Chapter 4: External Convection

Section 4.1: Introduction to Laminar Boundary Layers

4.1-1 Figure P4.1-1 illustrates a fluid flowing over a flat plate subjected to a constant heat flux, $\dot{q}_s''$. The free stream temperature is T_∞.

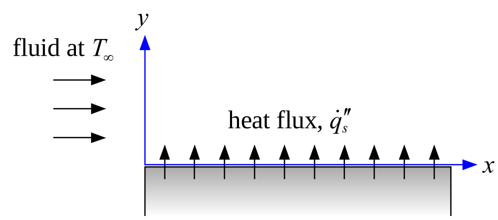


Figure P4.1-1: Fluid flowing over a plate that is subjected to a constant heat flux.

a.) Sketch the thermal boundary layer thickness (δ_t) as a function of position, x, from the leading edge of the plate. Assume that the flow is laminar.

b.) Sketch the surface temperature of the plate (T_s) as a function of position, x, from the leading edge of the plate. Assume that the flow is laminar.

4.1-2 (4-1 in text) Water at atmospheric pressure, free stream velocity u_∞ = 1.0 m/s and temperature T_∞ = 25°C flows over a flat plate with a surface temperature T_s = 90°C. The plate is L = 0.15 m long. Assume that the flow is laminar over the entire length of the plate.

a.) Estimate, using your knowledge of how boundary layers grow, the size of the momentum and thermal boundary layers at the trailing edge of the plate (i.e., at x = L). Do not use a correlation from your book, instead use the approximate model for boundary layer growth.

b.) Use your answer from (a) to estimate the shear stress at the trailing edge of the plate and the heat transfer coefficient at the trailing edge of the plate.

c.) You measure a shear stress of $\tau_{s,meas}$ = 1.0 Pa at the trailing edge of the plate; use the Modified Reynolds Analogy to predict the heat transfer coefficient at this location.

4.1-3 A sun roof is installed in the roof of a car, as shown in Figure P4.1-3.

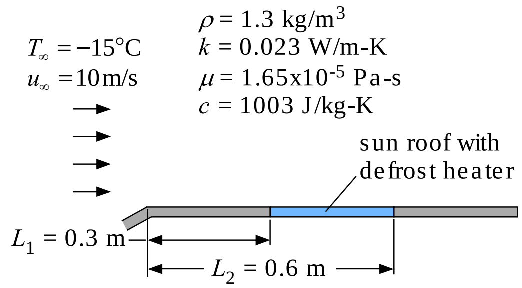


Figure P4.1-3: Sun roof.

A defrost heater is integrated with the sun roof glass in order to prevent ice from forming. The sun roof begins at L_1 = 0.3 m from the front of the roof and extends to L_2 = 0.6 m. The roof is W = 1.2 m wide. The car is driving at u_∞ = 10 m/s through air at T_∞ = -15°C. Assume that the properties of the air are conductivity k = 0.023 W/m-K, density ρ = 1.3 kg/m^3, viscosity μ = 1.65x10^{-5} Pa-s, and specific heat capacity c = 1003 J/kg-K. The defrost heater is a thin sheet of electrically resistive material that covers the entire sun roof and transfers a uniform heat flux to the glass. The convection coefficient between the interior surface of the roof and the cabin air is sufficiently low that convection heat transfer with the interior can be neglected. Assume that the flow over the roof remains laminar. Do not use a correlation to solve this problem; instead, use your conceptual knowledge of boundary layer behavior to estimate the answers.

a.) What is the total power required to keep the sun roof above T_{ice} = 0°C?

b.) Where is the hottest temperature on the sun roof surface? Estimate this temperature.

4.1-4 Air flows over two, thin flat plate arrangements (A and B in Figure P4.1-4). The single plate in arrangement A is twice as long as the two plates in arrangement B. The plates have the same width (into the page), the same uniform surface temperature, and are exposed to the same free-stream flow. The flow over the plates is laminar in both arrangements.

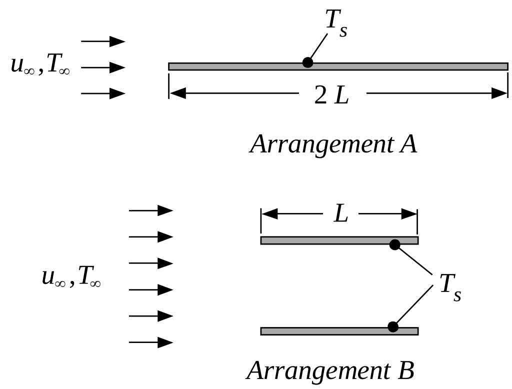


Figure P4.1-4: Air flowing over a single plate (Arrangement A) and two plates that are half the length (Arrangement B). The width of the plates (into the page), surface temperature, and free stream conditions are the same for both arrangements.

a.) Do you expect the total rate of heat transfer to the air to be higher for arrangement A or B? Justify your answer.

4.1-5 Two identical plates are oriented parallel to a flow, as shown in Figure P4.1-5.

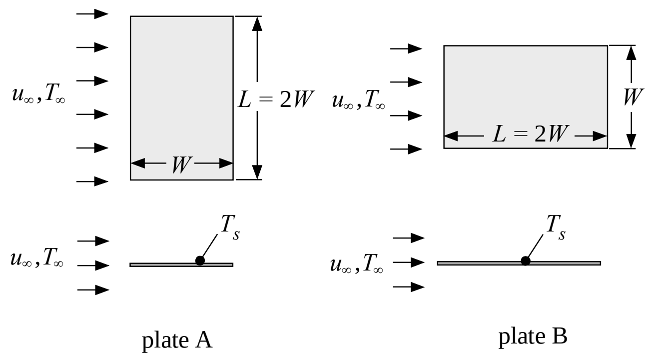


Figure P4.1-5: Plates A and B.

Plate A is oriented with its long dimension, L, oriented with the flow and its short dimension, W, oriented perpendicular to the flow. Plate B is oriented with its long dimension perpendicular to the flow. In both cases, $L = 2\,W$. Assume that the flow remains laminar in both cases. The free stream velocity and temperature, u_∞ and T_∞, and the plate temperature, T_s, are the same for both cases.

a.) Estimate the ratio of the total rate of heat transfer from plate A to the total rate of heat transfer from plate B. Do not use a correlation to answer this question.

4.1-6 (4-2 in text) Figure P4.1-6 illustrates the flow of a fluid with T_∞ = 0°C, u_∞ = 1 m/s over a flat plate.

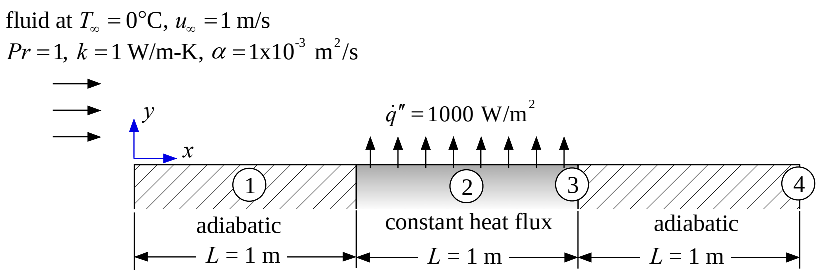


Figure P4.1-6: Flow over a flat plate.

