One Fluid Mixed, One Fluid Unmixed

Figure 8-62 illustrates a cross-flow heat exchanger in which the cold fluid is mixed whereas the hot fluid remains unmixed. In this situation, the temperature distribution associated with the hot fluid remains 2-D but the temperature distribution associated with the cold fluid becomes 1-D because mixing prevents any temperature gradient in the *y*-direction (i.e., the direction perpendicular to the flow). Therefore, a single cold fluid temperature node ($T_{C\,i}$) represents the temperature each *x*-location for all associated *y*-positions.

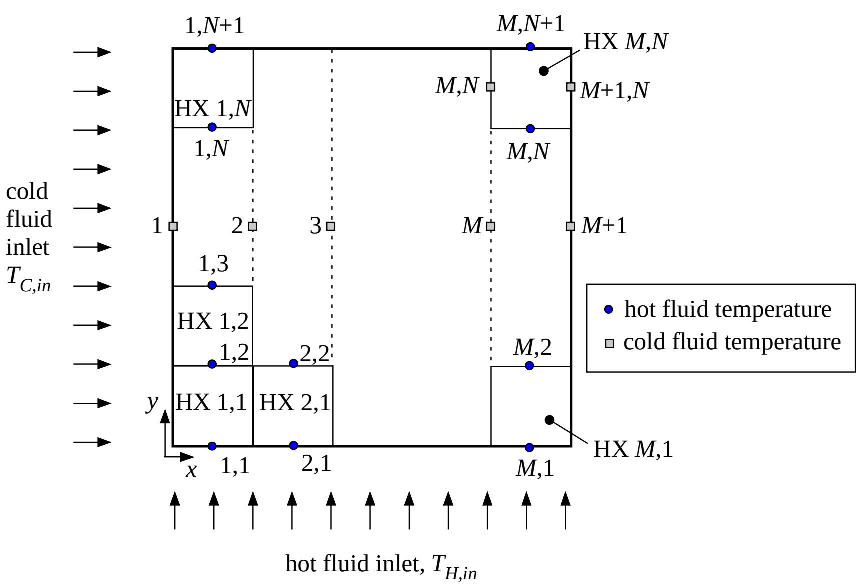

Figure 8-62: Cross-flow heat exchanger with the cold fluid mixed and the hot fluid unmixed.

The solution will be derived in the context of the constant property situation discussed previously. The hot fluid enters with temperature $T_{H,in}$ = 400 K, mass flow rate $\dot{m}_H$ = 0.05 kg/s and has specific heat capacity c_H = 1005 J/kg-K. The cold fluid enters with temperature $T_{C,in}$ = 300 K, mass flow rate $\dot{m}_C$ = 0.05 kg/s and has specific heat capacity c_C = 1005 J/kg-K. The total conductance of the heat exchanger is UA = 200 W/K. These inputs are entered in a MATLAB script:

```
clear all;
m_dot_H=0.05;          % hot-side mass flow rate (kg/s)
m_dot_C=0.05;          % cold-side mass flow rate (kg/s)
c_H=1005;              % hot-side specific heat capacity (J/kg-K)
c_C=1005;              % cold-side specific heat capacity (J/kg-K)
T_H_in=400;            % hot-side inlet temperature (K)
T_C_in=300;            % cold-side inlet temperature (K)
UA=200;                % conductance (W/K)
```

The dimensionless positions of the hot-side fluid nodes are:

$$\tilde{x}_{H\,i,j} = \frac{1}{M}\left(i - \frac{1}{2}\right) \quad \text{for } i = 1..M \quad \text{for } j = 1..\left(N+1\right) \tag{8-306}$$

$$\tilde{y}_{H\,i,j} = \frac{1}{N}\left(j-1\right) \text{ for } i = 1..M \text{ for } j = 1..\left(N+1\right) \tag{8-307}$$

```
M=20;        % number of grids in the cold-flow direction, x (-)
N=20;        % number of grids in the hot-flow direction, y (-)

% grid locations for hot-side temperatures
for i=1:M
    for j=1:(N+1)
        x_H(i,j)=(i-1/2)/M;
        y_H(i,j)=(j-1)/N;
    end
end
```

