EXAMPLE 10.4-1: Absorptivity and Emissivity of a Solar Selective Surface

A solar collector is a device that is designed to maximize the absorption of the irradiation that is received from the sun so that this energy can be used, for example for domestic water heating. A typical solar collector for water heating consists of a metallic collector plate that is bonded to tubes through which the water to be heated flows, as shown in Figure 1. The collector plate is heated when it is exposed to solar radiation and this energy transfer is conducted through the plate to the tubes where it heats the water by convection. The plate is contained in an insulated enclosure in order to reduce the rate at which energy is lost from the sides and back of the plate. Some solar collectors use one or more transparent glazings that are placed above the plate. However, the solar collector in Figure 1 is designed for a low temperature swimming pool heating application and therefore does not employ a cover. Solar radiation is incident on the collector surface with irradiation G = 800 W/m^2, assume that the irradiation is spectrally distributed as if it were emitted by a blackbody at T_{sun} = 5780 K. The ambient temperature is T_∞ = 25°C and the average convection heat transfer coefficient between the plate and the ambient air is $\overline{h}$ = 10 W/m^2-K. The collector plate has an average temperature of T_{plate} = 45°C. The plate also exchanges radiation with surroundings at T_∞.

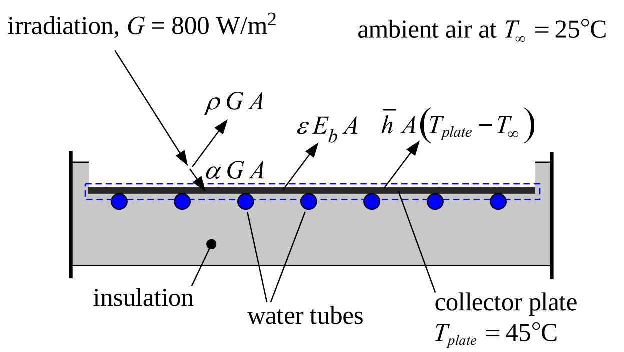

Figure 1: Solar collector used for swimming pool heating

The objective of the design of a solar collector is to maximize the absorption of solar radiation but minimize thermal loss from the plate to the surroundings due to radiation. Solar radiation is concentrated at relatively low wavelengths because it is emitted by a high temperature source, the sun. However, the collector the plate emits radiation at relatively high wavelengths because it is (by comparison to the sun) cold. Therefore, an ideal solar collector surface has an absorptivity (which, according to Kirchoff's Law, is equal to the emissivity) that is high at low wavelengths in order to capture the solar irradiation and an emissivity that is low at high wavelengths in order to minimize radiation heat loss. These types of selective surfaces are important for achieving high collector efficiency.

Figure 2 illustrates a semi-gray model of the selective surface that is used for the solar collector. The emissivity below λ_c = 5.0 μm is ε_{low} = 0.95 and the emissivity above λ_c is ε_{high} = 0.05; no real surface exhibits such a step-change behavior in its emissivity but this semi-gray model is useful to simulate some surfaces, such as black-chrome.

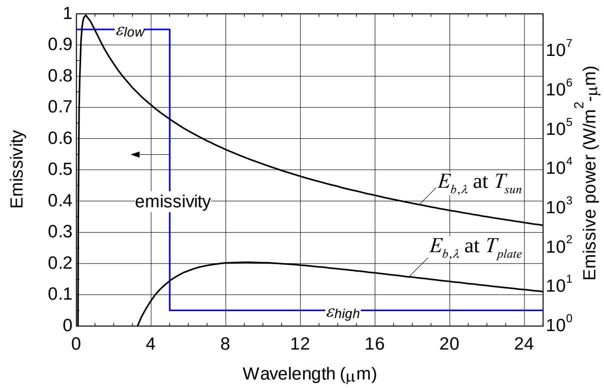

Figure 2: Emissivity as a function of wavelength for the selective surface used for the collector plate. Also shown is the blackbody emissive power at T_{plate} and T_{sun}.

a.) Calculate the steady-state rate of energy transfer to the water per unit area of collector surface.

The inputs are entered in EES:

```
"EXAMPLE 10.4-1"
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
epsilon_low=0.95 [-]                    "emissivity at low wavelengths"
epsilon_high=0.05 [-]                   "emissivty at high wavelengths"
lambda_c=5.0 [micron]                   "cut-off wavelength"
G=800 [W/m^2]                           "solar irradiation"
T_sun = 5780  [K]                       "temperature of the sun"
A=1 [m^2]                               "unit area"
T_plate=converttemp(C,K,45 [C])         "plate temperature"
T_infinity=converttemp(C,K,25 [C])      "temperature of surroundings"
h_bar=10 [W/m^2-K]                      "average heat transfer coefficient"
```

