Both Fluids Mixed

Figure E21-1 illustrates a cross-flow heat exchanger in which both the hot and cold fluids are mixed. In this situation, the temperature variation associated with either fluid is 1-D. The temperature of the hot fluid varies only in the *y*-direction (the direction of hot flow) and the temperature of the cold fluid varies only in the *x*-direction (the direction of cold flow). Therefore, a single hot fluid temperature node ($T_{H\,j}$) represents the hot fluid temperature at each *y*-location and a single cold fluid temperature node ($T_{C\,i}$) represents the cold fluid temperature at each *x*-location.

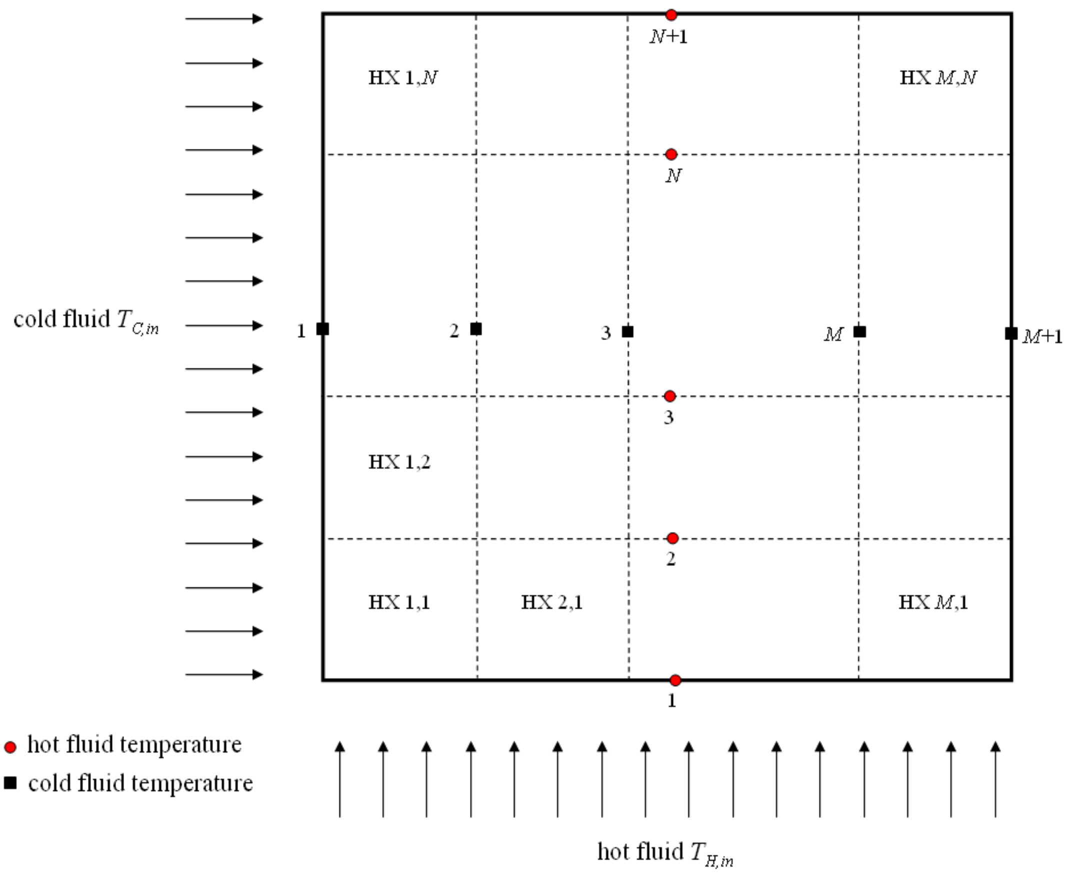

Figure E21-1: Cross-flow heat exchanger with both fluids mixed.