The dimensionless positions of the cold-side fluid nodes are given by:

$$\tilde{x}_{C\,i,j} = \frac{1}{M}\left(i-1\right) \text{ for } i = 1..\left(M+1\right) \text{ for } j = 1..N \tag{8-308}$$

$$\tilde{y}_{C\,i,j} = \frac{1}{N}\left(j-\frac{1}{2}\right) \text{ for } i = 1..\left(M+1\right) \text{ for } j = 1..N \tag{8-309}$$

Note that even though there is only one cold fluid temperature computed at each x location; the coordinates for the cold fluid are still distributed in both dimensions in order to enable the cold fluid temperature distribution to be plotted once the solution is obtained. That is, all of the cold fluid temperatures at each value of $\tilde{y}_{C\,i,j}$ for any value of i will be the same; however, in order to generate a plot similar to Figure 8-59 for the mixed/unmixed configuration it is necessary to place nodes over the entire 2-D area.

```
% grid locations for cold-side temperatures
for i=1:(M+1)
    for j=1:N
        x_C(i,j)=(i-1)/M;
        y_C(i,j)=(j-1/2)/N;
    end
end
```

The cross-flow heat exchanger is governed by energy balances on the hot fluid passing through each of the $M\,N$ heat exchanger control volumes and cold fluid energy balances on each of the M vertical columns of heat exchangers. The hot fluid energy balance for an arbitrary heat exchanger control volume (HX *i,j*, shown in Figure 8-63) can be expressed as:

$$\frac{\dot{m}_H\, c_H}{M}\left(T_{H\,i,j} - T_{H\,i,j+1}\right) = \frac{UA}{M\,N}\left[\frac{\left(T_{H\,i,j} + T_{H\,i,j+1}\right)}{2} - \frac{\left(T_{C\,i} + T_{C\,i+1}\right)}{2}\right] \text{ for } i = 1..M,\ j = 1..N \tag{8-310}$$

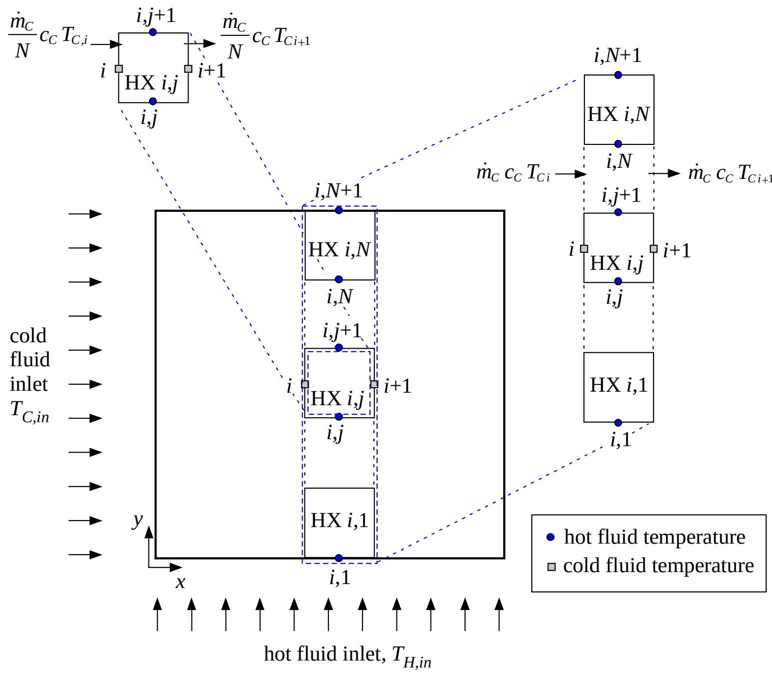

Figure 8-63: Energy balances used for the cross-flow heat exchanger with the cold fluid mixed and the hot fluid unmixed.

The cold fluid energy balance for a vertical column of heat exchangers (HX i,1 through *HX* i,N shown in Figure 8-63) can be expressed as:

$$\dot{m}_C\, c_C \left(T_{C\,i+1} - T_{C\,i}\right) = \frac{UA}{M\;N} \sum_{j=1}^{N} \left[\frac{\left(T_{H\,i,j} + T_{H\,i,j+1}\right)}{2} - \frac{\left(T_{C\,i} + T_{C\,i+1}\right)}{2} \right] \quad \text{for } i = 1..M \tag{8-311}$$

