus $r_{max} \approx 32\%$.

In (b) the windows will also have $T = 25\,^\circ\mathrm{C} \Longrightarrow r_{max} = 1.0$

(7.16) At T_w

$$e_s(T_w) = 17.65\,\mathrm{mb}$$

then

$$w_s(T_w) = \frac{\epsilon e_s(T_w)}{p - e_s(T_w)} = 11.2\,\mathrm{g\,kg^{-1}}.$$

Then from equation (7.22) we find that

$$T = 28.4\,^\circ\mathrm{C} = 301.4\,\mathrm{K}$$

Using Poisson's equation we can find $T_{900\,\mathrm{mb}}$

$$T_{900} = \theta \left(\frac{900}{1000}\right)^{0.268(1-0.269q)} \approx 293\,\mathrm{K}.$$

Now $e_{sw}(T_{900}) = 23.4\,\mathrm{mb}$ which gives $w_s(T_{900}) = 16.6\,\mathrm{g\,kg^{-1}}$. Thus $r_{900\,\mathrm{mb}} = 6/16.6 = 36.14\%$.

(7.17) We will assume that there is enough drops to cool the air down to $10\,^\circ\mathrm{C}$. In this case $T = 20\,^\circ\mathrm{C}, T_w = 10\,^\circ C, w_s(T_w) = 8\,\mathrm{g\,kg^{-1}}$ and equation (7.22) gives

$$w = 3.93\,\mathrm{g\,kg^{-1}}.$$

(7.18)

$$w = \frac{\mathrm{mass}\, H_2O}{\mathrm{mass\ dry\ air}} = \frac{0.01 n M_{\mathrm{H_2O}}}{0.99 n M_d} \approx 6.3\,\mathrm{g\,kg^{-1}}$$

$$\Longrightarrow T_v = T(1 + 0.61w) = T(1.0038)$$

(7.19)

$$\rho_w d_w = \int_0^\infty \rho_v dz$$

or

$$d_w = \frac{1}{\rho_w} \int_0^\infty \frac{e}{R_v T} dz. \quad (1)$$

An upper limit is obtained by assuming that for any $z, e \approx e_{sw}$

$$\frac{de_{sw}}{dz} = \frac{de_{sw}}{dT}\frac{dT}{dz} = -\frac{de_{sw}}{dT}\Gamma. \quad (2)$$

From C-C equation we have:

$$\frac{e_{sw}}{T} = \frac{R_v T}{l_v}\frac{de_s}{dT}. \quad (3)$$

Combining (1)–(3) yields

$$d_w = -\frac{1}{\rho_w l_v \Gamma} \int_0^\infty T \frac{de_s}{dz}.$$

Now, the upper limit can be obtained by assuming that $T(z) = T_0$ where T_0 is the surface temperature. In this case:

$$d_w \approx \frac{T_o e_{so}}{\rho_w l_v \Gamma}$$

(7.20)

$$1 - r = \frac{e_s - e}{e_s}$$

$$(1-r)\frac{w_s}{\epsilon} = (e_s - e)\frac{w_s}{\epsilon e} = (e_s - e)/(p - e_s)$$

$$\Longrightarrow 1 + (1-r)\frac{w_s}{\epsilon} = \frac{p-e}{p-e_s} \quad (1)$$

$$\left.\begin{aligned} w &= \epsilon\frac{e}{p-e} \\ w_s &= \epsilon\frac{e_s}{p-e_s} \end{aligned}\right\} \Longrightarrow \frac{p-e}{p-e_s} = \frac{w_s e}{w e_s} = \frac{w_s}{w} r \quad (2)$$

from (1) and (2), it follows that

$$w = \frac{r w_s}{1 + (1-r)\frac{w_s}{\epsilon}}$$

(7.21) Clear skies allow radiation to escape thus cooling the air near the surface which leads to dew or frost.

The role of wind is not as straightforward.