The flat plate is made up of three sections, each with length L = 1 m. The first and last sections are insulated and the middle section is exposed to a constant heat flux, $\dot{q}''$ = 1000 W/m^2. The properties of the fluid are Prandtl number Pr = 1, conductivity k = 1 W/m-K, and thermal diffusivity α = 1x10^{-3} m^2/s. Assume that the flow is laminar over the entire surface.

a.) Sketch the momentum and thermal boundary layers as a function of position, x. Do not worry about the qualitative characteristics of your sketch - get the quantitative characteristics correct.

b.) Sketch the temperature distribution (the temperature as a function of distance from the plate y) at the 4 locations indicated in Figure P4.1-6. Location 1 is half-way through the first adiabatic region, Location 2 is half-way through the heated region, Location 3 is at the trailing edge of the heated region (in the heated region), and Location 4 is at the trailing edge of the final adiabatic region. Again, focus on getting as many of the qualitative characteristics of your sketch correct as you can.

c.) Sketch the temperature of the surface of the plate as a function of position, x. Get the qualitative features of your sketch correct.

d.) Predict, approximately, the temperature of the surface at locations 1, 2, 3, and 4 in Figure P4.1-6. Do not use a correlation. Instead, use your conceptual understanding of how boundary layers behave to come up with very approximate estimates of these temperatures.

4.3-1 You have fabricated a $1/10^{th}$ scale model of a new type of automobile and placed it in a wind tunnel. You want to test it at an air velocity that will allow you to compute the drag force on the car when it travels at 55 miles per hour.

a.) What air velocity should you choose and how is the drag force that you measure on the model related to the drag force on the automobile?

4.3-2 (4-3 in text) You have fabricated a 1000x scale model of a microscale feature that is to be used in a microchip. The device itself is only 1 μm in size and is therefore too small to test accurately. However, you'd like to know the heat transfer coefficient between the device and an air flow that has a velocity of 10 m/s.

a.) What velocity should you use for the test and how will the measured heat transfer coefficient be related to the actual one?

4.3-3 (4-4 in text) Your company has come up with a randomly packed fibrous material that could be used as a regenerator packing. Currently there are no correlations available that would allow the prediction of the heat transfer coefficient for the packing. Therefore, you have carried out a series of tests to measure the heat transfer coefficient. A D_{bed} = 2 cm diameter bed is filled with these fibers with diameter d_{fiber} = 200 μm. The nominal temperature and pressure of the testing is T_{nom} = 20°C and p_{nom} = 1 atm, respectively. The mass flow rate of the test fluid, $\dot{m}$, is varied and the heat transfer coefficient is measured. Several fluids, including air, water, and ethanol, are used for testing. The data are shown in Table P4.3-3; the data can be downloaded from the resource website (www.cambridge.org/nellis&klein) as EES lookup tables (P4-4_air.lkt, P4-4_ethanol.lkt, and P4-4_water.lkt).

Table P4.3-3: Heat transfer data.

Air		Water		Ethanol	
Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)	Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)	Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)
0.0001454	170.7	0.00787	8464	0.009124	4162
0.0004073	311.9	0.02204	15470	0.02555	7607
0.0006691	413.7	0.0362	20515	0.04197	10088
0.0009309	491.8	0.05037	24391	0.05839	11993
0.001193	572.7	0.06454	28399	0.07481	13964
0.001454	631.1	0.0787	31296	0.09124	15388

a.) Plot the heat transfer coefficient as a function of mass flow rate for the three different different test fluids.

b.) Plot the Nusselt number as a function of the Reynolds number for the three different test fluids. Use the fiber diameter as the characteristic length and the free-flow velocity (i.e., the velocity in the bed if it were empty) as the characteristic velocity.

c.) Correlate the data for all of the fluids using a function of the form: $Nu = a\,Re^b\,Pr^c$. Note that you will want to transform the results using a natural logarithm and use the Linear Regression option from the Tables menu to determine *a*, *b*, and *c*.

d.) Use your correlation to estimate the heat transfer coefficient for 20 kg/s of oil passing through a 50 cm diameter bed composed of fibers with 2 mm diameter. The oil has density 875 kg/m^3, viscosity 0.018 Pa-s, conductivity 0.14 W/m-K, and Prandtl number 20.

4.3-4 The rate equations that govern the shear stress in a Newtonian fluid are given by Bejan (1993) and elsewhere. The tangential stress and excess normal stress are given by:

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \ \sigma_{xx} = 2\,\mu\frac{\partial u}{\partial x}$$

a.) Show that these rate equations can be substituted into the *x*-momentum balance and leads to the *x*-momentum equation.

4.3-5 Figure P4.3-5 illustrates a micro-scale feature that will be fabricated on a micro-electro-mechanical system (MEMS) in order to sense the characteristics of the flow of a fluid within a microchannel.

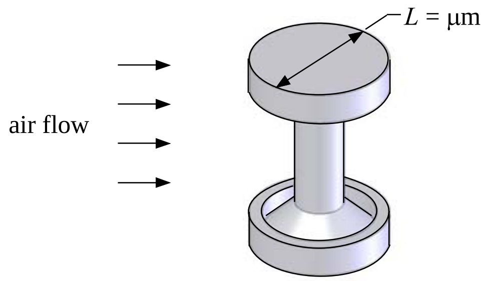


Figure P4.3-5: A MEMS feature for sensing flow characteristics.

In order to design the device it is necessary to understand the heat transfer coefficient associated with the flow of air across the feature. The device is not a commonly encountered shape (e.g., a cylinder or sphere) and therefore you are unable to locate appropriate correlations in the literature. Further, the device only L = 1 μm in size and is therefore too small to test accurately. Therefore, you have decided to build a scaled-up model of the feature that can be more easily tested in a wind tunnel. The scaled up model is 10,000x larger than the MEMS feature (i.e., L_{test} = 1 cm and all of the object dimensions scale proportionally). The model is mounted in a wind tunnel and electrical heaters are embedded in the model and apply a measured, constant heating of $\dot{q}_{test}$ = 0.5 W. The model is composed of a conductive material so that it has approximately a uniform surface temperature ($T_{s,test}$). The wind tunnel is used to provide a uniform flow of air at $T_{\infty,test}$ = 20°C and P_{test} = 1 atm over a range of velocities ($u_{\infty,test}$). Table P4.3-5 summarizes the experimental measurements; note that these data are available from the website as an EES Lookup Table (P4p3-5 Data.lkt).

Table P4.3-5: Experimental measurements.

Air velocity, $u_{\infty,test}$ (m/s)	Surface temperature, $T_{s,test}$ (°C)
0.15	84.39
0.30	69.82
0.45	62.88
0.60	58.55
0.75	55.50
1.13	50.55
1.50	47.47
2.25	43.64
3.00	41.25
4.50	38.29
6.00	36.45
7.50	35.14
11.25	33.03

Note that the total surface area of the model is $A_{s,test}$ = 6.28x10^{-4} m^2 and the surface area of the micro-scale device is (10,000)^{2}x smaller.

a.) Use the experimental measurements to prepare a figure that shows the Nusselt number of the micro-scale device as a function of the Reynolds number for a Prandtl number that is consistent with air.

b.) Prepare a figure that shows the heat transfer coefficient for the micro-scale device as a function of the gas velocity across the device for air with T_∞ = 80ºC and p = 10 atm.

4.3-6 (4-5 in text) Your company makes an extrusion that can be used as a lightweight structural member; the extrusion is long and thin and has an odd cross-sectional shape that is optimized for structural performance. This product has been used primarily in the air-craft industry; however, your company wants to use the extrusion in an application where it will experience cross-flow of water rather than air. There is some concern that the drag force experienced by the extrusion will be larger than it can handle. Because the cross-section of the extrusion is not simple (e.g., circular or square) you cannot go look up a correlation for the drag coefficient in the same way that you could for a cylinder. However, because of the extensive use of the extrusion in the air-craft industry you have an extensive amount of data relating the drag force on the extrusion to velocity when it is exposed to a cross-flow of air. These data have been collated and are shown graphically in Figure P4.3-6.