A steady state energy balance on the plate is shown in Figure 1:

$$\dot{q}_{water} = \alpha\, G\, A - \varepsilon\, \sigma\, A\left(T_{plate}^4 - T_\infty^4\right) - \bar{h}\, A\left(T_{plate} - T_\infty\right) \tag{1}$$

where ε is the total hemispherical emissivity of the plate (evaluated relative to the emissive power of the plate) and α is the total hemispherical absorptivity of the plate (evaluated relative to irradiation from the sun). These quantities can be evaluated using the techniques discussed in Section 10.2.3 for semi-gray surfaces. The total hemispherical emissivity is obtained according to:

$$\varepsilon = \frac{1}{\sigma T_{plate}^4} \int_0^\infty \varepsilon_\lambda\, E_{b,\lambda}\, d\lambda$$

where $E_{b,\lambda}$ is evaluated using Planck's law, Eq. (10-4) at T_{plate}. For the semi-gray surface shown in Figure 2, the integral is broken up according to the different wavelength bands:

$$\varepsilon = \frac{\int_0^{\lambda_c} \varepsilon_{low}\, E_{b,\lambda}\, d\lambda + \int_{\lambda_c}^{\infty} \varepsilon_{high}\, E_{b,\lambda}\, d\lambda}{\sigma\, T_{plate}^4}$$

Within each wavelength band, the emissivity is constant and can be removed from the integrand:

$$\varepsilon = \varepsilon_{low} \underbrace{\frac{\int_0^{\lambda_c} E_{b,\lambda}\, d\lambda}{\sigma\, T_{plate}^4}}_{F_{0-\lambda_c,T_{plate}}} + \varepsilon_{high} \underbrace{\frac{\int_{\lambda_c}^{\infty} E_{b,\lambda}\, d\lambda}{\sigma\, T_{plate}^4}}_{1-F_{0-\lambda_c,T_{plate}}}$$

which can be written in terms of the external fractional functions:

$$\varepsilon = \varepsilon_{low\lambda}\, F_{0-\lambda_c,T_{plate}} + \varepsilon_{high\lambda} \left(1 - F_{0-\lambda_c,T_{plate}}\right)$$

where $F_{0-\lambda_c,T_{plate}}$ is evaluated at T_{plate}. The external fractional functions are obtained using the Blackbody function in EES.

```
epsilon=epsilon_low*Blackbody(T_plate,0 [micron],lambda_c)+&
    epsilon_high*(1-Blackbody(T_plate,0 [micron],lambda_c))
        "total hemispherical emissivity relative to the spectral blackbody emissive power of the plate"
```

which leads to ε = 0.067. Note that the total hemispherical emissivity of the collector plate is weighted towards ε_{high} because the plate emits primarily at high wavelengths (see Figure 2). The total hemispherical absorptivity of the plate can be evaluated in a similar manner according to:

$$\alpha = \alpha_{low} \underbrace{\frac{\int_0^{\lambda_c} E_{b,\lambda}\, d\lambda}{\sigma\, T_{sun}^4}}_{F_{0-\lambda_c,T_{sun}}} + \alpha_{high} \underbrace{\frac{\int_{\lambda_c}^{\infty} E_{b,\lambda}\, d\lambda}{\sigma\, T_{sun}^4}}_{1-F_{0-\lambda_c,T_{sun}}}$$

where $E_{b,\lambda}$ is evaluated using Planck's law, Eq. (10-4) at T_{sun}. Note that α_{low} = ε_{low} and α_{high} = ε_{high} according to Kirchoff's law. Therefore:

$$\alpha = \varepsilon_{low}\, F_{0-\lambda_c,T_{sun}} + \varepsilon_{high} \left(1 - F_{0-\lambda_c,T_{sun}}\right)$$

where $F_{0-\lambda_c,T_{sun}}$ is evaluated at T_{sun}.

```
alpha=epsilon_low*Blackbody(T_sun,0 [micron],lambda_c)+&
    epsilon_high*(1-Blackbody(T_sun,0 [micron],lambda_c))
        "total hemispherical absorptivity relative to the irradiation from the sun"
```

which leads to α = 0.945. Note that the absorptivity is weighted towards ε_{low} because the irradiation from the sun is concentrated at low wavelengths. The energy balance, Eq. (1), is:

```
q_dot_water=G*A*alpha-epsilon*sigma#*A*(T_plate^4-T_infinity^4)-h_bar*A*(T_plate-T_infinity)
        "energy balance on plate"
```

which leads to $\dot{q}_{water}$ = 547 W for the assumed A = 1 m^2 collector area. This result corresponds to a collector efficiency of 68.4% relative to capturing the solar irradiation, G = 800 W/m^2.

b.) Compare your answer from (a) to the result that would be obtained if the plate had a constant spectral emissivity of 0.95 (i.e., if a selective surface were not used as the absorber plate but rather a plate with a uniform, high absorptivity were used instead).

The EES code is run again with ε_{high} = 0.95 which corresponds to a surface with a constant emissivity of 0.95 regardless of wavelength. The result is $\dot{q}_{water}$ = 433 W/m^2; this is a 20% reduction in performance and illustrates the importance of using selective surfaces for solar collectors.