The solution will be derived in the context of the constant property situation previously discussed in Section 8.9.2 for the case when both fluids are unmixed. The hot fluid enters with temperature $T_{H,in}$ = 400 K with mass flow rate $\dot{m}_H$ = 0.05 kg/s and specific heat capacity c_H = 1005 J/kg-K. The cold fluid enters with temperature $T_{C,in}$ = 300 K with mass flow rate $\dot{m}_C$ = 0.05 kg/s and specific heat capacity c_C = 1005 J/kg-K. The total conductance of the heat exchanger is UA = 200 W/K. These inputs are entered in a script:

```
clear all;
m_dot_H=0.05;          %hot-side mass flow rate (kg/s)
m_dot_C=0.05;          %cold-side mass flow rate (kg/s)
c_H=1005;              %hot-side specific heat capacity (J/kg-K)
```

```
c_C=1005;              %cold-side specific heat capacity (J/kg-K)
T_H_in=400;            %hot-side inlet temperature (K)
T_C_in=300;            %cold-side inlet temperature (K)
UA=200;                %conductance (W/K)
```

The dimensionless positions of the hot-side and cold-side fluid nodes are positioned according to Eqs. (8-308) through (8-311):

```
M=20;                       %number of grids in the cold-flow direction (x)
N=20;                       %number of grids in the hot-flow direction (y)

%grid locations for hot-side temperatures
for i=1:M
   for j=1:(N+1)
      x_H(i,j)=(i-1/2)/M;
      y_H(i,j)=(j-1)/N;
   end
end

%grid locations for cold-side temperatures
for i=1:(M+1)
   for j=1:N
      x_C(i,j)=(i-1)/M;
      y_C(i,j)=(j-1/2)/N;
   end
end
```

Even though T_H and T_C vary only in the *y*- and *x*-directions, respectively, the nodes are distributed throughout the heat exchanger in order to facilitate plotting the temperature distributions.

The cross-flow heat exchanger is governed by energy balances on the hot fluid passing through each of the N horizontal rows of heat exchangers (HX 1,j through HX M,j, as shown in E21-2) and energy balances on the cold fluid passing through each of the M vertical columns of heat exchangers (HX i,1 through HX i,N, also shown in Figure E21-2). The hot fluid energy balances are expressed as:

$$\dot{m}_H \, c_H \left(T_{H\,j} - T_{H\,j+1}\right) = \frac{UA}{M\;N} \sum_{i=1}^{M} \left[\frac{\left(T_{H\,j} + T_{H\,j+1}\right)}{2} - \frac{\left(T_{C\,i} + T_{C\,i+1}\right)}{2} \right] \text{ for } j = 1..N \qquad \text{(E21-1)}$$