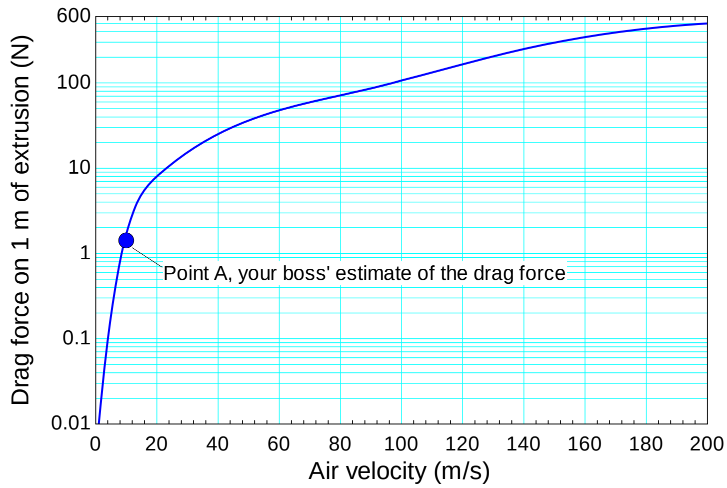


Figure P4.3-6: Drag force as a function of velocity for the extrusion when it is exposed to a cross-flow of air.

Your boss insists that the drag force for the extrusion exposed to water can be obtained by looking at Figure P4.3-6 and picking off the data at the point where V = 10 m/s (Pt. A in Figure P4.3-6); this corresponds to a drag force of about 1.7 N/m of extrusion.

a.) Is your boss correct? Explain why or why not.

b.) If you think that your boss is not correct, then explain how you could use the data shown in Figure P4.3-6 to estimate the drag force that will be experienced by the extrusion for a water cross-flow velocity of 10 m/s.

c.) Use the data in Figure P4.3-6 to estimate the drag force that will be experienced by the extrusion for a water cross-flow velocity of 10 m/s.

Sections 4.4: The Self-Similar Solution for Laminar Flow over a Flat Plate

4.4-1 (4-6 in text) The momentum and thermal boundary layer can be substantially affected by either injecting or removing fluid at the plate surface. For example, Figure P4.4-1 shows the surface of a turbine blade exposed to the free stream flow of a hot combustion gas with velocity u_∞ and temperature T_∞. The surface of the blade is protected by blowing gas through pores in the surface in a process called transpiration cooling.

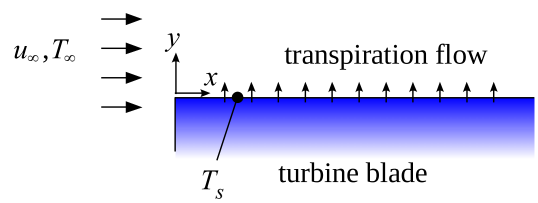


Figure P4.4-1: Transpiration cooled turbine blade.

The velocity of the injected gas is a function of x: $v_{y=0} = C\sqrt{\frac{u_\infty\, \upsilon}{x}}$, where C is a dimensionless constant and υ is the kinematic viscosity of the fluid. The gas is injected at the same temperature as the surface of the plate, T_s. The Prandtl number of the combustion gas is Pr = 0.7.

a.) Develop a self-similar solution to the momentum equation for this problem using a Crank-Nicolson numerical integration implemented in EES.

b.) Plot the dimensionless velocity (u/u_∞) as a function of the similarity parameter, η, for various values of C.

c.) The boundary layer will "blow-off" of the plate at the point where the shear stress at the plate surface becomes zero. What is the maximum value of C that can be tolerated before the boundary layer becomes unstable?

d.) Plot the ratio of the friction factor experienced by the plate with transpiration to the friction factor experienced by a plate without transpiration as a function of the parameter C.

e.) Develop a self-similar solution to the thermal energy equation for this problem using a Crank-Nicolson numerical integration implemented in EES.

f.) Plot the dimensionless temperature difference, $(T - T_s)/(T_\infty - T_s)$, as a function of the similarity parameter, η, for various values of C.

g.) Plot the ratio of the Nusselt number experienced by the plate with transpiration to the Nusselt number experienced by a plate without transpiration as a function of the parameter C.

4.4-2 (4-7 in text) Develop a self-similar solution for the flow over a flat plate that includes viscous dissipation. The ordinary differential equation governing the dimensionless temperature difference should include an additional term that is related to the Eckert number.

a.) Plot the dimensionless temperature difference, $(T - T_s)/(T_\infty - T_s)$, as a function of the similarity parameter, η, for various values of Ec with $Pr = 10$.

b.) Plot the ratio of the Nusselt number to the Nusselt number neglecting viscous dissipation (i.e., with $Ec = 0$) as a function of the Eckert number for various values of the Prandtl number.

4.7-1 (4-8 in text) Use the Spalding model to obtain a velocity and temperature law of the wall (your temperature law of the wall should be obtained numerically using the EES Integral command). Compare your result with the Prandtl-Taylor model. Use a molecular Prandtl number of Pr = 0.7 and a turbulent Prandtl number of Pr_{turb} = 0.9.

4.7-2 (4-9 in text) Use the van Driest model to obtain a velocity and temperature law of the wall. (Both of these results should be obtained numerically using the EES Integral command). Compare your result with the Prandtl-Taylor model. Use a molecular Prandtl number of $Pr = 0.7$ and a turbulent Prandtl number of $Pr_{turb} = 0.9$.

4.7-3 (4-10 in text) In Section 4.5, a conceptual model of a turbulent flow was justified based on the fact that the thermal resistance of the viscous sublayer (δ_{vs}/k) is larger than the thermal resistance of the turbulent boundary layer (δ_{turb}/k_{turb}). Estimate the magnitude of each of these terms for a flow of water over a smooth flat plate and evaluate the validity of this simplification. The free stream velocity is u_∞ = 10 m/s and the plate is L = 1 m long. The water is at 20°C and 1 atm.

4.7-4 A fluid with a low Prandtl number (Pr <<1) flows over a heated flat plate ($T_s > T_\infty$). Assume that the flow velocity, u_∞, is low enough that the flow remains laminar at a particular axial location x.

a.) Sketch the velocity and temperature distribution that you expect at location x. Indicate on your sketches the approximate extent of the momentum and thermal boundary layer thicknesses (δ_m and δ_t); make sure that the relative size of these boundary layer thicknesses is appropriate given the low Prandtl number of the fluid.

b.) Assume that the flow velocity, u_∞, is increased sufficiently that the flow transitions to turbulence. Sketch the temperature distribution that you would expect. Make sure that the qualitative differences between your answers to (a) and (b) are clear.

Section 4.8: Integral Solutions

4.8-1 Potential flow theory shows that the free stream velocity over a wedge or through an expansion, shown in Figures P4.8-1(a) and (b), will have a power-law functional form.

$$u_\infty = C_1\, x^m$$

where the exponent m is related to β according to:

$$m = \frac{\beta}{2-\beta}$$

Note that if $\beta > 0$, as shown in Figure P4.8-1(a), then the flow will be accelerating along the surface whereas if $\beta < 0$, as shown in Figure P4.8-1(b)),then the flow is decelerating. The surface temperature is constant, T_s, and the free stream temperature is T_∞.

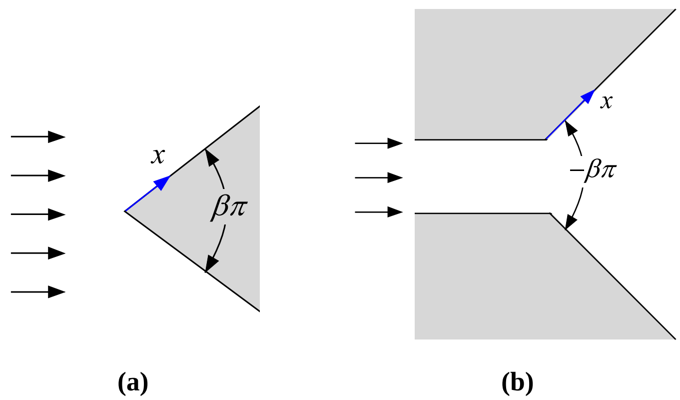


Figure P4.8-1: Flow (a) over a wedge of with angle $\beta\pi$ and (b) through an expansion with angle -$\beta\pi$.

a.) Use an integral technique with a second-order velocity distribution in order to determine a differential equation relating the momentum boundary layer thickness as a function of axial flow position.

