EXAMPLE 1.2-1: Liquid Oxygen Dewar

Figure 1 illustrates a spherical dewar containing saturated liquid oxygen that is kept at a pressure p_{LOx} = 25 psia; the saturation temperature of oxygen at this pressure is T_{LOx} = 95.6 K.

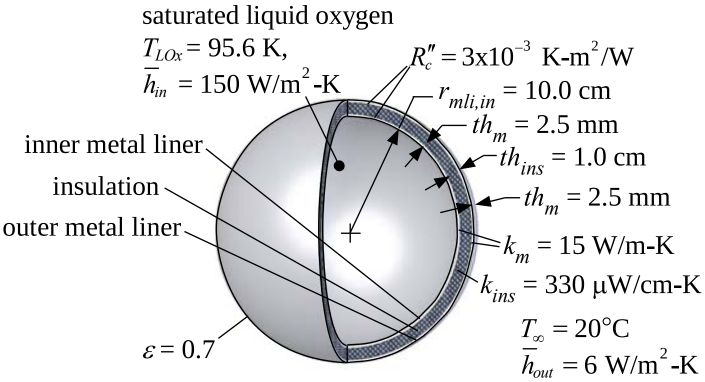


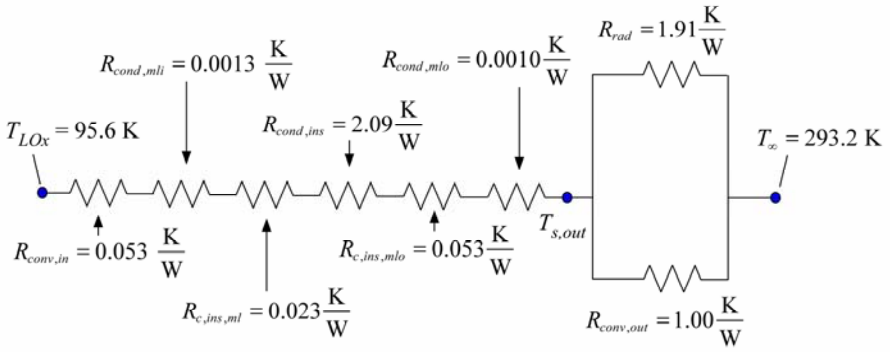
Figure 1: Spherical dewar containing saturated liquid oxygen.

The dewar consists of an inner and outer metal liner separated by polystyrene foam insulation. The inner metal liner has inner radius $r_{mli,in}$ = 10.0 cm and thickness th_m = 2.5 mm. The outer metal liner also has thickness th_m = 2.5 mm. The conductivity of both metal liners is k_m = 15 W/m-K. The heat transfer coefficient between the oxygen within the dewar and the inner surface of the dewar is $\overline{h}_{in}$ = 150 W/m^2-K. The outer surface of the dewar is surrounded by air at T_∞ = 20°C and radiates to surroundings that are also at T_∞ = 20°C. The emissivity of the outer surface of the dewar is ε = 0.7. The heat transfer coefficient between the outer surface of the dewar and the surrounding air is $\overline{h}_{out}$ = 6 W/m^2-K. The area-specific contact resistance that characterizes the interface between the insulation and the adjacent metal liners is R''_c = 3.0x10^{-3} K-m^2/W.

The thickness of the insulation between the two metal liners is th_{ins} = 1.0 cm. You are trying to evaluate the impact of using polystyrene foam insulation in place of the more expensive insulation that is currently used. Flynn (2005) suggests that the conductivity of polystyerene foam at cryogenic temperatures is approximately k_{ins} = 330 μW/cm-K.

a.) Draw a network that represents this situation using 1-D resistances.

The resistance network is illustrated in Figure 2.



The resistances include:
$R_{conv,in}$ = convection from inside surface
$R_{cond,mli}$ = conduction through inner metal liner
$R_{c,ins,mli}$ = contact between inner metal liner & insulation
$R_{cond,ins}$ = conduction through insulation
$R_{c,ins,mlo}$ = contact between outer metal liner & insulation
$R_{cond,mlo}$ = conduction through outer metal liner
R_{rad} = radiation
$R_{conv,out}$ = convection from outer surface

Figure 2: Resistance network representing the dewar.