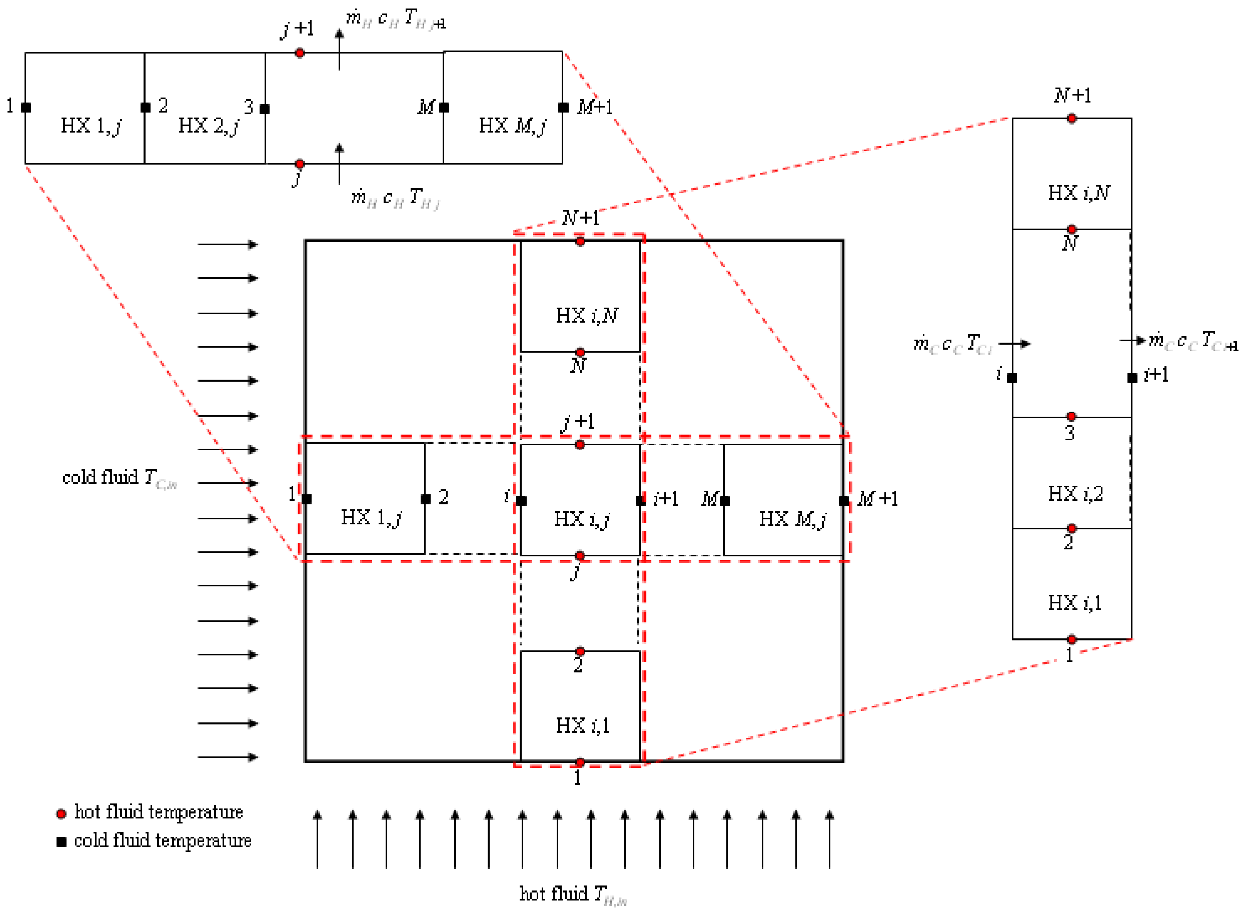

Figure E21-2: Energy balances used for the cross-flow heat exchanger where both fluids are mixed.

The cold fluid energy balances for each vertical column of heat exchangers are expressed as:

$$\dot{m}_C\, c_C \left(T_{C\,i+1} - T_{C\,i}\right) = \frac{UA}{M\,N} \sum_{j=1}^{N} \left[\frac{\left(T_{H\,j} + T_{H\,j+1}\right)}{2} - \frac{\left(T_{C\,i} + T_{C\,i+1}\right)}{2} \right] \quad \text{for } i = 1..M \tag{E21-2}$$

The boundary condition on the hot-side fluid temperature is:

$$T_{H1} = T_{H,in} \tag{E21-3}$$

and the boundary condition on the cold-side fluid temperature is:

$$T_{C1} = T_{C,in} \tag{E21-4}$$

Equations (E21-1) through (E21-4) represent $M\,N + 2$ equations in an equal number of unknown temperatures. In order to solve these equation in MATLAB, they must be placed in matrix format:

$$\underline{\underline{A}}\,\underline{X} = \underline{b} \tag{E21-5}$$

where $\underline{X}$ is a vector containing the unknown temperatures. One strategy for organizing the unknown temperatures in $\underline{X}$ is:

$$\underline{X} = \begin{bmatrix} T_{H\,1} \\ T_{H\,2} \\ \ldots \\ T_{H\,N+1} \\ T_{C1} \\ \ldots \\ T_{C\,M+1} \end{bmatrix} \tag{E21-6}$$

According to Eq. (E21-6), $T_{H\,j}$ is entry j of $\underline{X}$ and $T_{C\,i}$ is entry (N+1) + i of $\underline{X}$. The governing equations must be placed into the rows of the matrix $\underline{\underline{A}}$. One strategy for organizing these equations is:

$$\underline{\underline{A}} = \begin{bmatrix} \text{hot fluid energy balance for row HX 1,1 through HX } M\text{,1} \\ \ldots \\ \text{hot fluid energy balance for row HX 1,}N\text{ through HX } M, N \\ \text{cold fluid energy balance for column HX 1,1 through HX 1:}N \\ \ldots \\ \text{cold fluid energy balance for column HX } M\text{,1 through HX } M, N \\ \text{hot fluid boundary condition for } T_{H1} \\ \text{cold fluid boundary condition for } T_{C1} \end{bmatrix} \tag{E21-7}$$