4.8-2 A flat plate with a specified heat flux is exposed to a free stream. The flow remains laminar over the plate. Solve the problem using an integral technique with the following assumptions:

1. a 2nd order velocity profile,
2. a 2nd order temperature distribution,
3. the Prandtl number is greater than 1.0,
4. the heat flux at the surface of the plate ($\dot{q}_s''$, not the plate surface temperature) is a known function of x,
5. viscous dissipation is **not** negligible,
6. the free stream velocity, u_∞, is constant,
7. the ratio of the momentum to the thermal boundary layer thickness is: $\delta_m / \delta_t = Pr^{1/3}$

a.) Derive the ordinary differential equation for the thermal boundary layer thickness.

b.) Use your result from (a) to determine an equation for the local Nusselt number as a function of Reynolds number and Prandtl number when the heat flux on the plate surface is constant in the limit of no viscous dissipation.

c.) Compare your result from (b) with the solution provided in Section 4.9.2 by preparing a plot showing both solutions as a function of Reynolds number for Pr = 1.0 and Pr = 5.0.

d.) Use your result from (a) to determine an equation for the local Nusselt number as a function of Reynolds number and Prandtl number when the heat flux on the plate surface increases linearly from 0 at the leading edge in the limit of no viscous dissipation.

e.) Prepare a plot showing the Nusselt number for a constant heat flux (from (b)) and increasing heat flux (from (d)) as a function of Reynolds number for Pr = 2.0.

Problem 4.8-3: Flow over a Heated Region

A laser target is cooled by a flow of water, as shown in Figure P4.8-3.

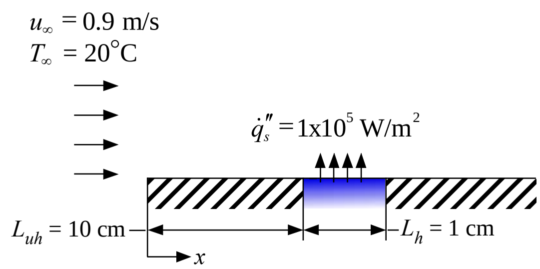


Figure P4.8-3: A laser target.

The target is L_{uh} = 10 cm from the edge of the plate and extends L_h = 1 cm in the flow direction. The heat flux from the laser is uniformly distributed, $\dot{q}_s''$ = 1x10^5 W/m^2, and entirely removed by the fluid (rather than being conducted into the plate). The free stream has velocity u_∞ = 0.9 m/s, temperature T_∞ = 20°C, and pressure p_∞ = 1 atm. You are to develop a solution to this problem using an integral technique. Clearly show your derivation for this homework so that partial credit can be awarded. Make the following assumptions:

1. the properties of water are constant and equal to their value at the free stream temperature,
2. the flow remains laminar over the plate,
3. use a 2nd order velocity and temperature profile,
4. viscous dissipation is negligible,
5. the free stream velocity, u_∞, is constant.

a.) Derive the ordinary differential equation for the momentum boundary layer thickness.

The second order velocity distribution in Table 4-4 is used and the terms associated with shear at the edge of the boundary layer are neglected. The resulting velocity distribution and surface shear stress are:

$$\frac{u}{u_\infty} = \left[2\frac{y}{\delta_m} - \frac{y^2}{\delta_m^2}\right] \tag{1}$$

and

$$\tau_s = 2\mu\frac{u_\infty}{\delta_m} \tag{2}$$

The momentum integral equation can be simplified for this problem:

$$\frac{d}{dx}\left[\int_{y=0}^{y=\delta_m}\left(u^2 - u\,u_\infty\right)dy\right] = -\frac{\tau_s}{\rho} \tag{3}$$

Substituting Eqs. (1) and (2) into Eq. (3) leads to:

$$u_\infty^2 \frac{d}{dx}\left[\int_{y=0}^{y=\delta_m}\left(\left(2\frac{y}{\delta_m}-\frac{y^2}{\delta_m^2}\right)^2-2\frac{y}{\delta_m}+\frac{y^2}{\delta_m^2}\right)dy\right]=-2\frac{\mu u_\infty}{\rho\,\delta_m} \tag{4}$$

or

$$u_\infty^2 \frac{d}{dx}\left[\int_{y=0}^{y=\delta_m}\left(-2\frac{y}{\delta_m}+5\frac{y^2}{\delta_m^2}-4\frac{y^3}{\delta_m^3}+\frac{y^4}{\delta_m^4}\right)dy\right]=-2\frac{\mu u_\infty}{\rho\,\delta_m} \tag{5}$$

Carrying out the integration leads to:

$$u_\infty^2 \frac{d}{dx}\left[\left(-\frac{y^2}{\delta_m}+\frac{5}{3}\frac{y^3}{\delta_m^2}-\frac{y^4}{\delta_m^3}+\frac{1}{5}\frac{y^5}{\delta_m^4}\right)_0^{\delta_m}\right]=-2\frac{\mu u_\infty}{\rho\,\delta_m} \tag{6}$$

Applying the limits leads to:

$$-\frac{2}{15}u_\infty^2\frac{d\delta_m}{dx}=-2\frac{\mu u_\infty}{\rho\,\delta_m} \tag{7}$$

Solving for the rate of change of the momentum boundary layer:

$$\frac{d\delta_m}{dx}=\frac{15\,\mu}{u_\infty\,\rho\,\delta_m} \tag{8}$$

which is the ordinary differential equation that governs the boundary layer growth.

b.) Solve the differential equation from (a) in order to obtain an explicit function for the momentum boundary layer thickness as a function of x.

The variation of the boundary layer thickness with position is the solution to the ordinary differential equation, Eq. (8), subject to the initial condition:

$$\delta_{m,x=0}=0 \tag{9}$$

The differential equation, Eq. (8), can be integrated analytically from 0 to x:

$$\int_0^{\delta_m}\delta_m\,d\delta_m=15\frac{\mu}{u_\infty\,\rho}\int_0^x dx \tag{10}$$

which leads to:

$$\delta_m = \sqrt{\frac{30\,\mu\,x}{u_\infty\,\rho}} \tag{11}$$

c.) Derive the ordinary differential equation for the thermal boundary layer thickness within the heated region. Note that the heat flux rather than the surface temperature is known.

The energy integral equation is simplified:

$$\frac{d}{dx}\left[\int_0^{\delta_t} u\left(T - T_\infty\right) dy\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{12}$$

and rearranged:

$$\frac{d}{dx}\left[u_\infty\left(T_\infty - T_s\right)\int_0^{\delta_t} \frac{u}{u_\infty}\frac{\left[T - T_s - \left(T_\infty - T_s\right)\right]}{\left(T_\infty - T_s\right)} dy\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{13}$$

The appropriate form of the second order temperature distribution from Table 4-5 is:

$$\frac{T - T_s}{T_\infty - T_s} = \left[2\frac{y}{\delta_t} - \frac{y^2}{\delta_t^2}\right] \tag{14}$$

Substituting Eqs. (1) and (14) into Eq. (13) leads to:

$$\frac{d}{dx}\left[u_\infty\left(T_\infty - T_s\right)\int_0^{\delta_t}\left(2\frac{y}{\delta_m} - \frac{y^2}{\delta_m^2}\right)\left(2\frac{y}{\delta_t} - \frac{y^2}{\delta_t^2} - 1\right) dy\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{15}$$

