EXAMPLE 1.7-2: Drawing a Wire

Figure 1 illustrates a wire drawn from a die. The wire diameter is D = 0.5 mm. The temperature of the material at the exit of the die is T_{draw} = 600ºC and it has a draw velocity of u = 10 mm/s. The properties of the wire are ρ = 2700 kg/m^3, k = 230 W/m-K, and c = 1000 J/kg-K. The wire is surrounded by air at T_∞ = 20ºC with an average heat transfer coefficient of $\bar{h}$ = 25 W/m^2-K. The wire travels for L = 25 cm before entering a pool of water that is kept at T_w = 20ºC; you may assume that the water-to-wire heat transfer coefficient is very high so that the wire equilibrates essentially instantaneously with the water as it enters the pool.

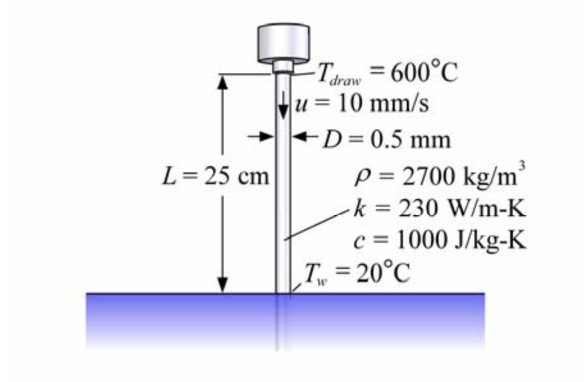

Figure 1: Wire drawn from a die.

a.) Develop an analytical model that can predict the temperature distribution in the wire.

The input parameters are entered in EES:

```
"EXAMPLE 1.7-2: Drawing a Wire"
$UnitSystem SI MASS  DEG PA C J
$Tabstops 0.2 0.4 0.6 0.8 3.5

"Inputs"
D=0.5 [mm]*convert(mm,m)                                    "diameter"
u=10 [mm/s]*convert(mm/s,m/s)                               "draw velocity"
c=1000 [J/kg-K]                                             "specific heat capacity"
k=230 [W/m-K]                                               "conductivity"
rho=2700 [kg/m^3]                                           "density"
h_bar=25 [W/m^2-K]                                          "heat transfer coefficient"
T_infinity=converttemp(C,K,20 [C])                          "air temperature"
T_draw=converttemp(C,K,600 [C])                             "draw temperature"
T_w=converttemp(C,K,20 [C])                                 "water temperature"
L=25 [cm]*convert(cm,m)                                     "length of wire"
```

The governing differential equation for a moving extended surface was derived in Section 1.7.3, Eq. (1-315):

$$\frac{d^2T}{dx^2} - \frac{u}{\alpha}\frac{dT}{dx} - m^2\,T = -m^2\,T_\infty$$

where α is the thermal diffusivity:

$$\alpha = \frac{k}{\rho\, c}$$

m is the fin constant:

$$m = \sqrt{\frac{\bar{h}\; per}{k\; A_c}}$$

and per and A_c are the perimeter and cross-sectional area, respectively, of the moving surface:

$$per = \pi\, D$$

$$A_c = \pi \frac{D^2}{4}$$

```
A_c=pi*D^2/4                                          "cross-sectional area"
per=pi*D                                              "perimeter"
alpha=k/(rho*c)                                       "thermal diffusivity"
m=sqrt(h_bar*per/(k*A_c))                             "fin parameter"
```

The general solution derived in Section 1.7.3 is:

$$T = C_1 \exp\left(\lambda_1\, x\right) + C_2 \exp\left(\lambda_2\, x\right) + T_\infty \tag{1}$$

where C_1 and C_2 are undetermined constants and:

$$\lambda_1 = \frac{u}{2\,\alpha} + \sqrt{\frac{1}{4}\left(\frac{u}{\alpha}\right)^2 + m^2}$$

$$\lambda_2 = \frac{u}{2\,\alpha} - \sqrt{\frac{1}{4}\left(\frac{u}{\alpha}\right)^2 + m^2}$$

```
lambda_1=u/(2*alpha)+sqrt((u/alpha)^2/4+m^2)          "solution parameter 1"
lambda_2=u/(2*alpha)-sqrt((u/alpha)^2/4+m^2)          "solution parameter 2"
```

The constants are evaluated using the boundary conditions. The temperatures at x = 0 and x = L are specified:

$$T_{x=L} = T_w \tag{2}$$

$$T_{x=0} = T_{draw} \tag{3}$$

Substituting Eq. (1) into Eqs. (2) and (3) leads to two algebraic equations for C_1 and C_2:

$$C_1 \exp\left(\lambda_1 L\right) + C_2 \exp\left(\lambda_2 L\right) + T_\infty = T_w$$

$$C_1 + C_2 + T_\infty = T_{draw}$$

which are entered in EES:

```
C_1*exp(lambda_1*L)+C_2*exp(lambda_2*L)+T_infinity=T_w      "boundary condition at x=L"
C_1+C_2+T_infinity=T_draw                                   "boundary condition at x=0"
```

The solution is evaluated in EES and converted to Celsius.

```
x=x_bar*L                                                   "position"
T=C_1*exp(lambda_1*x)+C_2*exp(lambda_2*x)+T_infinity        "solution"
T_C=converttemp(K,C,T)                                      "in C"
```

A parametric table in EES can be used to provide the temperature as a function of position. It is convenient to define the variable `x_bar`, the axial position normalized by the length of the wire. Including `x_bar` in the table and varying it from 0 to 1 is equivalent to varying the position from 0 to *L*. One advantage of using the variable `x_bar` is that as the length is changed, it is not necessary to adjust the Parametric table, only to re-run it. Figure 2 illustrates the temperature as a function of dimensionless position for various values of the length.

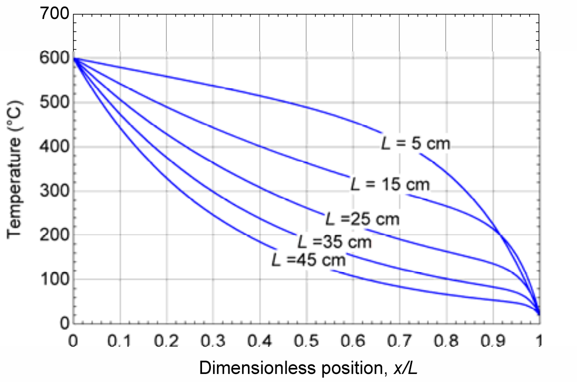

Figure 2: Temperature as a function of dimensionless position for various values of length.