According to Eq. (E21-7), the hot-side energy balance for the row of heat exchangers corresponding to HX 1,j through HX M,j is placed in row j of $\underline{\underline{A}}$ and the cold-side energy balance for the column of heat exchangers corresponding to *HX i*,1 through *HX i*,N is placed in row $N + i$ of $\underline{\underline{A}}$. The hot fluid boundary condition for node $T_{H\,1}$ corresponds to row $M\,N + 1$ of $\underline{\underline{A}}$ and the cold fluid boundary condition for node $T_{C\,1}$ corresponds to row $M\,N + 2$ of $\underline{\underline{A}}$.

Matrix $\underline{\underline{A}}$ and vector $\underline{b}$ are initialized. Note that matrix $\underline{\underline{A}}$ is defined as a sparse matrix and the number of non-zero coefficients is estimated by inspection of Eqs. (E21-1) and (E21-2).

```
%initialize matrices
A=spalloc(M+N+2,M+N+2,N*(M+3)+M*(N+3)+2);
b=zeros(M+N+2,1);
```

Equations (E21-1) through (E21-4) are rearranged so that the coefficients multiplying the unknown temperatures are clear. The hot fluid energy balances, Eq. (E21-1), become:

$$T_{H\,j}\underbrace{\left[\dot{m}_H\,c_H-\frac{UA}{2\,N}\right]}_{A_{j,j}}+T_{H\,j+1}\underbrace{\left[-\dot{m}_H\,c_H-\frac{UA}{2\,N}\right]}_{A_{j,j+1}}+$$
$$T_{C1}\underbrace{\left[\frac{UA}{2M\,N}\right]}_{A_{j,(N+1)+1}}+\sum_{i=2}^{M}T_{C\,i}\underbrace{\left[\frac{UA}{M\,N}\right]}_{A_{j,(N+1)+i}}+T_{C\,M+1}\underbrace{\left[\frac{UA}{2M\,N}\right]}_{A_{j,(N+1)+M+1}}=0 \qquad \text{(E21-8)}$$
$$\text{for } j=1..N$$

```
%hot-side energy balances
for j=1:N
    A(j,j)=m_dot_H*c_H-UA/(2*N);
    A(j,j+1)=-m_dot_H*c_H-UA/(2*N);
    A(j,N+1+1)=UA/(2*M*N);
    for i=2:M
        A(j,N+1+i)=UA/(M*N);
    end
    A(j,N+1+M+1)=UA/(2*M*N);
end
```

The cold fluid energy balances, Eq. (E21-2), become:

$$T_{C\,i+1}\underbrace{\left[\dot{m}_C\,c_C+\frac{UA}{2\,M}\right]}_{A_{N+i,N+1+i+1}}+T_{C\,i}\underbrace{\left[-\dot{m}_C\,c_C+\frac{UA}{2\,M}\right]}_{A_{N+i,N+1+i}}+$$
$$T_{H1}\underbrace{\left[-\frac{UA}{2M\,N}\right]}_{A_{N+i,1}}+\sum_{j=2}^{N}T_{H\,j}\underbrace{\left[-\frac{UA}{M\,N}\right]}_{A_{N+i,j}}+T_{H\,N+1}\underbrace{\left[-\frac{UA}{2M\,N}\right]}_{A_{N+i,N+1}}=0 \qquad \text{(E21-9)}$$
$$\text{for } i=1..M$$

```
%cold-side energy balances
for i=1:M
    A(N+i,N+1+i+1)=m_dot_C*c_C+UA/(2*M);
    A(N+i,N+1+i)=-m_dot_C*c_C+UA/(2*M);
    A(N+i,1)=-UA/(2*M*N);
    for j=2:N
        A(N+i,j)=-UA/(M*N);
    end
    A(N+i,N+1)=-UA/(2*M*N);
end
```

The hot-side boundary condition, Eq. (E21-3), becomes:

$$T_{H1} \underbrace{[1]}_{A_{M\,N+1,1}} = \underbrace{T_{H,in}}_{b_{M\,N+1}} \quad \text{(E21-10)}$$

```
%hot-side inlet fluid temperature boundary condition
A(M*N+1,1)=1;
b(M*N+1,1)=T_H_in;
```