Equation (15) can be expanded:

$$\frac{d}{dx}\left[u_\infty\left(T_\infty - T_s\right)\int_0^{\delta_t}\left(4\frac{y^2}{\delta_t\,\delta_m} - 2\frac{y^3}{\delta_t^2\,\delta_m} - 2\frac{y}{\delta_m} - 2\frac{y^3}{\delta_t\,\delta_m^2} + \frac{y^4}{\delta_t^2\,\delta_m^2} + \frac{y^2}{\delta_m^2}\right) dy\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{16}$$

The integrations are carried out:

$$\frac{d}{dx}\left[u_\infty\left(T_\infty - T_s\right)\left(\frac{4}{3}\frac{y^3}{\delta_t\,\delta_m} - \frac{1}{2}\frac{y^4}{\delta_t^2\,\delta_m} - \frac{y^2}{\delta_m} - \frac{1}{2}\frac{y^4}{\delta_t\,\delta_m^2} + \frac{1}{5}\frac{y^5}{\delta_t^2\,\delta_m^2} + \frac{1}{3}\frac{y^3}{\delta_m^2}\right)_0^{\delta_t}\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{17}$$

Applying the limits:

$$\frac{d}{dx}\left[u_\infty\left(T_\infty - T_s\right)\left(\frac{4}{3}\frac{\delta_t^2}{\delta_m} - \frac{1}{2}\frac{\delta_t^2}{\delta_m} - \frac{\delta_t^2}{\delta_m} - \frac{1}{2}\frac{\delta_t^3}{\delta_m^2} + \frac{1}{5}\frac{\delta_t^3}{\delta_m^2} + \frac{1}{3}\frac{\delta_t^3}{\delta_m^2}\right)\right] = \frac{\dot{q}_s''}{\rho\, c} \tag{18}$$

or

$$\frac{d}{dx}\left[\left(T_\infty - T_s\right)\left(-0.167\frac{\delta_t^2}{\delta_m} + 0.0333\frac{\delta_t^3}{\delta_m^2}\right)\right] = \frac{\dot{q}_s''}{\rho\, c\, u_\infty} \tag{19}$$

The relationship between heat flux and the surface-to-fluid temperature difference from Table 4-5 is:

$$\dot{q}_s'' = 2k\frac{\left(T_s - T_\infty\right)}{\delta_t} \tag{20}$$

Substituting Eq. (20) into Eq. (19) leads to:

$$\frac{d}{dx}\left[\frac{\dot{q}_s''\,\delta_t}{2\,k}\left(0.167\frac{\delta_t^2}{\delta_m} - 0.0333\frac{\delta_t^3}{\delta_m^2}\right)\right] = \frac{\dot{q}_s''}{\rho\, c\, u_\infty} \tag{21}$$

The heat flux in the heated region is constant, therefore:

$$\frac{d}{dx}\left[0.167\frac{\delta_t^3}{\delta_m} - 0.0333\frac{\delta_t^4}{\delta_m^2}\right] = \frac{2\,\alpha}{u_\infty} \tag{22}$$

d.) Substitute the ratio of the thermal to momentum boundary layer thickness, $r = \delta_t/\delta_m$, into your solution from (c) to obtain a differential equation for r rather than δ_t.

Substituting the definition of r into Eq. (22) leads to:

$$\frac{d}{dx}\left[\delta_m^2\left(0.167\, r^3 - 0.0333\, r^4\right)\right] = \frac{2\,\alpha}{u_\infty} \tag{23}$$

e.) Simplify your equation from (d) by neglecting any terms that are obviously small.

The second term on the left side of Eq. (23) will be small relative to the first term because r is necessarily small and the coefficient is also small:

$$\frac{d}{dx}\left(\delta_m^2\, r^3\right) = \frac{11.98\,\alpha}{u_\infty} \tag{24}$$

which leads to:

$$\frac{d}{dx}\left(\delta_m^2 r^3\right) = \frac{11.98\,\alpha}{u_\infty} \tag{25}$$

f.) Solve your equation from (e) in order to obtain an analytical expression for r as a function of x.

Equation (25) is separated and integrated:

$$\int_0^{\delta_m^2 r^3} d\left(\delta_m^2 r^3\right) = \frac{11.98\,\alpha}{u_\infty}\int_{L_{uh}}^{x} dx \tag{26}$$

which leads to:

$$\delta_m^2 r^3 = \frac{11.98\,\alpha}{u_\infty}\left(x - L_{uh}\right) \tag{27}$$

where δ_m is given by Eq. (11).

g.) Use your solution from (f) to prepare a plot of the surface temperature as a function of x within the heated region.

The inputs are entered in EES:

```
"Problem P4.8-3"
$UnitSystem SI MASS RAD PA K J
$TabStops 0.2 0.4 3.5 in

u_infinity=0.9 [m/s]                                    "free stream velocity"
T_infinity=converttemp(C,K,20[C])                       "free stream temperature"
p_infinity=1 [atm]*convert(atm,Pa)                      "pressure"
L_uh=10 [cm]*convert(cm,m)                              "unheated length"
L_h=1 [cm]*convert(cm,m)                                "heated length"
q``_s=100000 [W/m^2]                                    "laser flux"

k=conductivity(Water,T=T_infinity,P=p_infinity)         "conductivity"
rho=density(Water,T=T_infinity,P=p_infinity)            "density"
mu=viscosity(Water,T=T_infinity,P=p_infinity)           "viscosity"
c=cP(Water,T=T_infinity,P=p_infinity)                   "specific heat capacity"
alpha=k/(rho*c)                                         "thermal diffusivity"
nu=mu/rho                                               "kinematic viscosity"
Pr=nu/alpha                                             "Prandtl number"
```

The momentum boundary layer is calculated according to Eq. (11):

```
x=0.105 [m]
delta_m=sqrt(30*nu*x/u_infinity) "momentum boundary layer thickness"
```

Equation (27) provides r:

```
delta_m^2*r^3=11.98*alpha*(x-L_uh)/u_infinity                "ratio"
```

The thermal boundary layer thickness is calculated:

$$\delta_t = r\,\delta_m \tag{28}$$

and the surface temperature is computed using Eq. (20):

```
delta_t=r*delta_m                                   "thermal boundary layer thickness"
q``_s=2*k*(T_s-T_infinity)/delta_t                  "surface temperature"
T_s_C=converttemp(K,C,T_s)                          "in C"
```

Figure P4.8-3(b) illustrates the temperature as a function of position predicted using the integral solution.

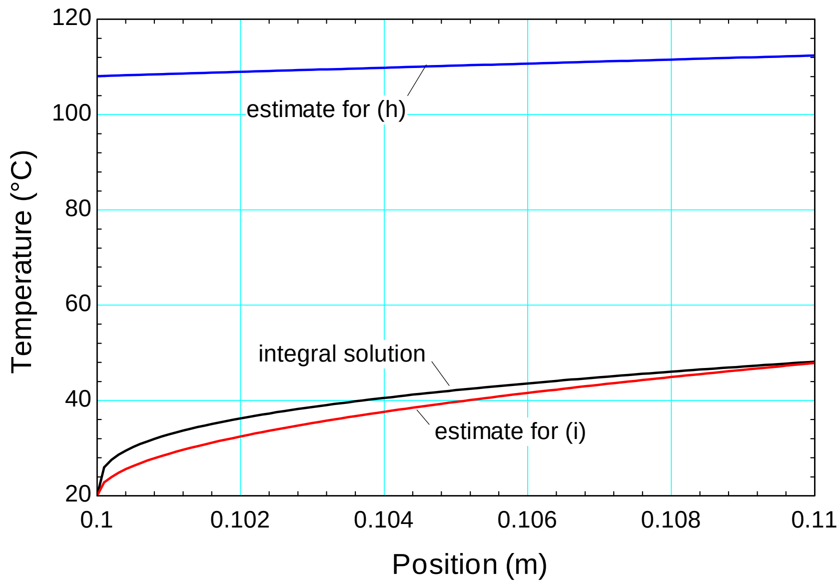


Figure P4.8-3(b): Temperature as a function of position in the heated region.

h.) Your co-worker suggests that you could simply have used the solution for a plate with a constant surface temperature to compute a local heat transfer coefficient and therefore the surface temperature. Evaluate how well this idea would work by overlaying the surface temperature predicted using this method on your plot from (g).

