## EXAMPLE 10.5-4 Radiation Exchange in a Duct with Semi-gray Surfaces

The long triangular enclosure, shown in Figure 1, has one wall (surface 1) that is maintained at $T_1$ = 300 K and another (surface 2) at $T_2$ = 1000 K. The third wall (surface 3) is adiabatic. The duct is very long (into the page) and therefore this is a 2-D problem. The width of each wall is $W$ = 0.1 m.

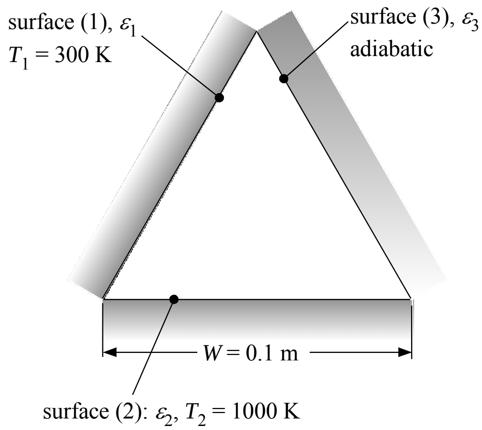


**Figure 1: Long triangular duct with semi-gray surfaces**

The duct surfaces are not gray but can be modeled as being semi-gray. The emissivity of each of the 3 surfaces is illustrated in Figure 2.

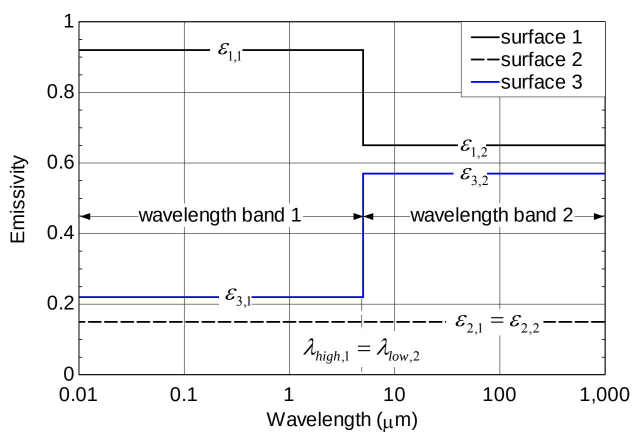


**Figure 2: Emissivities of the surfaces**

There are two wavelength bands for this problem. Wavelength band 1 extends from $\lambda_{low,1}$ = 0 μm to $\lambda_{high,1}$ = 5.0 μm and wavelength band 2 extends from $\lambda_{low,2}$ = 5.0 μm to $\lambda_{high,2} = \infty$. In wavelength band 1, the emissivity of surfaces 1, 2, and 3 are $\varepsilon_{1,1}$ = 0.92, $\varepsilon_{2,1}$ = 0.15, and $\varepsilon_{3,1}$ = 0.22, respectively. In wavelength band 2, the emissivity of surfaces 1, 2, and 3 are $\varepsilon_{1,2}$ = 0.65, $\varepsilon_{2,2}$ = 0.15, and $\varepsilon_{3,2}$ = 0.57, respectively.

a.) Determine the temperature of the adiabatic wall and the rate of heat transfer per unit length required to maintain the other two walls at their specified temperatures. Assume that only radiation heat transfer is occurring in the enclosure.

The known information is entered into EES; note that $\lambda_{high,2}$ is set to a value large enough to include essentially all of the radiation emitted by each of the surfaces (see Figure 10-3).

```
"EXAMPLE 10.5-4"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
W=0.1 [m]                              "width of the duct"
L=1 [m]                                "per unit length"
T[1]=300 [K]                           "temperature of surface 1"
T[2]=1000 [K]                          "temperature of surface 2"

"wavelength band 1"
lambda_low[1]=0.0 [micron]             "lower limit of wavelength band 1"
lambda_high[1]=5 [micron]              "upper limit of wavelength band 1"
epsilon[1,1]=0.92 [-]                  "emissivity of surface 1 in wavelength band 1"
epsilon[2,1]=0.15 [-]                  "emissivity of surface 2 in wavelength band 1"
epsilon[3,1]=0.22 [-]                  "emissivity of surface 3 in wavelength band 1"

"wavelength band 2"
lambda_low[2]=5 [micron]               "lower limit of wavelength band 2"
lambda_high[2]=1000 [micron]           "upper limit of wavelength band 2"
epsilon[1,2]=0.65 [-]                  "emissivity of surface 1 in wavelength band 2"
epsilon[2,2]=0.15 [-]                  "emissivity of surface 2 in wavelength band 2"
epsilon[3,2]=0.57 [-]                  "emissivity of surface 3 in wavelength band 2"
```

The area of each surface is the same:

$$A_1 = A_2 = A_3 = W\,L$$

```
"areas"
A[1]=W*L                                          "area of surface 1"
A[2]=W*L                                          "area of surface 2"
A[3]=W*L                                          "area of surface 3"
```