The boundary condition associated with the entering hot-fluid is:

$$T_{H\,i,1} = T_{H,in} \quad \text{for } i = 1..M \tag{8-312}$$

The boundary condition associated with the entering cold-fluid is:

$$T_{C1} = T_{C,in} \tag{8-313}$$

Equations (8-310) through (8-313) represent M (N+1) + M +1 equations in an equal number of unknown temperatures. In order to solve these equations in MATLAB, they must be placed in matrix format:

$$\underline{\underline{A}}\,\underline{X} = \underline{b} \tag{8-314}$$

where $\underline{X}$ is a vector containing the unknown temperatures. One strategy for organizing the unknown temperatures in $\underline{X}$ is:

$$\underline{X} = \begin{bmatrix} X_1 = T_{H\,1,1} \\ X_2 = T_{H\,2,1} \\ \ldots \\ X_M = T_{H\,M,1} \\ X_{M+1} = T_{H\,1,2} \\ \ldots \\ X_{M(N+1)} = T_{H\,N+1,M} \\ X_{M(N+1)+1} = T_{C1} \\ \ldots \\ X_{M(N+1)+M+1} = T_{C\,M+1} \end{bmatrix} \tag{8-315}$$

According to Eq. (8-315), $T_{H\,i,j}$ is entry (j-1) M + i of $\underline{X}$ and $T_{C\,i}$ is entry (N+1)M + i of $\underline{X}$. The governing equations must be placed into the rows of the matrix $\underline{\underline{A}}$. One strategy for organizing these equations is:

$$\underline{\underline{A}} = \begin{bmatrix} \text{row 1 = hot fluid energy balance for HX 1,1} \\ \text{row 2 = hot fluid energy balance for HX 2,1} \\ \ldots \\ \text{row } M = \text{hot fluid energy balance for HX } M,1 \\ \text{row } M+1 = \text{hot fluid energy balance for HX 1,2} \\ \ldots \\ \text{row } M\,N = \text{hot fluid energy balance for HX } M,N \\ \text{row } M\,N+1 = \text{cold fluid energy balance for column HX 1,1 through HX 1:}N \\ \ldots \\ \text{row } M\,N+N = \text{cold fluid energy balance for column HX } M,1 \text{ through HX } M,N \\ \text{row } M\,N+N+1 = \text{hot fluid boundary condition for } T_{H1,1} \\ \ldots \\ \text{row } M\,N+N+M = \text{hot fluid boundary condition for } T_{H\,M,1} \\ \text{row } M\,N+N+M+1 = \text{cold fluid boundary condition for } T_{C1} \end{bmatrix} \tag{8-316}$$

According to Eq. (8-316), the hot-side energy balance for HX i,j corresponds to row (j-1) M + i of $\underline{\underline{A}}$ and the cold-side energy balance for the column of heat exchangers corresponding to *HX* i,1 through *HX* i,N corresponds to row $M\,N$ + i of $\underline{\underline{A}}$. The hot fluid boundary condition for node $T_{H\,i,1}$ corresponds to row $M\,N$ + M + i of $\underline{\underline{A}}$ and the cold fluid boundary condition for node $T_{C\,1}$ corresponds to row $M\,N$ + 2 M + 1 of $\underline{\underline{A}}$.

Matrix $\underline{\underline{A}}$ and vector $\underline{b}$ are initialized. Matrix $\underline{\underline{A}}$ is defined as a sparse matrix and the number of non-zero coefficients is estimated by inspection of Eqs. (8-310) and (8-311).

```
% initialize matrices
A=spalloc(M*N+2*M+1, M*N+2*M+1, 4*M*N+M*(2*N+2)+2);
b=zeros(M*N+2*M+1,1);
```

Equations (8-310) through (8-313) are rearranged so that the coefficients multiplying the unknown temperatures are clear. The hot fluid energy balances, Eq. (8-310), become:

$$T_{H\,i,j}\underbrace{\left[\frac{\dot{m}_H\,c_H}{M}-\frac{UA}{2\,M\,N}\right]}_{A_{(j-1)M+i,(j-1)M+i}}+T_{H\,i,j+1}\underbrace{\left[-\frac{\dot{m}_H\,c_H}{M}-\frac{UA}{2\,M\,N}\right]}_{A_{(j-1)M+i,(j+1-1)M+i}}+T_{C\,i}\underbrace{\left[\frac{UA}{2\,M\,N}\right]}_{A_{(j-1)M+i,(N+1)M+i}}+T_{C\,i+1}\underbrace{\left[\frac{UA}{2\,M\,N}\right]}_{A_{(j-1)M+i,(N+1)M+i+1}}=0$$
$$\text{for } i=1..M,\ j=1..N \tag{8-317}$$

```
% hot-side energy balances
for i=1:M
    for j=1:N
        A((j-1)*M+i,(j-1)*M+i)=m_dot_H*c_H/M-UA/(M*N*2);
        A((j-1)*M+i,(j+1-1)*M+i)=-m_dot_H*c_H/M-UA/(M*N*2);
        A((j-1)*M+i,(N+1)*M+i)=UA/(M*N*2);
        A((j-1)*M+i,(N+1)*M+i+1)=UA/(M*N*2);
    end
end
```