The resistance network interacts with the surrounding air and surroundings (T_{∞}) and the saturated liquid oxygen (T_{LOx}).

b.) Estimate the rate of heat transfer to the liquid oxygen.

The solution will be carried out using EES. It is assumed that you have already been exposed to the EES software by completing the self-guided tutorial contained in Appendix A.1. The first step in preparing a successful solution to any problem with EES is to enter the inputs to the problem and set their units. Experience has shown that it is generally best to work exclusively in SI units (m, J, K, kg, Pa, etc.). This unit system is entirely self-consistent. If the problem statement includes parameters in other units, they can be converted to SI units within the "Inputs" section of the code. The upper section of your EES code should look something like:

```
"EXAMPLE 1.2-1: Liquid Oxygen Dewar"
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
p_LOx=25 [psia]*convert(psia,Pa)                    "pressure of liquid oxygen"
T_LOx=95.6 [K]                                      "temperature of liquid oxygen"
h_bar_in=150 [W/m^2-K]
    "heat transfer coefficient between the liquid oxygen and the inner wall"
r_mli_in=10 [cm]*convert(cm,m)                      "inner radius of the inner metal liner"
th_m=2.5 [mm]*convert(mm,m)                         "thickness of inner metal liner"
th_ins_cm=1.0 [cm]                                  "thickness of insulation, in cm"
th_ins=th_ins_cm*convert(cm,m)                      "thickness of insulation"
```

```
e=0.7 [-]                                                "emissivity of outside surface"
T_infinity=converttemp(C,K,20 [C])
    "temperature of surroundings and surrounding air"
R``_c=3.0e-3 [K-m^2/W]                                   "area-specific contact resistance"
k_ins=330 [microW/cm-K]*convert(microW/cm-K,W/m-K)    "mean conductivity of insulation"
k_m=15 [W/m-K]                                           "conductivity of metal"
h_bar_out=6 [W/m^2-K]
    "heat transfer coefficient between outer wall and surrounding air"
```

The resistance to convection between the inner surface of the dewar and the oxygen is:

$$R_{conv,in} = \frac{1}{\bar{h}_{in}\, 4\,\pi\, r_{mli,in}^2}$$

```
R_conv_in=1/(4*pi*r_mli_in^2*h_bar_in) "convection resistance to liquid oxygen"
```

The inner radius of the insulation is:

$$r_{ins,in} = r_{mli,in} + th_m$$

The resistance to conduction through the inner metal liner is:

$$R_{cond,mli} = \frac{1}{4\,\pi\, k_m}\left(\frac{1}{r_{mli,in}} - \frac{1}{r_{ins,in}}\right)$$

and the contact resistance resistance between the inner metal liner and the insulation is:

$$R_{c,ins,mli} = \frac{R''_c}{4\,\pi\, r_{ins,in}^2}$$

```
r_ins_in=r_mli_in+th_m                          "inner radius of insulation"
R_cond_mli=(1/r_mli_in-1/r_ins_in)/(4*pi*k_m)   "conduction resistance of inner metal liner"
R_c_ins_mli=R``_c/(4*pi*r_ins_in^2)
    "contact resistance between inner metal liner and insulation"
```

The outer radius of the insulation is:

$$r_{ins,out} = r_{ins,in} + th_{ins}$$

The resistance to conduction through the insulation is:

$$R_{cond,ins} = \frac{1}{4\,\pi\, k_{ins}}\left(\frac{1}{r_{ins,in}} - \frac{1}{r_{ins,out}}\right)$$

and the contact resistance resistance between the insulation and the outer metal liner is:

$$R_{c,ins,mlo} = \frac{R''_c}{4\,\pi\,r_{ins,out}^2}$$

```
r_ins_out=r_ins_in+th_ins                           "outer radius of insulation"
R_cond_ins=(1/r_ins_in-1/r_ins_out)/(4*pi*k_ins)  "conduction resistance of insulation"
R_c_ins_mlo=R``_c/(4*pi*r_ins_out^2)
    "contact resistance between insulation and outer metal liner"
```

