Counter-Flow Configuration

The plate heat exchanger operating in a counter-flow configuration, shown schematically in Figure 8-36, can also be solved using the sub-heat exchanger modeling approach. Because the sub-heat exchanger modeling methodology is necessarily iterative, the differences between the parallel- and counter-flow configurations are not significant. The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
W=35 [cm]*convert(cm,m)                    "width of heat exchanger"
L=70 [cm]*convert(cm,m)                    "length of heat exchanger in flow direction"
N_ch=100 [-]                               "number of channel pairs"
th_H=2.2 [mm]*convert(mm,m)                "channel width on hot-side"
th_C=2.2 [mm]*convert(mm,m)                "channel width on cold-side"
th_m=0.5 [mm]*convert(mm,m)                "thickness of plate"
p_H=7.5 [MPa]*convert(MPa,Pa)              "hot-side pressure"
p_C=100 [kPa]*convert(kPa,Pa)              "cold-side pressure"
m_dot_H=1.5 [kg/s]                         "hot-side mass flow rate"
m_dot_C=1.5 [kg/s]                         "cold-side mass flow rate"
H$='Air_ha'                                "hot-side fluid"
C$='Air_ha'                                "cold-side fluid"
T_H_in=300 [K]                             "hot-side inlet temperature"
T_C_in=90 [K]                              "cold-side inlet temperature"
```

The sub-heat exchanger model for the counter-flow configuration is shown in Figure 8-41.

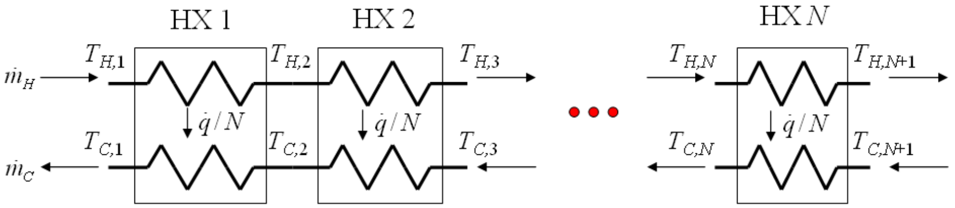

Figure 8-41: Sub-heat exchanger model of a counter-flow heat exchanger.

The technique proceeds using the same general steps discussed for a parallel-flow configuration. The hot-side exit temperature ($T_{H,out}$) is assumed

```
T_H_out=250 [K]          "assumed outlet temperature"
```

and used to compute the total heat transfer rate in the heat exchanger:

$$\dot{q} = \dot{m}_H \left(i_{H,in} - i_{H,out} \right) \tag{8-176}$$

where $i_{H,in}$ and $i_{H,out}$ are the specific enthalpy of the hot fluid at the inlet and outlet states, respectively, calculated using EES' internal property routine:

```
i_H_in=enthalpy(H$,T=T_H_in,P=p_H)                "enthalpy of hot inlet fluid"
i_H_out=enthalpy(H$,T=T_H_out,P=p_H)              "enthalpy of hot outlet fluid"
```

```
q_dot=m_dot_H*(i_H_in-i_H_out)                                    "total heat transfer rate"
```

The specific enthalpy of the cold fluid leaving the heat exchanger ($i_{C,out}$) is computed from an energy balance:

$$i_{C,out} = i_{C,in} + \frac{\dot{q}}{\dot{m}_C} \tag{8-177}$$

where $i_{C,in}$ is the specific enthalpy of the cold fluid entering the heat exchanger. The cold-side outlet temperature ($T_{C.out}$) is obtained using EES' internal property routine:

```
i_C_in=enthalpy(C$,T=T_C_in,P=p_C)                               "enthalpy of cold inlet fluid"
i_C_out=i_C_in+q_dot/m_dot_C                                     "enthalpy of cold outlet fluid"
T_C_out=temperature(C$,h=i_C_out,P=p_C)                          "temperature of cold outlet fluid"
```

The total heat transferred in the heat exchanger increases as you move from left-to-right in Figure 8-41 according to:

$$\dot{q}_i = \frac{\dot{q}}{N}\left(i-1\right) \quad \text{for } i=1..\left(N+1\right) \tag{8-178}$$

```
N=10 [-]                                                         "number of sub-heat exchangers"
duplicate i=1,N
   q_dot[i]=i*q_dot/N                                            "total heat transfer rate"
end
```

The temperatures $T_{H,1}$ and $T_{C,1}$ are the hot-side inlet and cold-side outlet fluid temperatures, respectively, for the counter-flow configuration. The enthalpies $i_{H,1}$ and $i_{C,1}$ are the associated specific enthalpies associated with these states.