The Reynolds number based on the position relative to the front of the plate is computed:

$$Re_{x,1} = \frac{x\,\rho\,u_\infty}{\mu} \tag{29}$$

The local Nusselt number is computed using the correlation provided in Section 4.9.1:

$$Nu_{x,1} = \frac{0.3387\, Re_{x,1}^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}} \tag{4-30}$$

and used to calculate the heat transfer coefficient:

$$h_1 = Nu_{x,1}\frac{k}{x} \tag{4-31}$$

The surface temperature is computed according to:

$$T_{s,1} = T_\infty + \frac{\dot{q}_s''}{h_1} \tag{4-32}$$

```
Re_x1=x*rho*u_infinity/mu                              "Reynolds number"
Nusselt_x1_uniform1=0.3387*sqrt(Re_x1)*Pr^(1/3)/(1+(0.0468/Pr)^(2/3))^(1/4)
                                                       "Nusselt number based on uniform temperature"
h_uniform1=Nusselt_x1_uniform1*k/x                     "heat transfer coefficient based on uniform temperature"
T_s_uniform1=T_infinity+q``_s/h_uniform1                  "surface temperature based on uniform temperature"
T_s_uniform1_C=converttemp(K,C,T_s_uniform1)                "in C"
```

The solution for $T_{s,1}$ is overlaid onto Figure P4.8-3(b). Note that the estimate is not very good because it substantially over-predicts the thermal boundary layer thickness, the thermal resistance of the boundary layer and therefore the surface temperature.

i.) Your co-worker is somewhat irritated by how bad his idea from (h) was and has now proposed an alternative approach. He suggests that you could have used the solution for a plate with a constant surface temperature to compute a local heat transfer coefficient based on position with respect to the start of the heated region. Evaluate how well this idea would work by overlaying the surface temperature predicted using this method on your plot from (g).

The Reynolds number based on the position relative to the front of the heated region is computed:

$$Re_{x,2} = \frac{\left(x - L_{uh}\right)\rho\, u_\infty}{\mu} \tag{33}$$

The local Nusselt number is computed using the correlation provided in Section 4.9.1:

$$Nu_{x,2} = \frac{0.3387\, Re_{x,2}^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}} \tag{4-34}$$

and used to calculate the heat transfer coefficient:

$$h_2 = Nu_{x,2}\,\frac{k}{x} \tag{4-35}$$

The surface temperature is computed according to:

$$T_{s,2} = T_\infty + \frac{\dot{q}_s''}{h_2} \tag{4-36}$$

```
Re_x2=(x-L_uh)*rho*u_infinity/mu                "Reynolds number"
Nusselt_x2_uniform2=0.3387*sqrt(Re_x2)*Pr^(1/3)/(1+(0.0468/Pr)^(2/3))^(1/4)
                                                "Nusselt number based on uniform temperature"
h_uniform2=Nusselt_x2_uniform2*k/(x-L_uh)"heat transfer coefficient based on uniform temperature"
T_s_uniform2=T_infinity+q``_s/h_uniform2        "surface temperature based on uniform temperature"
T_s_uniform2_C=converttemp(K,C,T_s_uniform2)           "in C"
```

The solution for $T_{s,2}$ is overlaid onto Figure P4.8-3(b). Note that the estimate is better, but still not that great.

4.8-4 (4-11 in text) Figure P4.8-4 illustrates a flat plate that has an unheated starting length (ε); the hydrodynamic boundary layer grows from the leading edge of the plate while the thermal boundary layer grows from $x > L_{uh}$. Assume that the plate has a constant surface temperature, T_s, for $x > L_{uh}$.

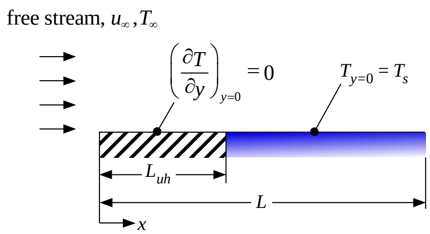


Figure P4.8-4: Plate with an unheated starting length.

Determine a correlation for the local Nusselt number in this situation using the integral technique. Use a third order velocity and temperature distribution. Neglect viscous dissipation. You may find it useful to solve for the ratio of the thermal to momentum boundary layer thickness.

4.8-5 (4-12 in text) A flow of a liquid metal over a flat plate, shown in Figure 4.8-5, is being considered during the design of an advanced nuclear reactor.

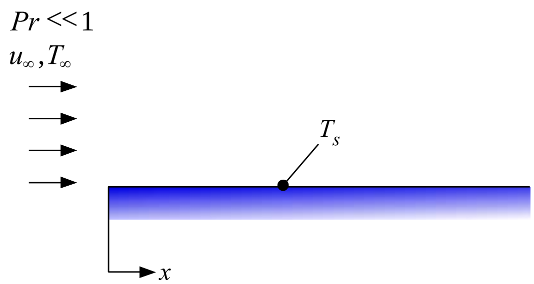


Figure P4.8-5: Flow of a low Prandtl number liquid metal over a flat plate.

You are to develop a solution to this problem using an integral technique. Because the Prandtl number is much less than one, it is appropriate to assume that $\delta_m << \delta_t$ and therefore the velocity is constant and equal to u_∞ throughout the thermal boundary layer. Use a 2nd order temperature distribution and neglect viscous dissipation.

a.) If the temperature of the plate is constant and equal to T_s then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

b.) Plot your result from (a) as a function of *Re* for *Pr* = 0.001. Overlay on your plot the correlation for the local number for flow over a constant temperature flat plate found in Section 4.9.

c.) Plot your result from (a) and the correlation from Section 4.9 as a function of *Pr* for *Re* = 1x10^4. Your Prandtl number range should be from 0.001 to 100 and it should be clear from your plot that the solutions begin to diverge as the Prandtl number approaches unity.

d.) If the heat flux at the surface of the plate is constant and equal to q_s'' then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

e.) If the heat flux at the surface of the plate varies linearly with position and is zero at the leading edge of the plate, then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

f.) Plot the solutions from (a), (d), and (e) as a function of Reynolds number for *Pr* = 0.001.

4.8-6 (4-13 in text) Determine the local friction coefficient as a function of Reynolds number for laminar flow over a flat plate using the momentum integral technique. Assume a velocity distribution of the form: $u/u_{\infty} = a\sin\left(b\,y/\delta_m + c\right)$ where a, b, and c are undetermined constants. Compare your answer to the Blasius solution from Section 4.4.

4.8-7 (4-14 in text) A flat plate that is L = 0.2 m long experiences a heat flux given by:

$$\dot{q}_s'' = \dot{q}_{max}'' \cos\left(\frac{\pi x}{4L}\right)$$

where $\dot{q}_{max}''$ = 2x10^4 W/m^2. The free stream velocity is u_∞ = 20 m/s and the free stream temperature is T_∞ = 20°C. The fluid passing over the plate has thermal diffusivity α = 1x10^{-4} m^2/s, conductivity k = 0.5 W/m-K, and Prandtl number Pr = 2.0.

a.) Use a linear temperature distribution and a linear velocity distribution in order to obtain an ordinary differential equation for the thermal boundary layer thickness.

b.) Solve the ordinary differential equation from (a) numerically. At the leading edge of the plate there is a singularity; obtain an analytical solution in this region and start your numerical solution at the x position where the analytical solution is no longer valid.

c.) Plot the surface temperature of the plate as a function of axial position.

d.) Overlay on your plot from (c) the surface temperature calculated using the correlation for the local heat transfer coefficient on a flat plate.