The cold-side boundary condition, Eq. (E21-4), becomes:

$$T_{C1} \underbrace{[1]}_{A_{M\,N+2,N+1+1}} = \underbrace{T_{C,in}}_{b_{M\,N+2}} \quad \text{(E21-11)}$$

```
%cold-side inlet fluid temperature boundary condition
A(M*N+2,N+1+1)=1;
b(M*N+2,1)=T_C_in;
```

The solution is obtained:

```
X=A\b;
for i=1:M
    for j=1:(N+1)
        T_H(i,j)=X(j);
    end
end
for i=1:(M+1)
    for j=1:N
        T_C(i,j)=X(N+1+i);
    end
end
```

Figure E21-3 illustrates the temperature distribution for the hot and the cold fluid within the heat exchanger; notice that the temperature of the cold fluid is constant in the *y*-direction and the hot fluid is constant in the *x*-direction. Also notice that the temperatures actually cross within the heat exchanger; that is, towards the hot fluid exit, the hot fluid is simultaneously being heated by the cold fluid near the cold exit and cooled by the cold fluid near the cold inlet; the net heat transfer rate to the hot fluid is near zero and therefore it ceases to change temperature in the flow direction. This behavior implies that the energy obtained near the cold fluid exit is transferred by mixing and lost near the cold fluid inlet. This counter-intuitive result agrees with the published ε-*NTU* solutions, does not violate the second law of thermodynamics, and is consistent with the assumptions underlying a completely mixed flow. However, it is questionable how well such a situation represents any real heat exchanger.