The outer radius of the outer metal liner is:

$$r_{mlo,out} = r_{ins,out} + th_m$$

The resistance to conduction through the outer metal liner is:

$$R_{cond,mlo} = \frac{1}{4\,\pi\,k_m}\left(\frac{1}{r_{ins,out}} - \frac{1}{r_{mlo,out}}\right)$$

and the convection resistance between the outer surface of the dewar and the air is:

$$R_{conv,out} = \frac{1}{\bar{h}_{out}\,4\,\pi\,r_{mli,out}^2}$$

```
r_mlo_out=r_ins_out+th_m                            "outer radius of outer metal liner"
R_cond_mlo=(1/r_ins_out-1/r_mlo_out)/(4*pi*k_m)   "conduction resistance of outer metal liner"
R_conv_out=1/(4*pi*r_mlo_out^2*h_bar_out)          "convection resistance to surrounding air"
```

The surface temperature on the outside of the dewar ($T_{s,out}$ in Figure 2) cannot be known until the problem is solved and yet it must be used to calculate the resistance to radiation, R_{rad}. One of the nice things about using the Engineering Equation Solver (EES) software to solve this problem is that the software can deal with this type of nonlinearity and provide the solution to the implicit equations. It is this capability that simultaneously makes EES so powerful and yet sometimes, ironically, difficult to use. EES should be able to solve equations regardless of the order in which they are entered. However, you should enter equations in a sequence that allows you to solve them as you enter them; this is exactly what you would be forced to do if you were to solve the problem using a typical programming language (e.g., MATLAB, FORTRAN, etc.). This technique of entering your equations in a systematic order provides you with the opportunity to debug each subset of equations as you move along, rather than waiting until all of the equations have been entered to try to solve them. Another benefit of approaching a problem in this sequential manner is that you can consistently update the guess values associated with the variables in your problem; EES solves your equations using a nonlinear relaxation technique and therefore the closer the guess values of the variables are to “reasonable” values, the more likely EES will find the correct solution

To proceed with the solution to this problem using EES, it is helpful to initially assume a reasonable surface temperature (e.g., the average of the surrounding and the liquid oxygen temperatures) so that it is possible to estimate the radiation resistance:

$$R_{rad} = \frac{1}{4\,\pi\,r_{mli,out}^2\,\sigma\,\varepsilon\left(T_{s,out}^2 + T_\infty^2\right)\left(T_{s,out} + T_\infty\right)}$$

and continue with the solution. The next few lines in your EES code should look something like:

```
T_s_out=(T_LOx+T_infinity)/2          "guess for the surface temperature (removed to complete problem)"
R_rad=1/(4*pi*r_mlo_out^2*sigma#*e*(T_s_out^2+T_infinity^2)*(T_s_out+T_infinity))
          "radiation resistance"
```

Solve the equations that have been entered (select Calculate from the Solve menu) and check that your answers make sense. Verify that the variables and equations have a consistent set of units by setting the units for each of the variables. The best way to do this is to go to the Variable Information window (select Variable Info from the Options menu) and enter the unit for each variable in the Units column. Once this is done, check the units for your problem (select Check Units from the Calculate menu) in order to make sure that all of the units are consistent with the equations.