4.8-8 Figure 4.8-8 illustrates the laminar flow of a very low Prandtl number liquid metal (k = 10 W/m-K, c = 100 J/kg-K, ρ = 1000 kg/m^3) over a heated surface. The length of the surface is L = 0.7 m. The free stream velocity and temperature are u_∞ = 10 m/s and T_∞ = 400 K, respectively.

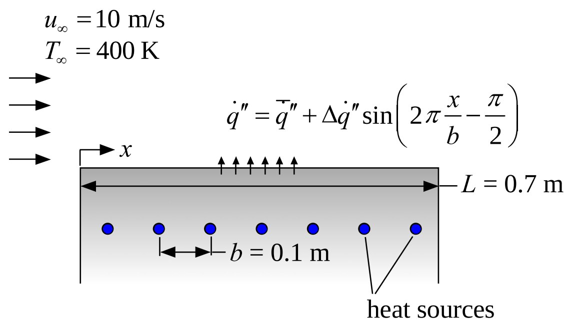


Figure P4.8-8: Low Prandtl number liquid metal flowing over a heated wall.

There are discrete heat sources buried within the wall separated by a distance b = 0.1 m so that the heat flux from the wall to the fluid is not uniform but rather varies sinusoidally according to:

$$\dot{q}''(x) = \bar{\dot{q}}'' + \Delta\dot{q}''\sin\left(2\pi\frac{x}{b} - \frac{\pi}{2}\right)$$

where $\bar{\dot{q}}''$ = 1x10^5 W/m^2 is the average heat flux and $\Delta\dot{q}''$ = 0.5x10^5 W/m^2 is the amplitude of the variation. Develop a solution to this problem using the integral technique that will allow you to predict the temperature of the wall as a function of position (x). Follow the steps outlined below and clearly indicate your work.

a.) The liquid metal has a low Prandtl number and therefore the thermal boundary layer will be much larger than the momentum boundary layer. Therefore, it is appropriate to assume that the velocity is constant within the thermal boundary layer: $u(y) = u_\infty$

You should assume a linear temperature distribution to implement your integral technique. Using this approach, develop a differential equation that describes the thermal boundary layer thickness.

b.) Using EES, numerically integrate your expression from (a) in order to determine the boundary layer thickness as a function of x. Prepare a plot of your result and turn in your EES file.

c.) Using your EES solution, generate a plot that shows the wall temperature as a function of position.

Section 4.9: External Flow Correlations

4.9-1 Figure P4.9-1 illustrates a hot film anemometer used to measure air velocity. The anemometer is a thin plate of metal that is oriented parallel to the oncoming air flow. The free stream temperature (T_∞) and the plate surface temperature (T_s - assume that the plate is at a constant temperature) are both measured. The plate temperature is controlled by electrically heating the material. The plate temperature is always kept ΔT = 5 K higher than the free stream temperature by varying the electrical power provided, $\dot{q}_e$. The heat transfer coefficient between the surface of the plate and the air flow depends on air velocity and therefore it is possible to relate $\dot{q}_e$ to the free stream velocity, u_∞. The length of the plate in the direction of flow is L = 10.0 mm and the width of the plate is W = 25 mm. The plate is placed in an air flow at pressure p_∞ = 1 atm and T_∞ = 20°C and the power required by the hot film is $\dot{q}_e$ = 0.20 W. You can neglect radiation for this problem. Note that both sides of the plate (the top and the bottom in Figure 1) convect to the free stream.

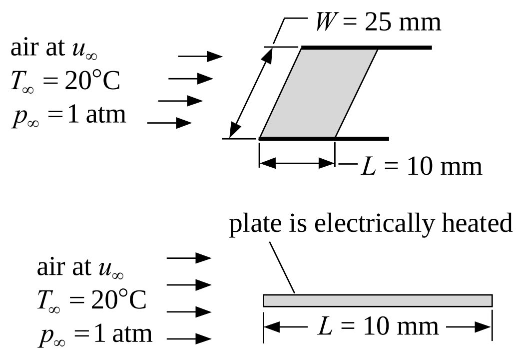


Figure P4.9-1: A hot film anemometer.

a.) What is the air velocity, u_∞?

b.) Plot the air velocity as a function of $\dot{q}_e$. This is the calibration curve for the anemometer; if you were using the anemometer you would measure $\dot{q}_e$ and use this curve to determine u_∞.

c.) Assess the limits of the operating temperature range for the anemometer. If you change the operating temperature (T_∞) by ±50 K will the calibration curve from (b) still be applicable? Justify your answer with a plot.

4.9-2 (4-15 in text) You and your friend are looking for an apartment in a high-rise building. You have your choice of 4 different south-facing units (units #2 through #5) in a city where the wind is predominately from west-to-east, as shown in Figure P4.9-2. You are responsible for paying the heating bill for your apartment and you have noticed that the exterior wall is pretty cheaply built. Your friend has taken heat transfer and therefore is convinced that you should take unit #5 in order to minimize the cost of heating the unit because the boundary layer will be thickest and heat transfer coefficient smallest for the exterior wall of that unit. Prepare an analysis that can predict the cost of heating each of the 4 units so that you can (a) decide whether the difference is worth considering, and (b) if it is, choose the optimal unit. Assume that the heating season is *time* = 4 months long (120 days) and the average outdoor air temperature during that time is T_∞ = 0°C and average wind velocity is u_∞ = 5 mph. The dimensions of the external walls are provided in Figure P4.9-2; assume that no heat loss occurs except through the external walls. Further, assume that the walls have a total thermal resistance on a unit area basis (not including convection) of R''_w = 1 K-m^2/W. The internal heat transfer coefficient is $\overline{h}_{in}$ = 10 W/m^2-K. You like to keep your apartment at T_{in} = 22°C and use electric heating at a cost of *ec* = 0.15$/kW-hr. You may use the properties of air at T_∞ for your analysis and neglect the effect of any windows.

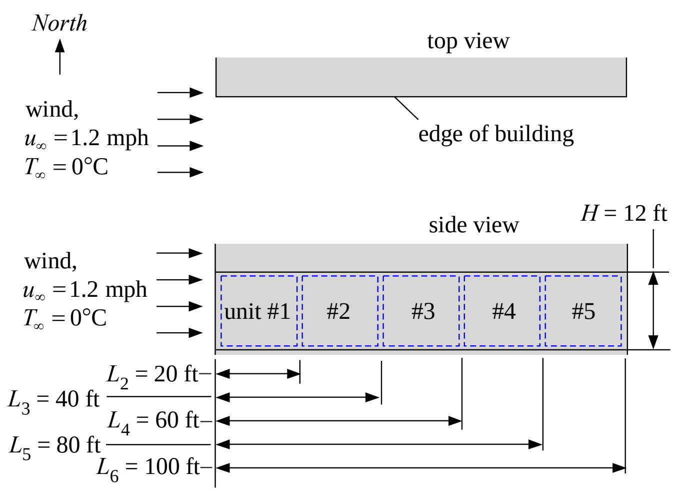


Figure P4.9-2: Location of the external wall of units 2 through 5 relative to the wind direction.

a.) Determine the average yearly heating cost for each of the 4 units and discuss which apartment is best and why.

b.) Prepare a plot showing the heating cost for unit #2 and unit #5 as a function of the wind velocity for the range 0.5 mph to 5.0 mph. Explain any interesting characteristics that you observe.

4.9-3 (4-16 in text) A solar photovoltaic panel is mounted on a mobile traffic sign in order to provide power without being connected to the grid. The panel is W = 0.75 m wide by L = 0.5 m long. The wind blows across the panel with velocity u_∞ = 5 miles/hr and temperature T_∞ = 90°F, as shown in Figure P4.9-3. The back side of the panel is insulated. The panel surface has an emissivity of ε = 1.0 and radiates to surroundings at T_∞. The PV panel receives a solar flux of $\dot{q}''_s$ = 490 W/m^2. The panel produces electricity with an efficiency η (that is, the amount of electrical energy produced by the panel is the product of the efficiency, the solar flux, and the panel area). The efficiency of the panel is a function of surface temperature; at 20°C the efficiency is 15% and the efficiency drops by 0.25%/K as the surface temperature increases (i.e., if the panel surface is at 40°C then the efficiency has been reduced to 10%). All of the solar radiation absorbed by the panel and not transformed into electrical energy must be either radiated or convected to its surroundings.