The cold fluid energy balances, Eq. (8-311), become:

$$T_{C\,i+1}\underbrace{\left[\dot{m}_C\,c_C+\frac{UA}{2\,M}\right]}_{A_{M\,N+i,(N+1)M+i+1}}+T_{C\,i}\underbrace{\left[-\dot{m}_C\,c_C+\frac{UA}{2\,M}\right]}_{A_{M\,N+i,(N+1)M+i}}+$$
$$T_{H\,i,1}\underbrace{\left[-\frac{UA}{2\,M\,N}\right]}_{A_{M\,N+i,(1-1)M+i}}+\sum_{j=2}^{N}T_{H\,i,j}\underbrace{\left[-\frac{UA}{M\,N}\right]}_{A_{M\,N+i,(j-1)M+i}}+T_{H\,i,N+1}\underbrace{\left[-\frac{UA}{2\,M\,N}\right]}_{A_{M\,N+i,(N+1-1)M+i}}=0 \tag{8-318}$$
$$\text{for } i=1..M$$

```
% cold-side energy balances
for i=1:M
    A(M*N+i,(N+1)*M+i+1)=m_dot_C*c_C+UA/(2*M);
    A(M*N+i,(N+1)*M+i)=-m_dot_C*c_C+UA/(2*M);
    A(M*N+i,(1-1)*M+i)=-UA/(2*M*N);
    for j=2:N
        A(M*N+i,(j-1)*M+i)=-UA/(M*N);
    end
    A(M*N+i,(N+1-1)*M+i)=-UA/(2*M*N);
```

```
end
```

The hot-side boundary condition, Eq. (8-312), becomes:

$$T_{H\,i,1} \underbrace{[1]}_{A_{M\,N+M+i,(1-1)M+i}} = \underbrace{T_{H,in}}_{b_{M\,N+M+i}} \text{ for } i = 1..M \tag{8-319}$$

```
% hot-side inlet fluid temperature boundary condition
for i=1:M
     A(M*N+M+i,(1-1)*M+i)=1;
     b(M*N+M+i,1)=T_H_in;
end
```

The cold-side boundary condition, Eq. (8-313), becomes:

$$T_{C\,1} \underbrace{[1]}_{A_{M\,N+2M+1,(N+1)M+1}} = \underbrace{T_{C,in}}_{b_{M\,N+2M+1}} \tag{8-320}$$

```
A(M*N+2*M+1,(N+1)*M+1)=1;
b(M*N+2*M+1,1)=T_C_in;
```

The solution is obtained and copied into matrices $\underline{\underline{T_H}}$ and $\underline{\underline{T_C}}$ for convenience. Note that the 1-D solution for the temperature distribution in the cold fluid is used to fill the entire 2-D matrix $\underline{\underline{T_C}}$. Again, this is done in order to allow the solutions to be plotted.

```
X=A\b;
for i=1:M
     for j=1:(N+1)
          T_H(i,j)=X((j-1)*M+i);
     end
end
for i=1:(M+1)
     for j=1:N
          T_C(i,j)=X((N+1)*M+i);
     end
end
```

Figure 8-64 illustrates the temperature distribution for the hot and the cold fluid within the heat exchanger. Notice that the temperature of the cold fluid is constant in the *y*-direction which prevents it from conforming to the shape of the hot fluid completely and therefore limits the effectiveness of the heat exchanger.