CHAPTER 8

(8.2)

$$\frac{dT_{virt}}{dz} = -T_{virt,\,0}\frac{a}{(a+z)^2}$$
$$\Longrightarrow \Gamma_{virt} = T_{virt,\,0}\frac{a}{(a+z)^2}$$
$$\Gamma_{virt} - \Gamma_d = \frac{aT_{virt,\,0}}{(a+z)^2} - \Gamma_d$$

For

$$\Gamma_{virt} - \Gamma_d = 0 \Longrightarrow$$
$$z_c = \sqrt{\frac{aT_{virt,\,0}}{\Gamma_d}} - a$$

For $z > z_c$, $\Gamma_v - \Gamma_d < 0$ stable
For $z < z_c$, $\Gamma_v - \Gamma_d > 0$ unstable

(8.3)

$$\frac{1}{\theta_{virt,\,0}}\frac{d\theta_v}{dz} = \frac{1}{a_1}e^{z/a_1} - \frac{1}{a_2}$$

For

$$\frac{d\theta_v}{dz} = 0 \Longrightarrow z_c = a_1 \ln\frac{a_1}{a_2}$$

For $z > z_c$, $\frac{d\theta_v}{dz} > 0$ stable
For $z < z_c$, $\frac{d\theta_v}{dz} < 0$ unstable

(8.4)

$$\tau = \frac{2\pi\sqrt{T_{virt,\,0}}}{\sqrt{g\Gamma'_{virt}}} = \frac{2\pi\sqrt{T_{virt,\,0}}}{\sqrt{g\Gamma_d}}$$

Period increases with temperature but the relationship is *not* linear.

(8.5)

$$d\phi = gdz \Longrightarrow \phi = g\int_0^z dz = gz$$
$$\Longrightarrow \Delta\phi = g\Delta z$$

$$\left.\begin{aligned} \frac{dp}{dz} &= -\rho g \\ d\phi &= g dz \end{aligned}\right\} \Longrightarrow -adp = d\phi \Longrightarrow$$

$$\Delta\phi = -\int_1^2 adp = g\Delta z \Longrightarrow -\int_1^2 \frac{RT}{p} dp = g\Delta z \Longrightarrow$$

$$\left.\begin{aligned} &\Delta z = \frac{R\overline{T}}{g} \ln\frac{p_1}{p_2} = \frac{R_d \overline{T}_{virt}}{g} \ln\frac{p_1}{p_2} \\ \text{After lifting:} \qquad &\Delta z = \frac{R_d \overline{T}_v}{g} \ln\frac{p_1 + dp_1}{p_2 + dp_2} \end{aligned}\right\} \Longrightarrow$$

$$\ln\frac{p_1 + dp_1}{p_2 + dp_2} = \ln\frac{p_1}{p_2} \Longrightarrow \frac{dp_1}{p_1} = \frac{dp_2}{p_2}$$

(8.6)

$$T_{virt} = T_{virt,\,0} - \Gamma_{virt} z$$

$$dp = -\frac{pg}{R_d T_{virt}} dz$$

$$\frac{dp}{p} = -\frac{g}{R_d} \frac{dz}{T_{virt,\,0} - \Gamma_{virt} z}$$

$$\int_{p_0}^{p} \frac{dp}{p} = +\frac{g}{R_d \Gamma_{virt}} \int_0^z \frac{d(T_{virt,\,0} - \Gamma_{virt} z)}{T_{virt,\,0} - \Gamma_{virt} z}$$

$$\ln\frac{p}{p_o} = \frac{g}{R_d \Gamma_{virt}} \ln\frac{T_{virt,\,0} - \Gamma_{virt} z}{T_{virt,\,0}}$$

$$p = p_o \left(1 - \frac{\Gamma_{virt} z}{T_{virt,\,0}}\right)^{\frac{g}{R_d \Gamma_{virt}}}$$

(8.7)

$$dp = -\frac{pg}{R_d T_{virt}} dz$$

$$\int_{p_o}^{p} \frac{dp}{p} = -\frac{g}{R_d T_{virt,\,0}} \int_0^z dz$$

$$\frac{p}{p_o} = -\frac{gz}{R_d T_{virt,\,0}} \Longrightarrow p = p_o e^{-\frac{gz}{R_d T_{virt,\,0}}}$$

(8.9)

$$z(t) = A(e^{\lambda t} - e^{-\lambda t}) \qquad (1)$$

$$v = \frac{dz}{dt} = \lambda A(e^{\lambda t} + e^{-\lambda t})$$

P	T_{virt}	T_{dew}	e	w	q	T	θ_{virt}	e_{s}	e_{i}	r	$T\,(\text{LCL})$	θ_{ep}
1000	303	295	26.5019	0.01693	0.01665	300	303	35.8897		0.73842	294	350
950	298	294	24.8972	0.01674	0.01646	295	302	26.5628		0.9373	294	348
900	291	291	20.59	0.01456	0.01435	288	300	17.4932		1.17703	292	340
850	289	288	16.9607	0.01266	0.01251	287	303	15.6896		1.08102	288	338
800	293	283	12.1653	0.0096	0.00951	291	313	21.0013		0.57926	281	340
750	283	278	8.62198	0.00723	0.00718	282	308	11.1867		0.77073	277	328
700	268	263	2.5762	0.0023	0.00229	268	297		3.85731	0.66787	261	303
650	263	258	1.63841	0.00157	0.00157	263	298		2.51924	0.65036	256	302
600	253	243	0.37689	0.00039	0.00039	253	293		1.01761	0.37037	240	294