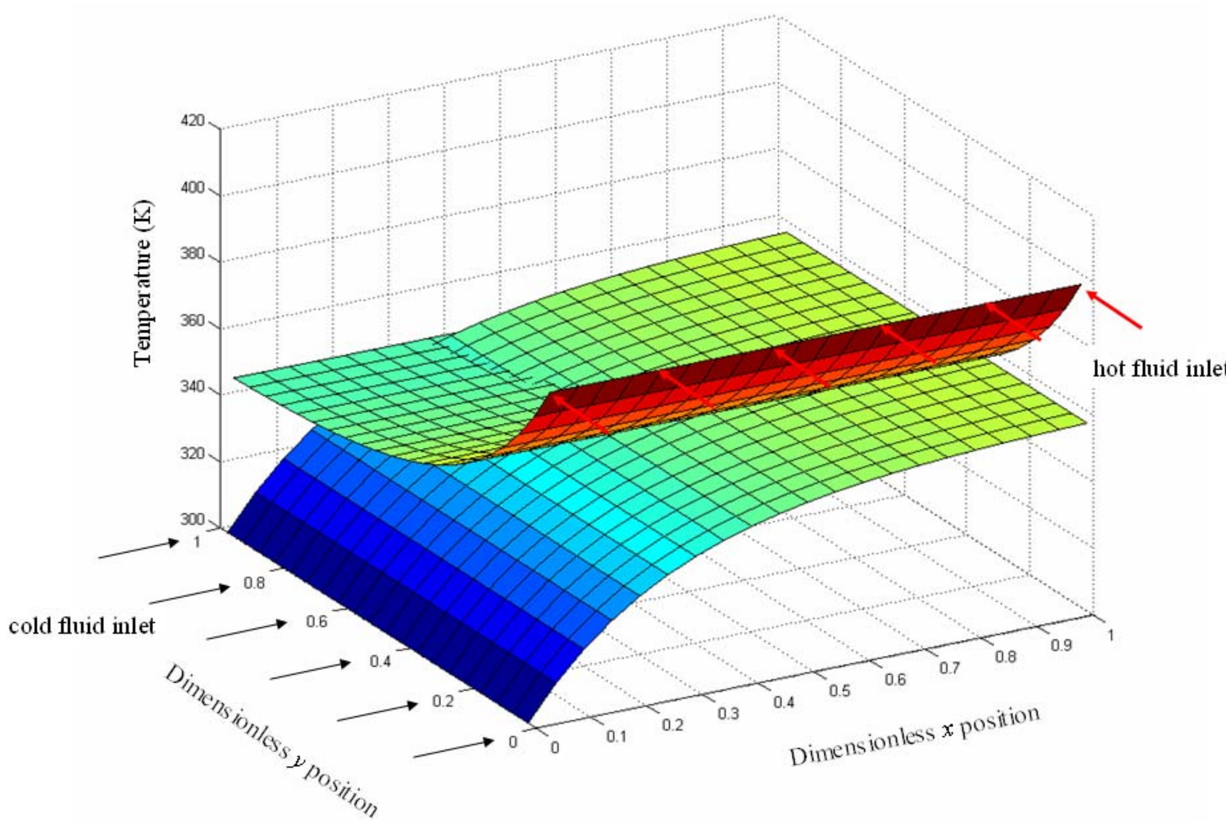

Figure E21-3: Temperature distribution associated with the hot- and cold-fluids for a cross-flow heat exchanger with both fluids mixed.

The total heat transfer rate from the hot fluid may be computed according to:

$$\dot{q}_H = \dot{m}_H \, c_H \left(T_{H,in} - T_{H\,N+1} \right) \qquad \text{(E21-12)}$$

```
%total hot-side heat transfer rate
q_dot_H=m_dot_H*c_H*sum(T_H_in-T_H(:,N+1))/M;
```

The total heat transfer rate to the cold fluid is:

$$\dot{q}_C = \dot{m}_C \, c_C \left(T_{C\,M+1} - T_{C,in} \right) \qquad \text{(E21-13)}$$

```
%total cold-side heat transfer rate
q_dot_C=m_dot_C*c_C*sum(T_C(M+1,:)-T_C_in)/N;
```

which leads to $\dot{q}_H$ = 2.813 kW and $\dot{q}_C$ =2.813 kW. The heat transfer rate in the mixed/mixed configuration is less than the mixed/unmixed or unmixed/unmixed configurations. The numerical solution can be compared directly with the ε-*NTU* solution discussed in Section 8.3. The minimum and maximum capacitance rates, $\dot{C}_{min}$ and $\dot{C}_{max}$, are computed:

```
C_dot_min=min([m_dot_H*c_H,m_dot_C*c_C]);          %minimum capacitance rate
```

```
C_dot_max=max([m_dot_H*c_H,m_dot_C*c_C]);          %maximum capacitance rate
```

The maximum possible rate of heat transfer is:

$$\dot{q}_{max} = \dot{C}_{min}\left(T_{H,in} - T_{C,in}\right) \tag{E21-14}$$

and the effectiveness of the heat exchanger, predicted by the numerical model, is:

$$\varepsilon = \frac{\dot{q}_H}{\dot{q}_{max}} \tag{E21-15}$$

```
q_dot_max=C_dot_min*(T_H_in-T_C_in);       %maximum possible capacitance rate
eff=q_dot_H/q_dot_max;                     %effectiveness
```

which leads to ε = 0.5598. The effectiveness of the mixed/mixed configuration is less than the effectiveness of either the mixed/unmixed and unmixed/unmixed configurations.

The number of transfer units associated with the heat transfer is computed according to:

$$NTU = \frac{UA}{\dot{C}_{min}} \tag{E21-16}$$

The capacity ratio is:

$$C_R = \frac{\dot{C}_{min}}{\dot{C}_{max}} \tag{E21-17}$$

```
%eff-NTU solution
NTU=UA/C_dot_min;                          %number of transfer units
C_R=C_dot_min/C_dot_max;                   %capacitance ratio
```

The effectiveness predicted by the ε-NTU solution for a cross-flow heat exchanger with both fluids mixed ($\varepsilon_{crossflow\text{-}mixed/mixed}$) is obtained using the formula listed in Table 8-1:

$$\varepsilon = \left[\frac{1}{1-\exp\left(-NTU\right)} + \frac{C_R}{1-\exp\left(-C_R\,NTU\right)} - \frac{1}{NTU}\right]^{-1} \tag{E21-18}$$

```
eff_crossflowmixedmixed=(1/(1-exp(-NTU))+C_R/(1-exp(-C_R*NTU))-1/NTU)^(-1);
```

which leads to $\varepsilon_{crossflow\text{-}mixed/mixed}$ = 0.5597, within 0.02% of the numerical solution.