

EXAMPLE 2.1-1: Magnetic Ablative Power Measurement

This example revisits the magnetic ablation concept that was previously described in EXAMPLES 1.3-1 and 1.8-2. You want to measure the power generated by the thermoseed that is used for the ablation process. Recall that the radius of the thermoseed is $r_{ts} = 1.0$ mm. The sphere is placed $W = 5$ cm below the surface of a solution of agar, as shown in Figure 1. Agar is a material with well-known thermal properties that resembles gelatin and is sometimes used as a surrogate for tissue in biological experiments. The agar is allowed to solidify around the sphere and the container of agar is large enough to be considered semi-infinite. The surface of the agar is exposed to an ice-water bath in order to keep it at a constant temperature, $T_{ice} = 0^\circ\text{C}$. The sphere is heated using an oscillating magnetic field and its surface temperature is measured using a thermocouple. The conductivity of agar is $k = 0.35$ W/m-K.

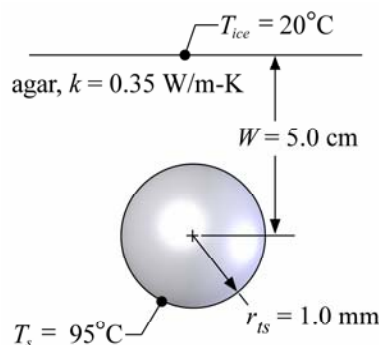


Figure 1: Test setup to measure the heat generated by magnetic ablation.

- a.) If the measured surface temperature of the sphere is $T_s = 95^\circ\text{C}$, how much energy is generated in the thermoseed?

The inputs are entered in EES:

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"EXAMPLE 2.1-1: Magnetic Ablative Power Measurement"
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in
```

"Inputs"

```
r_ts=1.0 [mm] *convert(mm,m)
W = 5 [cm]*convert(cm,m)
k = 0.35 [W/m-K]
T_ice=converttemp(C,K,0 [C])
T_s=converttemp(C,K,95 [C])
```

```
"thermoseed radius"
"depth of sphere"
"conductivity of agar"
"ice bath temperature"
"surface temperature"
```

The shape factor associated with the buried sphere (S) is accessed from the EES library of shape factors:

```
S=SF_1(2*r_ts,W) "shape factor for buried sphere"
```

The thermal resistance between the surface of the sphere and the semi-infinite body (R) is:

$$R = \frac{1}{k S}$$

The heat transfer rate (\dot{q}) is computed using the thermal resistance and the known temperatures:

$$\dot{q} = \frac{T_s - T_{ice}}{R}$$

R=1/(k*S) "thermal resistance"
 q_dot=(T_s-T_ice)/R "heat transfer rate"

which leads to a generation rate of 0.422 W.

- b.) Estimate the uncertainty in your measurement of the power. Assume that your temperature measurements (ice and thermoseed surface temperature) are accurate to $\delta T = 1.0^\circ\text{C}$, the conductivity of agar is known to within 10% (i.e., $\delta k = 0.035 \text{ W/m-K}$), the depth measurement has an uncertainty of $\delta W = 2.0 \text{ mm}$, and the sphere radius is known to within $\delta r_{ts} = 0.1 \text{ mm}$.

It is possible to separately estimate the uncertainty introduced by each of the parameters listed above. For example, to evaluate the effect of the uncertainty in the conductivity, simply increase the conductivity by δk :

deltak=0.035 [W/m-K] "uncertainty in conductivity"
 k = 0.35 [W/m-K] + deltak "conductivity of agar"

and re-run the model. The result is a change in the generation rate from 0.422 W to 0.464 W which translates into an uncertainty in the power of 0.042 W or 10%. This process can be repeated for each of the independent variables in order to identify the uncertainty that is introduced into the dependent variable calculation. These contributions should be combined using the root-sum-square technique in order to obtain an overall uncertainty. This process can be carried out automatically in EES; select Uncertainty Propagation from the Calculate Menu to access the dialog shown in Figure 2.

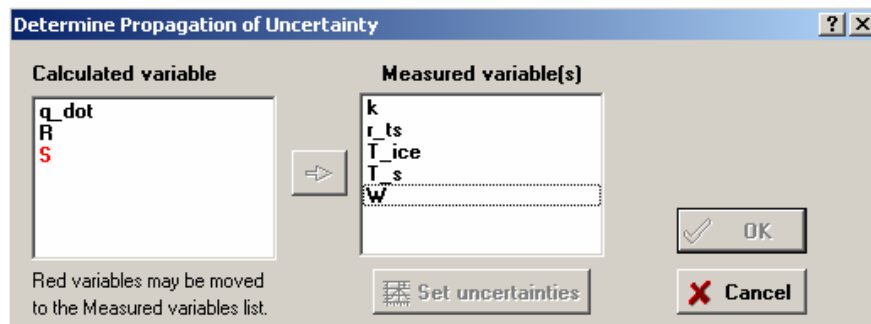


Figure 2: Propagation of uncertainty dialog.

The possible dependent (calculated) and independent (measured) variables are listed; highlight the independent variables that have some uncertainty (all of them for this problem). The calculated variable that you want to examine is $q_{\dot{}}$. Select the Set uncertainties button to reach the window shown in Figure 3. Set each of the uncertainties either in absolute or relative terms; note that each of these quantities can also be set using variables that are defined in the main equation window.

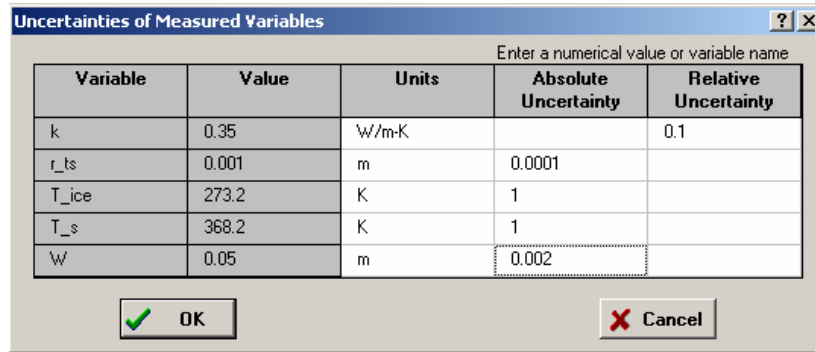


Figure 3: Uncertainty of measured variables dialog.

Select OK twice in order to initiate the calculations; the results appear the Uncertainty Results window (Figure 4) which shows the total uncertainty in the variable $q_{\dot{}}$ (0.06 W or 14%) as well as a delineation of the source of the uncertainty.

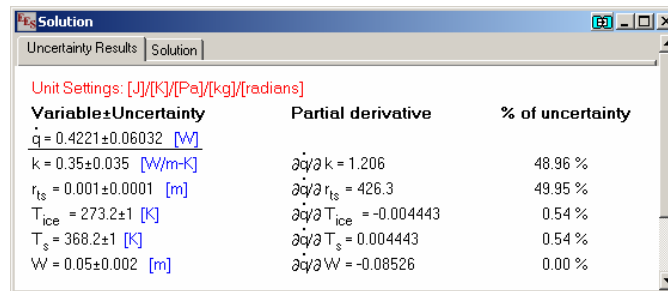


Figure 4: Uncertainty of Results window.

Notice that the dominant sources of uncertainty for this problem are the conductivity of agar and the radius of the sphere (each contributing approximately 50% of the total). The temperature measurements are adequate and the depth does not matter at all since the shape factor becomes nearly independent of the depth provided W is much larger than the diameter (see Eq. (2-5)).