STABILITY CONDITIONS

Layer	$d\theta_{\mathrm{virt}}/dz$	Condition
1000–950	< 0	Unstable
950–900	< 0	Unstable
900–850	< 0	Unstable // supersaturated
850–800	> 0	*Stable* // saturated use $d\theta_{\mathrm{ep}}/dz$
800–750	< 0	Unstable
750–700	< 0	Unstable
700–650	> 0	Stable
650–600	< 0	Unstable

CONVECTIVE STABILITY CONDITIONS

Layer	$d\theta_{\mathrm{ep}}/dz$	Condition
1000–950	< 0	Unstable
950–900	< 0	Unstable
900–850	< 0	Unstable
850–800	> 0	Stable
800–750	< 0	Unstable
750–700	< 0	Unstable
700–650	< 0	UnStable
650–600	< 0	Unstable

for

$$t = 0, \quad v = 1 \Longrightarrow$$

$$A = \frac{1}{2\lambda} \text{ where } \lambda = \sqrt{\frac{g}{T_{virt,\,0}}(\Gamma_{virt} - \Gamma_d)} = 0.023$$

or

$$A = 21.74$$

Then from equation (1), $z(t = 60\,\mathrm{sec}) \approx 81\,\mathrm{m}$ and $v = \frac{dz}{dt} = 2.1\,\mathrm{m/sec}$.

(8.10) From problem 8.7 we have that

$$p_l(z) = p_l\, e^{-\frac{gz}{R_d T_{virt,\,l}}}\,,\; p_w(z) = p_w\, e^{\frac{-gz}{R_d T_{virt,\,w}}}\,.$$

At $z = h$,

$$p_l(h) = p_w(h) \Longrightarrow$$

$$\frac{p_l}{p_w} = \frac{e^{-\frac{gh}{R_d T_{virt,\,w}}}}{e^{-\frac{gh}{R_d T_{virt,\,l}}}} \Longrightarrow$$

$$h = \frac{R_d}{g} \frac{\ln(\frac{p_w}{p_l})}{\frac{1}{T_{virt,\,w}} - \frac{1}{T_{virt,\,l}}}$$

(8.11)

Problem 8.6: $p = p_o \left[1 - \frac{\Gamma z}{T_o}\right]^{\frac{g}{R\Gamma}}$

Problem 8.7: $p = p\, e^{-\frac{gz}{RT_o}}$

Problem 8.10: $h = \frac{R\,\ln(\frac{p_w}{p_l})}{g(\frac{1}{T_w} - \frac{1}{T_l})}$

where R is the gas constant of the environment and Γ is the environmental lapse rate.

CHAPTER 9

(9.1)

$$\left.\begin{aligned} u &= \ln T \\ w &= -T \ln p \end{aligned}\right\} \Longrightarrow e^u = T$$

$$-\frac{w}{T} = \ln p \longrightarrow e^{-\frac{w}{T}} = p$$

$$\begin{aligned}
x &= Re^{(u+we^{-u})} \\
y &= -e^{-we^{-u}} \\
\frac{\partial x}{\partial u} &= R(1 - we^{-u})e^{(u+we^{-u})} \\
\frac{\partial x}{\partial w} &= Re^{-u}e^{u+we^{-u}} = Re^{we^{-u}} \\
\frac{\partial y}{\partial u} &= -we^{-u}e^{-we^{-u}} \\
\frac{\partial y}{\partial w} &= e^{-u}e^{-we^{-u}} = e^{-(u+we^{-u})}.
\end{aligned}$$

It follows that

$$\begin{aligned}
J &= \frac{\partial x}{\partial u}\frac{\partial y}{\partial w} - \frac{\partial x}{\partial w}\frac{\partial y}{\partial u} \\
&= R(1 - we^{-u}) + Rwe^{-u} = R = \text{constant}.
\end{aligned}$$

Since $J = \text{constant}$ we conclude that this diagram is an area-equivalent diagram.

Since $p = e^{-we^{-u}}$ isobars are not straight lines.