The total resistance separating the liquid oxygen from the surroundings is:

$$R_{total} = R_{conv,in} + R_{cond,mli} + R_{c,ins,mli} + R_{cond,ins} + R_{c,ins,mlo} + R_{cond,mlo} + \left(\frac{1}{R_{conv,out}} + \frac{1}{R_{rad}}\right)^{-1}$$

and the heat transfer rate from the surroundings to the liquid oxygen can be estimated:

$$\dot{q} = \frac{\left(T_\infty - T_{LOx}\right)}{R_{total}}$$

```
R_total=R_conv_in+R_cond_mli+R_c_ins_mli+R_cond_ins+R_c_ins_mlo+R_cond_mlo+&
    (1/R_conv_out+1/R_rad)^(-1)                          "total resistance"
q_dot=(T_infinity-T_LOx)/R_total                         "heat flow"
```

At this point, we can use the heat transfer rate to recalculate the surface temperature (as opposed to assuming it).

$$T_{s,out} = T_{LOx} + \dot{q}\left(R_{conv,in} + R_{cond,mli} + R_{c,ins,mli} + R_{cond,ins} + R_{c,ins,mlo} + R_{cond,mlo}\right)$$

It is necessary to comment out or delete the equation that provided the assumed surface temperature and instead calculate the surface temperature correctly. This step creates an implicit set of nonlinear equations. Before you ask EES to solve the set of equations, it is a good idea to upate the guess values for each variable (select Update Guesses from the Calculate menu).

```
{T_s_out=(T_LOx+T_infinity)/2} "guess for the surface temperature"
T_s_out=T_LOx+q_dot*(R_conv_in+R_cond_mli+R_c_ins_mli+R_cond_ins+R_c_ins_mlo+R_cond_mlo)
          "surface temperature"
```

The rate of heat transfer to the liquid oxygen is $\dot{q}$ = 69.4 W.

Resistance networks provide substantial insight into the problem. Figure 2 shows the magnitude of each of the resistances in the network. The resistances associated with conduction through the insulation, radiation from the surface of the dewar, and convection from the surface of the dewar are of the same order of magnitude and large relative to the others in the circuit. Conduction through the insulation is much more important than conduction through the metal liners, convection to the liquid oxygen or the contact resistance; these resistances can probably be neglected in a rough analysis and certainly very little effort should be expended to better understand these aspects of the problem.

Both radiation and convection from the outer surface are important, as they are of similar magnitude. The convection resistance is smaller and therefore more heat will be transferred by convection from the surface than is radiated. If the radiation resistance had been much larger than the convection resistance (as is often the case in forced convection problems where the convection coefficient is much larger) then radiation could be neglected. The smallest resistance in a parallel network will dominate the problem because most of the energy will tend to flow through that resistance.

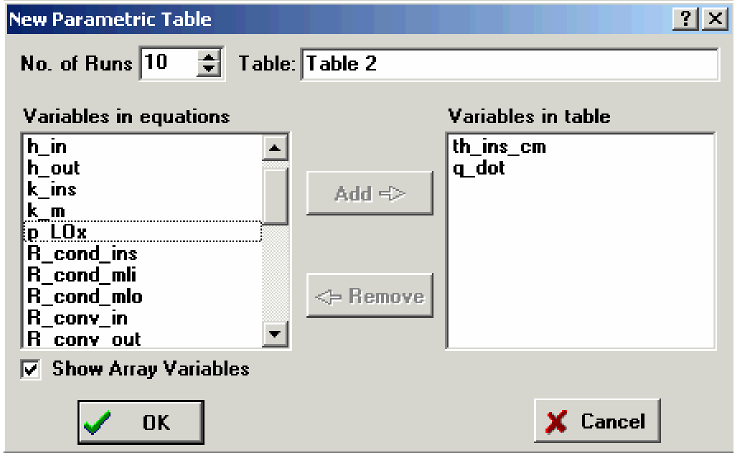
It is almost always a good idea to estimate the size of the resistances in a heat transfer problem prior to solving it; often it is possible to simplify the problem considerably and the size of the resistances can certainly be used to guide your efforts. For the problem here, a detailed analysis of conduction through the metal liner and the value of the thermal conductivity of the metal would be a misguided use of time whereas a more accurate measurement of the conductivity of the insulation would be very important.

c.) Plot the rate of heat transfer to the liquid oxygen as a function of the insulation thickness.

In order to generate the requested plot, it is necessary to parametrically vary the insulation thickness. The specified value of the insulation thickness is commented out

```
{th_ins_cm=1.0 [cm]}     "thickness of insulation, in cm"
```

and a parametric table is generated (select New Parametric Table from the Tables menu) that includes the variables `th_ins_cm` and `q_dot` (Figure 3).