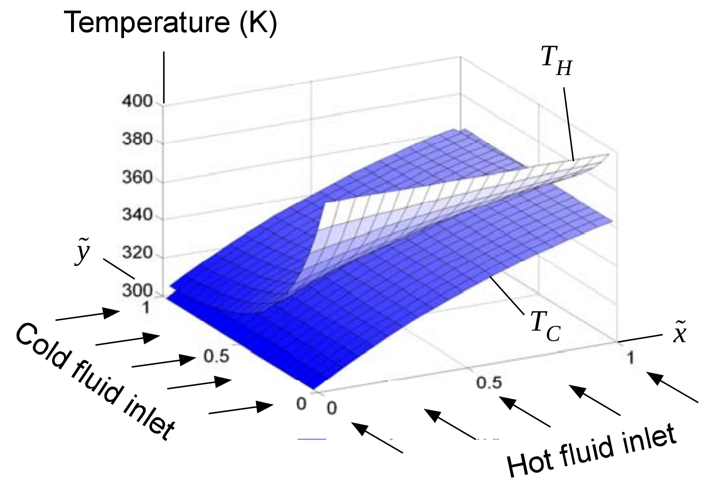

Figure 8-64: Temperature distribution associated with the hot- and cold-fluids for a cross-flow heat exchanger with the hot fluid unmixed and the cold fluid mixed.

The total heat transfer rate from the hot fluid is computed according to:

$$\dot{q}_H = \frac{\dot{m}_H\, c_H}{M} \sum_{i=1}^{M} \left(T_{H,in} - T_{H\,i,N+1} \right) \tag{8-321}$$

```
% total hot-side heat transfer rate
q_dot_H=m_dot_H*c_H*sum(T_H_in-T_H(:,N+1))/M;
```

The total heat transfer rate to the cold fluid is:

$$\dot{q}_C = \dot{m}_C\, c_C \left(T_{C\,M+1} - T_{C,in} \right) \tag{8-322}$$

```
% total cold-side heat transfer rate
q_dot_C=m_dot_C*c_C*sum(T_C(M+1,:)-T_C_in)/N;
```

which leads to $\dot{q}_H$ = 3.142 kW and $\dot{q}_C$ = 3.142 kW. Notice that the rate of heat transfer in the mixed/unmixed configuration is less than the unmixed/unmixed configuration. The numerical solution can be compared directly with the ε-*NTU* solution discussed in Section 8.3. The minimum and maximum capacitance rates, $\dot{C}_{min}$ and $\dot{C}_{max}$, are computed:

```
C_dot_min=min([m_dot_H*c_H,m_dot_C*c_C]);          % minimum capacitance rate
C_dot_max=max([m_dot_H*c_H,m_dot_C*c_C]);          % maximum capacitance rate
```

The maximum possible rate of heat transfer is:

$$\dot{q}_{max} = \dot{C}_{min} \left(T_{H,in} - T_{C,in} \right) \tag{8-323}$$

and the effectiveness of the heat exchanger, predicted by the numerical model, is:

$$\varepsilon = \frac{\dot{q}_H}{\dot{q}_{max}} \tag{8-324}$$

```
q_dot_max=C_dot_min*(T_H_in-T_C_in);            % maximum possible capacitance rate
eff=q_dot_H/q_dot_max;                           % effectiveness
```

which leads to ε = 0.6253; the effectiveness of the mixed/unmixed configuration is less than the effectiveness of the unmixed/unmixed configuration.

The number of transfer units associated with the heat transfer is computed according to:

$$NTU = \frac{UA}{\dot{C}_{min}} \tag{8-325}$$

The capacity ratio is:

$$C_R = \frac{\dot{C}_{min}}{\dot{C}_{max}} \tag{8-326}$$

```
% eff-NTU solution
NTU=UA/C_dot_min;                                % number of transfer units
C_R=C_dot_min/C_dot_max;                         % capacitance ratio
```

The effectiveness predicted by the ε-*NTU* solution for a cross-flow heat exchanger with one fluid mixed and the other unmixed ($\varepsilon_{crossflow\text{-}mixed/unmixed}$) is obtained using the formula listed in Table 8-1. Note that because C_R = 1, it doesn't matter whether the maximum or minimum capacitance rate fluid is mixed:

$$\varepsilon_{crossflow-mixed/unmixed} = \frac{1 - \exp\left[C_R\left\{\exp\left(-NTU\right) - 1\right\}\right]}{C_R} \tag{8-327}$$

```
eff_crossflowmixedunmixed=(1-exp(C_R*(exp(-NTU)-1)))/C_R;   % solution
```

which leads to $\varepsilon_{crossflow\text{-}mixed/unmixed}$ = 0.6252; this is within 0.02% of the numerical solution.