From Poisson's equation we have that for the dry adiabats

$$\ln \theta = \ln T + k_d \ln 1000 - k_d \ln p$$

or

$$-\ln p = -\frac{1}{k_d} \ln T + \text{constant}.$$

This equation is a straight line in a $(-\ln p, \ln T)$ diagram but *not* in a $(-T \ln p, \ln T)$ diagram.

(9.2)

$$\left.\begin{aligned} u &= T \\ w &= -p^{k_d} \end{aligned}\right\} \Longrightarrow \quad \begin{aligned} u &= T \\ -w^{\frac{1}{k_d}} &= p \end{aligned}$$

$$\begin{aligned} x = a = \frac{RT}{p} &= -Ru\,w^{-\frac{1}{k_d}} \\ y = -p &= w^{\frac{1}{k_d}} \end{aligned}$$

$$\left.\begin{aligned} \frac{\partial x}{\partial u} &= -Rw^{-\frac{1}{k_d}} \\ \frac{\partial x}{\partial w} &= Ru\frac{1}{k_d}w^{-\frac{1}{k_d}-1} \\ \frac{\partial y}{\partial u} &= 0 \\ \frac{\partial y}{\partial w} &= \frac{1}{k_d}w^{\frac{1}{k_d}-1} \end{aligned}\right\} \Longrightarrow J = \frac{R}{k_d}w^{-1}$$

Since $J = \frac{R}{k_d w} \neq$ constant, it follows that this diagram is *not* an area-equivalent diagram. Since $w = -p^{k_d}$ and $u = T$ isobars and isotherms are constant w and u lines respectively, and thus they are straight lines.

From Poisson's equation

$$\begin{aligned} \theta = T\left(\frac{1000}{p}\right)^{k_d} &\Longrightarrow \\ p^k = T \times &\text{ constant,} \end{aligned}$$

which in a (p^k, T) diagram is a straight line.

(9.3)

$$\begin{aligned} u &= T \\ w &= c_p \ln \theta = c_p \ln T - c_p k_d \ln p + \text{constant} \Longrightarrow \\ &w - c_p \ln u - \text{constant} = -c_p k_d \ln p \Longrightarrow \\ \ln p &= A \ln u - Bw + C \end{aligned}$$

where

$$A = \frac{1}{k_d}, B = \frac{1}{c_p k_d}, C = \text{constant} \times \frac{1}{c_p k_d}.$$

It follows that

$$p = e^{A \ln u - Bw + C}$$

$$x = a = \frac{RT}{p} = \frac{Ru}{e^{A \ln u - Bw + C}} = \frac{Ru}{e^{A \ln u} e^{[C - Bw]}} = \frac{Ru}{u^A e^{[C - Bw]}} = \frac{Ru^{1-A}}{e^{[C-Bw]}}$$

$$y = -p = -e^{A \ln u - Bw + C} = -u^A e^{[C-Bw]}$$

$$\left.\begin{aligned} \frac{\partial x}{\partial u} &= \frac{R(1-A)}{e^{[C-Bw]}} u^{-A} \\ \frac{\partial x}{\partial w} &= \frac{BRu^{1-A}}{e^{[C-Bw]}} \\ \frac{\partial y}{\partial u} &= -Ae^{[C-Bw]} u^{A-1} \\ \frac{\partial y}{\partial w} &= Bu^A e^{[C-Bw]} \end{aligned}\right\} \Longrightarrow \quad J = R(1-A)B + \tfrac{BR}{A} = \text{constant}$$

Q.E.D.

(9.4) Since at 950 mb, $T = 22.5\,^\circ\mathrm{C}$ and $T_{dew} = 15.7\,^\circ\mathrm{C}$ it follows

$$\left.\begin{aligned} &w = 12.4 \Longrightarrow e = \tfrac{wp}{w+\epsilon} = \tfrac{11.78}{0.6344} = 18.6\,\mathrm{mb} \\ &e_{sw}(22.5\,^\circ\mathrm{C}) = 27.4\,\mathrm{mb} \end{aligned}\right\} \Longrightarrow \quad r = 0.68$$

From the diagram we find that

$$\theta = 27.5$$
$$T_{wp} = 18\,^\circ\mathrm{C}$$
$$\theta_{wp} = 20\,^\circ\mathrm{C}$$

Now:

(1) $T_w = T + \dfrac{l_v}{c_{pd}}(w - w_{sw})$

$$T_w \approx T + \frac{l_v}{c_{pd}} \left(w - \frac{\epsilon}{p} 6.11 \exp\left[19.83 - \frac{5417}{T_w}\right]\right)$$

Numerical solution provides

$$T_w \approx 292\,\mathrm{K} \approx 19\,^\circ\mathrm{C}$$

(2) $T_e = T + \dfrac{l_v w}{c_{pd} + wc_w} \approx 324\,\mathrm{K} = 51\,^\circ\mathrm{C}$

(3) From equation (7.63) with $p = 1000, T = 20\,^\circ\mathrm{C}$, $w = 0.016$, it follows that $\theta_{ep} = 66\,^\circ\mathrm{C}$.

