EXAMPLE 2.8-1: Resistance of a Bracket

You may be faced with trying to understand the heat transfer through a complex, 2-D or 3-D geometry, such as the bracket illustrated in Figure 1. It is beyond the scope of any technique discussed in this book to analytically determine the heat flow through this geometry and therefore it will be necessary to use a finite element software package for this purpose. However, it is possible to use the resistance concept represented by Eq. (2-232) to bound and estimate the heat flow through the bracket. Doing so is a useful exercise for several reasons. If you determine that the heat flow cannot possibly be important to the larger application (whatever that is) then the time and money required to generate the finite element model can be saved. If a finite element model is generated, then the simple thermal resistance estimate can provide a sanity check on the results.

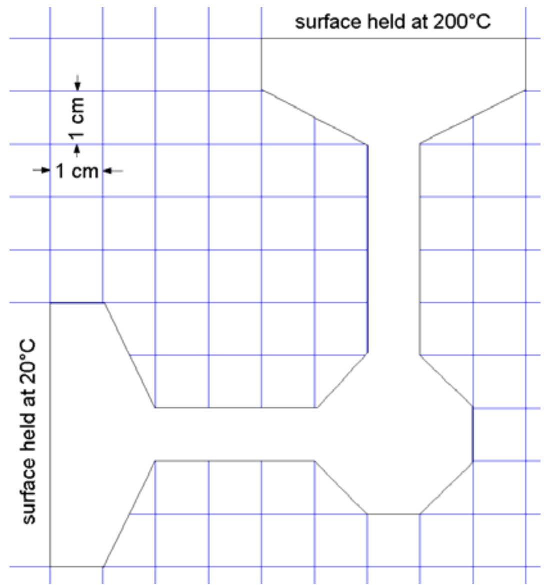

Figure 1: A bracket with a complex, 2-D geometry made of steel with *k* = 14 W/m-K and thickness 1 cm (into the page).

The bracket is made of steel having a thermal conductivity k = 14 W/m-K. One surface of the bracket is held at T_H = 200°C and the other is at T_C = 20°C.

a.) Estimate the rate of heat transfer through the bracket using a resistance approximation.

The length that the heat must be conducted in order to go from the surface at T_H to the surface at T_C is approximately L = 14 cm and the area for conduction is approximately A_c = 1 cm^2. Clearly these are not exact values because the problem is two-dimensional; some energy must flow a longer distance to reach the more proximal regions of the bracket and there are several portions of the bracket where the area is larger than 1 cm^2. However, it is possible to estimate the resistance of the bracket with these approximations:

$$R_{bracket} \approx \frac{L}{k\,A} = \frac{14\,\text{cm}}{}\left|\frac{\text{m K}}{14\text{ W}}\right|\frac{}{1\text{ cm}^2}\left\|\frac{100\text{ cm}}{\text{m}}\right. = 100\,\frac{\text{K}}{\text{W}}$$

which provides an estimate of the heat flow:

$$\dot{q} \approx \frac{\left(T_H - T_C\right)}{R_{bracket}} = \frac{\left(200^{\circ}\text{C} - 20^{\circ}\text{C}\right)}{}\left|\frac{\text{W}}{100\ \text{K}}\right. = 1.8\,\text{W}$$

It may be that 1.8 W is a trivial rate of energy loss from whatever is being supported by the bracket and therefore the bracket does not require a more detailed analysis. However, if a more exact answer is required then a finite element solution would be necessary.

b.) Use FEHT to determine the rate of heat transfer through the bracket.

The geometry from Figure 1 can be entered in FEHT; set a scale where 1 cm on the screen corresponds to 0.01 m and use the Outline selection from the Draw menu to approximately trace out the bracket. Then, right-click on each of the corner nodes and enter the exact position in the Node Information Dialog. The boundary conditions should be set as well as the material properties. Create a crude mesh and refine it.

The problem is solved and the solution is shown in Figure 2.

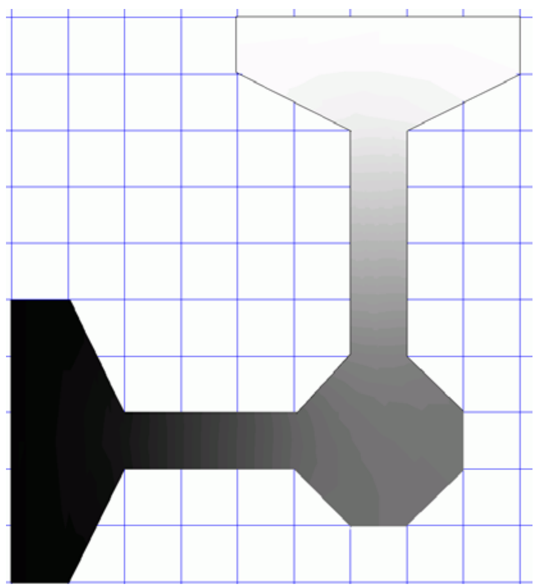

Figure 2: Solution

The total heat flux at either the 200°C or the 20°C boundaries can be obtained by selecting Heat Flows from the View menu and then selecting all of the nodal boundaries along these boundaries. (Left-click and drag a selection rectangle.) At the T_H boundary, the total heat flow is reported as 249.8 W/m (Figure 3) or, for a 1 cm thick bracket, 2.5 W.

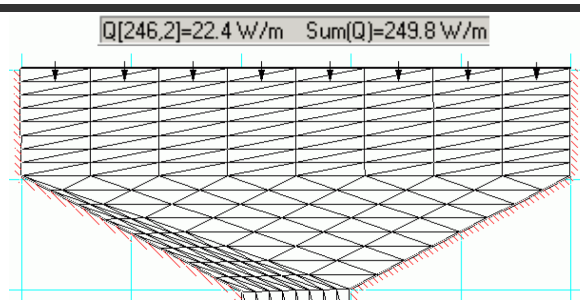

Figure 3: Heat flow along the top boundary.

The same calculation along the T_C boundary leads to 2.5 W. Therefore, the total heat flow is within 40% of the 1.8 W value predicted by the simple resistance approximation. The sanity check is valuable; if the finite element model had predicted 10's or 100's of W then it would be almost certain that there is an error in the one of the two solutions (perhaps a unit conversion or a material property entered incorrectly). Furthermore, the 1-D solution was an underestimate of the heat transfer because it did not account for the regions of the bracket that have larger cross-section. In the next sections, several methods are presented that can be used to bound the thermal resistance of a multi-dimensional object using 1-D resistances that are calculated with specific assumptions.