```
"Obtain temperature distribution"
T_H[1]=T_H_in                                                    "hot-side inlet temperature"
T_C[1]=T_C_out                                                   "cold-side outlet temperature"
i_H[1]=i_H_in                                                    "hot_side inlet enthalpy"
i_C[1]=i_C_out                                                   "cold-side outlet enthalpy"
```

An energy balance on the hot-side provides the enthalpy leaving each of the sub-heat exchangers:

$$i_{H,i+1} = i_{H,i} - \frac{\dot{q}}{N\,\dot{m}_H} \quad \text{for } i=1..N \tag{8-179}$$

The temperature of the hot-side fluid leaving each sub-heat exchanger ($T_{H,i}$) is obtained from the enthalpy and pressure using EES' internal property routine:

```
duplicate i=2,(N+1)
   i_H[i]=i_H[i-1]-q_dot/(N*m_dot_H)          "energy balance on hot-side of each sub-heat exchanger"
   T_H[i]=temperature(H$,h=i_H[i],P=p_H)  "temperature leaving hot-side of each sub-heat exchanger"
```

```
end
```

An energy balance on the cold-side provides the enthalpy leaving each of the sub-heat exchangers:

$$i_{C,i+1} = i_{C,i} - \frac{\dot{q}}{N\,\dot{m}_C} \quad \text{for } i = 1..N \tag{8-180}$$

Notice the change of sign in Eq. (8-180) as compared to Eq. (8-165) due to the change in the direction of the flow. The temperature of the cold-side fluid leaving each sub-heat exchanger ($T_{C,i}$) is obtained from the enthalpy and pressure using EES' internal property routine:

```
duplicate i=2,(N+1)
    i_C[i]=i_C[i-1]-q_dot/(N*m_dot_C)         "energy balance on cold-side of each sub-heat exchanger"
    T_C[i]=temperature(C$,h=i_C[i],P=p_C)  "temperature leaving cold-side of each sub-heat exchanger"
end
```

The ε-*NTU* solution is applied to each of the sub-heat exchangers. The capacitance rates on the hot- and cold-side within each sub-heat exchanger are calculated:

$$\dot{C}_{H,i} = \dot{m}_H \frac{\left(i_{H,i} - i_{H,i+1}\right)}{\left(T_{H,i} - T_{H,i+1}\right)} \quad \text{for } i = 1..N \tag{8-181}$$

$$\dot{C}_{C,i} = \dot{m}_C \frac{\left(i_{C,i} - i_{C,i+1}\right)}{\left(T_{C,i} - T_{C,i+1}\right)} \quad \text{for } i = 1..N \tag{8-182}$$

```
"Apply effectiveness-NTU solution"
duplicate i=1,N
    C_dot_H[i]=m_dot_H*(i_H[i]-i_H[i+1])/(T_H[i]-T_H[i+1])          "hot-side capacitance rate"
    C_dot_C[i]=m_dot_C*(i_C[i]-i_C[i+1])/(T_C[i]-T_C[i+1])          "cold-side capacitance rate"
end
```

The effectiveness of each sub-heat exchanger can be computed:

$$\varepsilon_i = \frac{\dot{q}/N}{\text{MIN}\left(\dot{C}_{C,i}, \dot{C}_{H,i}\right)\left(T_{H,i} - T_{C,i+1}\right)} \quad \text{for } i = 1..N \tag{8-183}$$

Note that the cold-side inlet temperature is $T_{C,i+1}$ (rather than $T_{C,i}$ as it was in Eq. (8-170) for the parallel-flow configuration). The number of transfer units required by each sub-heat exchanger (NTU_i) is obtained using the ε-*NTU* solution for a counter-flow heat exchanger, implemented by the function `HX`. The conductance required in each sub-heat exchanger is:

$$UA_i = NTU_i\, \text{MIN}\left(\dot{C}_{C,i}, \dot{C}_{H,i}\right) \tag{8-184}$$