(4) From equation (7.64) $\Longrightarrow T_{ep} = 333.3\,\mathrm{K} \approx 60.3\,^\circ\mathrm{C}$.

(9.5) From the sounding we find that

$$p_{LFC} \approx 700\,\text{mb}$$
$$p_{LNB} \approx 300\,\text{mb}$$

if we assume:

$$< T' - T > \approx 1\,\text{K}$$

it follows (from the equation at the bottom of page 153) that

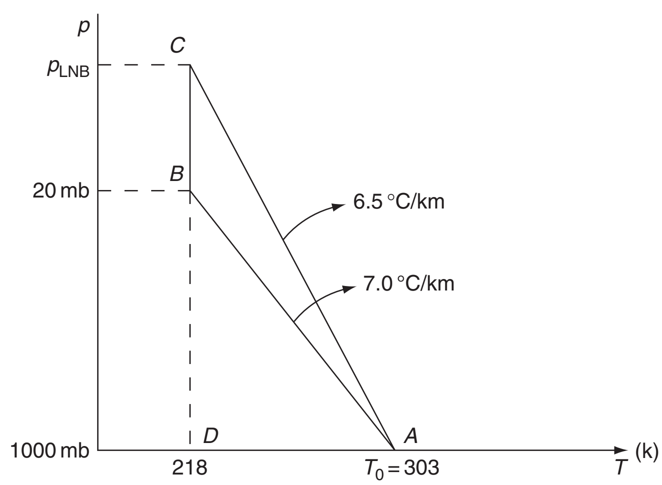
$$\text{CAPE} \approx 244\,\text{J}\,\text{kg}^{-1}$$

or

$$u_{max} = \sqrt{2\,\text{CAPE}} = 22.1\,\text{m/sec.}$$

This is not a very large value. So, even though some convection may develop, deep convection most likely will not.

(9.6)



For constant lapse rate:

$$p(z) = p_o \left[1 - \frac{\Gamma z}{T_0}\right]^{\frac{g}{R_d T}}$$

or

$$z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{R_d \Gamma}{g}}\right]$$

Then

(1) $z_{200\,\text{mb}} = 12\,164\,\text{m}$

(2) $T_{200}\,\text{mb} = T_0 - \Gamma z = 218\,\text{K}$

(3) $218 = T_0 - \Gamma_s z_{LNB} \Longrightarrow z_{LNB} = 13\,077\,\text{m}$

(4) $p_{LNB} = p_o \left(1 - \dfrac{\Gamma_s z}{T_0}\right)^{\frac{g}{R_d \Gamma_s}} \approx 178\,\text{mb}.$

Now

$$\begin{aligned}\text{CAPE} &= -R_d \int_{1000}^{178} (T_{200\text{mb}} - T_0)\,d\ln p \\ &= R_d(\text{Area of } ADC - \text{Area of } ABD)\end{aligned}$$

Note here that we are dealing with $d\ln p$ not dp. It follows that

$$\text{CAPE} = \frac{1}{2} \times 287 \times 85(1.72 - 1.6) = 1464\,\text{J}\,\text{kg}^{-1}$$

which corresponds to

$$u_{max} = 54\,\text{m/sec}.$$

(9.8) from 800 to 550 mb, $\Delta z \approx 2.8\,\text{km}$. The mass of dry air in a volume $2800 \times 1 \times 1\ \text{m}^3$ is $2800 \times 1.293 = 3620\,\text{kg}$. This means that there is $3620 \times 0.0027 = 9.8\,\text{kg}$ of water in that volume. If all precipitates it will occupy a volume $m_w/\rho_w = 0.0098\,\text{m}^3$. Then, since $1 \times 1 \times z = 0.0098 \Longrightarrow z \approx 1\,\text{cm}$. From formula (7.19) we find $d_w \simeq 3\,\text{cm}$. The difference might arise from the fact that in (7.19) all the atmospheric vapor is assumed to condense.

(9.7,9.9,9.10) Use a diagram to obtain the answers.