Determination of the view factors is easy for this symmetric geometry and can be done by inspection; the view factor from any surface to itself must be zero (they are all flat) and the view factor from each surface to each of the other surfaces must be 0.5 by symmetry.

```
"view factors"
"surface 1"
F[1,1]=0 [-]
F[1,2]=0.5 [-]
F[1,3]=0.5 [-]

"surface 2"
F[2,1]=0.5 [-]
F[2,2]=0 [-]
F[2,3]=0.5 [-]
```

```
"surface 3"
F[3,1]=0.5 [-]
F[3,2]=0.5 [-]
F[3,3]=0 [-]
```

It is easiest to solve gray body problems by assuming a temperature for each surface (even those without specified temperatures) in order to allow the emissive power of each surface to be computed within each wavelength band. The assumed surface temperatures must later be relaxed (i.e., commented out) in order to enforce the boundary conditions for the problem. For this problem, the temperature of surface 3 (the adiabatic surface) is initially assumed:

```
T[3]=500 [K]                    "guess for temperature of surface 3 (this will be removed)"
```

The emissive power for each surface within each wavelength band is determined according to Eq. (10-110); the integral is obtained using the Blackbody function in EES:

$$E_{b,i,w} = \int_{\lambda_{low,w}}^{\lambda_{high,w}} E_{b,\lambda}\, d\lambda \quad \text{evaluated at } T_i \quad \text{for } i = 1..3 \text{ and } w = 1..2$$

```
"blackbody emissive power for each surface in each wavelength band"
duplicate i=1,3
    duplicate w=1,2
        E_b[i,w]=Blackbody(T[i],lambda_low[w],lambda_high[w])*sigma#*T[i]^4
    end
end
```

The rates of heat transfer for each surface within each wavelength band are computed using the same technique discussed in Section 10.5.5; Eqs. (10-108) and (10-109) are written for each surface and wavelength band:

$$\dot{q}_{i,w} = \frac{\varepsilon_{i,w} A_i \left(E_{b,i,w} - J_{i,w}\right)}{\left(1 - \varepsilon_{i,w}\right)} \quad \text{for } i = 1...3 \text{ and } w = 1..2$$

$$\dot{q}_{i,w} = A_i \sum_{j=1}^{3} F_{i,j} \left(J_{i,w} - J_{j,w}\right) \quad \text{for } i = 1...3 \text{ and } w = 1..2$$

```
"heat transfer rates for each surface in each wavelength band"
duplicate i=1,3
    duplicate w=1,2
        q_dot[i,w]=epsilon[i,w]*A[i]*(E_b[i,w]-J[i,w])/(1-epsilon[i,w])
        q_dot[i,w]=A[i]*sum(F[i,j]*(J[i,w]-J[j,w]),j=1,3)
    end
end
```

The net rate of radiation heat transfer for each surface is obtained by summing the radiation heat transfer within each wavelength band, according to Eq. (10-111):

$$\dot{q}_i = \sum_{w=1}^{2} \dot{q}_{i,w} \quad \text{for } i = 1..3$$

```
"total heat transfer to each surface"
duplicate i=1,3
    q_dot_total[i]=sum(q_dot[i,w],w=1,2)
end
```

Solving the problem will provide the heat transfer rate within each wavelength band as well as the net heat transfer for each surface, given the assumed temperature for surface 3 (Figure 3).

| $\dot{q}_{i,1}$ [W] | $\dot{q}_{i,2}$ [W] | $\dot{q}_{total_i}$ [W] | $T_i$ [K] |
|---|---|---|---|
| -447.4 | -243.7 | -691.1 | 300 |
| 514 | 273.4 | 787.4 | 1000 |
| -66.61 | -29.69 | -96.3 | 500 |

**Figure 3: Arrays table showing heat transfer rate within each wavelength band as well as the net heat transfer rate from each surface.**

Note that surface 3 is not adiabatic for the temperature that was assumed to solve the problem ($\dot{q}_3$ is 787.4 W according to Figure 3). The guess values are updated, the assumed value of $T_3$ is commented out and the adiabatic boundary condition for surface 3 is enforced:

```
{T[3]=500 [K]}          "guess for temperature of surface 3 (this will be removed)"
q_dot_total[3]=0        "enforce that surface 3 is adiabatic"
```

which leads to $T_3$ = 579.8 K, $\dot{q}_1$ = -773.2 W (i.e., 773.2 W must be removed from surface 1), and $\dot{q}_2$ = 773.2 W (i.e., 773.2 W must be provided to surface 2). The arrays table containing the details of the solution is shown in Figure 4.

| Sort | $\lambda_{high,i}$ [micron] | $\lambda_{low,i}$ [micron] | $E_{b,i,2}$ [W/m2] | $E_{b,i,1}$ [W/m2] | $J_{i,2}$ [W/m2] | $J_{i,1}$ [W/m2] | $\dot{q}_{i,1}$ [W] | $\dot{q}_{i,2}$ [W] | $\dot{q}_{total_i}$ [W] | $T_i$ [K] |
|---|---|---|---|---|---|---|---|---|---|---|
| [1] | 5 | 0 | 452.9 | 5.901 | 2105 | 411.4 | -466.3 | -306.9 | -773.2 | 300 |
| [2] | 1000 | 5 | 20706 | 35931 | 5894 | 6932 | 511.8 | 261.4 | 773.2 | 1000 |
| [3] | | | 4797 | 1604 | 4454 | 3217 | -45.48 | 45.48 | 0 | 579.8 |

**Figure 4: Arrays table showing the solution within each wavelength band for each surface.**