```
duplicate i=1,N
```

```
    eff[i]=q_dot/(N*MIN(C_dot_H[i],C_dot_C[i])*(T_H[i]-T_C[i+1]))  "effectiveness of sub-heat exchanger"
    NTU[i]=HX('counterflow', eff[i], C_dot_H[i], C_dot_C[i], 'NTU') "NTU required by sub-heat exchanger"
    UA[i]=NTU[i]*MIN(C_dot_H[i],C_dot_C[i])                         "conductance in sub-heat exchanger"
end
```

The conductance for each of the sub-heat exchangers is translated into its physical size based on the geometry and operating conditions. The physical distance, Δx_i, that each sub-heat exchanger occupies is:

$$\Delta x_i = \frac{UA_i}{2\,N_{ch}\,W}\left(\frac{1}{h_{C,i}}+\frac{th_m}{k_{m,i}}+\frac{1}{h_{H,i}}\right) \quad \text{for } i=1..N \tag{8-185}$$

The local heat transfer coefficients are obtained using the DuctFlow_local function. Notice that average values of temperature are used and that the local heat transfer coefficient for the hot-side is evaluated at x_i whereas the local heat transfer coefficient for the cold-side is evaluated at x_{N+1} - x_i because the cold-side flow has had this length to develop. The ending position of each sub-heat exchanger is:

$$x_{i+1} = x_i + \Delta x_i \quad \text{for } i=1..N \tag{8-186}$$

```
"determine length of each sub-heat exchanger"
x[1]=0 [m]                                          "starting position of 1st sub-heat exchanger"
duplicate i=1,N
    call DuctFlow_local(H$,(T_H[i]+T_C[i])/2,p_H,m_dot_H/N_ch,th_H,W,x[i]+0.001 [m],0 [-]: &
        h_H[i], h_H_H[i], dPHdx[i])                 "hot-side local heat transfer coefficient"
    call DuctFlow_local(C$,(T_H[i]+T_C[i])/2,p_C,m_dot_C/N_ch,th_C,W,x[N+1]-x[i]+0.001 [m],0 [-]: &
        h_C[i], h_C_H[i], dPCdx[i])                 "cold-side local heat transfer coefficient"
    k_m[i]=k_('Aluminum', (T_H[i]+T_C[i])/2)        "metal conductivity at local average temperature"
    DELTAx[i]=UA[i]*(1/h_H[i]+th_m/k_m[i]+1/h_C[i])/(2*N_ch*W) "length of sub-heat exchanger"
    x[i+1]=x[i]+DELTAx[i]
end
```

The error between the predicted and specified heat exchanger length is defined:

$$err = \frac{\left|x_{N+1} - L\right|}{L} \tag{8-187}$$

```
err=abs(x[N+1]-L)/L      "objective function"
```

The value of the variable err is minimized by adjusting the variable T_H_out. The temperature of the streams as a function of position is shown in Figure 8-42; the solution is consistent with the numerical solution discussed in Section 8.6.2 and shown in Figure 8-37.

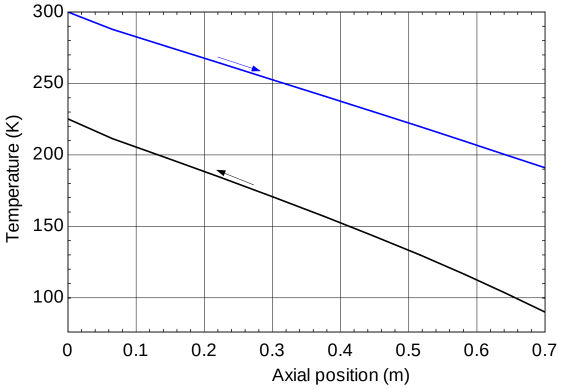

Figure 8-42: Temperature distribution for the plate heat exchanger in a counter-flow configuration.