New Parametric Table

No. of Runs 10

Table: Table 2

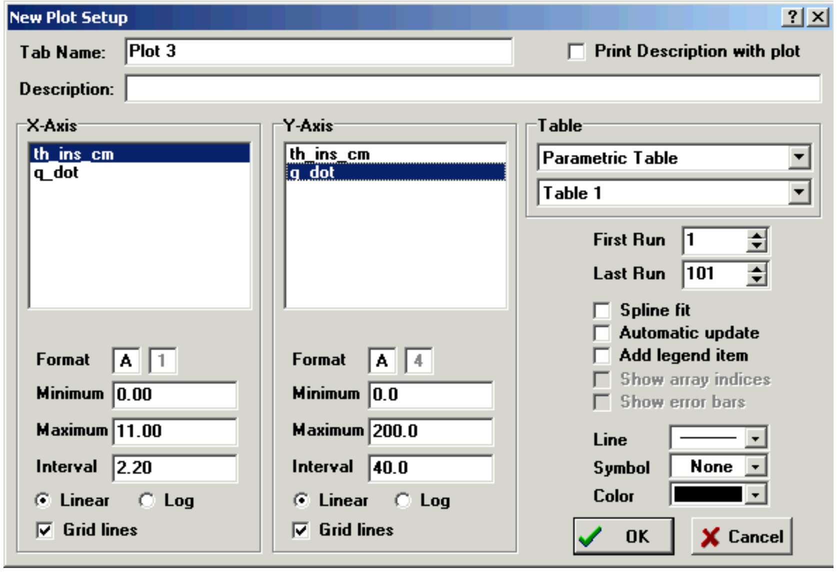
Variables in equations:
h_in
h_out
k_ins
k_m
p_LOx
R_cond_ins
R_cond_mli
R_cond_mlo
R_conv_in
R_conv_out

Variables in table:
th_ins_cm
q_dot

☑ Show Array Variables

Figure 3: New Parametric Table Window.

Right-click on the th_ins_cm column and select Alter Values; vary the thickness from 0 cm to 10 cm and solve the table (select Solve Table from the Calculate menu). Prepare a plot of the results (select New Plot Window from the Plots menu and then select X-Y Plot) by selecting the variable th_ins_cm for the X-Axis and q_dot for the Y-Axis (Figure 4).



New Plot Setup

Tab Name: Plot 3

☐ Print Description with plot

Description:

X-Axis
th_ins_cm
q_dot
Format A 1
Minimum 0.00
Maximum 11.00
Interval 2.20
● Linear ○ Log
☑ Grid lines

Y-Axis
th_ins_cm
q_dot
Format A 4
Minimum 0.0
Maximum 200.0
Interval 40.0
● Linear ○ Log
☑ Grid lines

Table: Parametric Table
Table 1

First Run 1
Last Run 101

☐ Spline fit
☐ Automatic update
☐ Add legend item
☐ Show array indices
☐ Show error bars

Line
Symbol None
Color

Figure 4: New Plot Setup Window.

Figure 5 illustrates the rate of heat transfer as a function of the insulation thickness.

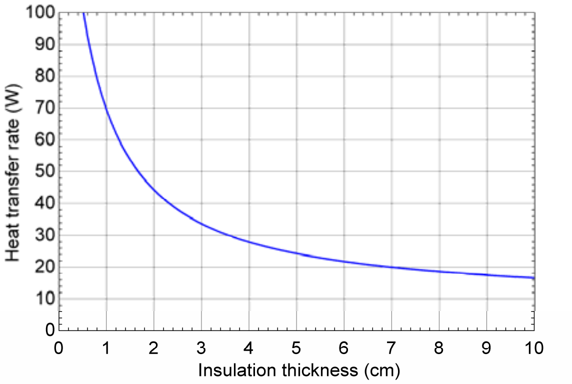


Figure 5: Heat transfer rate as a function of insulation thickness.