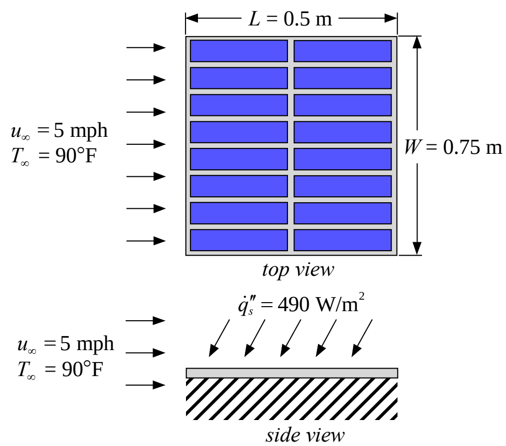


Figure P4.9-3: Solar panel for providing power to a remote traffic sign.

a.) Determine the panel surface temperature, T_s, and the amount of electrical energy generated by the panel.
b.) Prepare a plot of the electrical energy generated by the panel as a function of the solar flux for $\dot{q}''_s$ ranging from 100 W/m^2 to 700 W/m^2. Your plot should show that there is an optimal value for the solar flux – explain this result.
c.) Prepare a plot of the electrical energy generated by the panel as a function of the wind velocity (with $\dot{q}''_s$ = 490 W/m^2) for u_∞ ranging from 5 mph to 50 mph – explain any interesting aspects of your plot.
d.) Prepare a plot of the shear force experienced by the panel due to the wind as a function of wind velocity for u_∞ ranging from 5 mph to 50 mph – explain any interesting aspects of your plot.

4.9-4 (4-17 in text) The wind chill temperature is loosely defined as the temperature that it "feels like" outside when the wind is blowing. More precisely, the wind chill temperature is the temperature of still air that would produce the same bare skin temperature that you experience on a windy day. If you are alive then you are always transferring thermal energy (at rate $\dot{q}$) from your skin (at temperature T_{skin}). On a windy day, this heat loss is resisted by a convection resistance where the heat transfer coefficient is related to forced convection ($R_{conv,fc}$), as shown in Figure P4.9-4(a). The skin temperature is therefore greater than the air temperature (T_{air}). On a still day, this heat loss is resisted by a lower convection resistance because the heat transfer coefficient is related to natural convection ($R_{conv,nc}$), as shown in Fig. P4.9-4(b). For a given heat loss, air temperature, and wind velocity, the wind chill temperature (T_{WC}) is the temperature of still air that produces the same skin temperature.

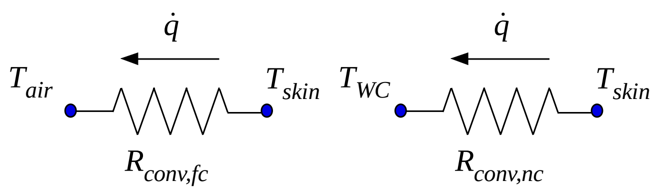


Figure P4.9-4: Resistance network for a body losing heat on (a) a windy and (b) a still day.

It is surprisingly complicated to compute the wind chill temperature because it requires that you know the rate at which the body is losing heat and the heat transfer coefficient between a body and air on both a windy and still day. At the same time, the wind chill temperature is important and controversial because it affects winter tourism in many places. The military and other government agencies that deploy personnel in extreme climates are also very interested in the wind chill temperature in order to establish allowable exposure limits. This problem looks at the wind chill temperature using your heat transfer background. It has been shown that the heat transfer coefficient for most animals can be obtained by treating them as if they were spherical with an equivalent volume.

a.) What is the diameter of a sphere that has the same volume as a man weighing M = 170 lb_m (assume that the density of human flesh is ρ_f = 64 lb_m/ft^3)?
b.) Assuming that the man can be treated as a sphere, compute the skin temperature for the man on a day when the wind blows at V = 10 mph and the air temperature is T_{air} = 0°F. Assume that the metabolic heat generation for the man is $\dot{q}$ = 150 W.
c.) Assume that the natural convection heat transfer coefficient that would occur on a day with no wind is h_{nc} = 8.0 W/m^2-K. What is the wind chill temperature?

According to the National Weather Service (http://www.weather.gov/om/windchill/), the wind chill temperature can be computed according to:

$$T_{WC} = 35.74 + 0.6215\,T_{air} - 35.75\,V^{0.16} + 0.4275\,T_{air}\,V^{0.16}$$

where T_{air} is the air temperature in °F and V is the wind velocity in mph.

d.) Use the National Weather Service equation to compute T_{WC} on a day when T_{air} = 0°F and V = 10 mph.

e.) Plot the wind chill temperature on a day with T_{air} = 0°F as a function of the wind velocity; show the value predicted by your model and by the National Weather Service equation for wind velocities ranging from 5 to 30 mph.

4.9-5 (4-18 in text) A soldering iron tip can be approximated as a cylinder of metal with radius r_{out} = 5.0 mm and length L = 20 mm. The metal is carbon steel; assume that the steel has constant density ρ = 7854 kg/m^3 and constant conductivity k = 50.5 W/m-K, but a specific heat capacity that varies with temperature according to:

$$c = 374.9\left[\frac{\text{J}}{\text{kg-K}}\right] + 0.0992\left[\frac{\text{J}}{\text{kg-K}^2}\right]T + 3.596\text{x}10^{-4}\left[\frac{\text{J}}{\text{kg-K}^3}\right]T^2$$

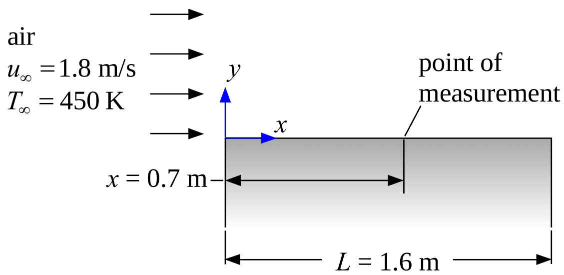
The surface of the iron radiates and convects to surroundings that have temperature T_{amb} = 20°C. Radiation and convection occur from the sides of the cylinder (the top and bottom are insulated). The soldering iron is exposed to an air flow (across the cylinder) with a velocity V = 3.5 m/s at T_{amb} and P_{amb} = 1 atm. The surface of the iron has an emissivity ε = 1.0. The iron is heated electrically by ohmic dissipation; the rate at which electrical energy is added to the iron is $\dot{g}$ = 35 W.

a.) Assume that the soldering iron tip can be treated as a lumped capacitance. Develop a numerical model using the Euler technique that can predict the temperature of the soldering iron as a function of time after it is activated. Assume that it is activated when the tip is at ambient temperature. Be sure to account for the fact that the heat transfer coefficient, the radiation resistance, and the heat capacity of the soldering iron tip are all a function of the temperature of the tip**.**

b.) Plot the temperature of the soldering iron as a function of time. Make sure that your plot covers sufficient time that your soldering iron has reached steady state.

c.) Verify that the soldering iron tip can be treated as a lumped capacitance.

4.9-6 Air at u_∞ = 1.8 m/s and T_∞ = 450 K flows over a flat plate in a wind tunnel. The plate is L = 1.6 m in length in the flow direction and 1 m wide. Measurements are made of the air temperature at various positions above the plate (various values of y) using a small thermocouple that is designed to minimize the disturbance to the flow. The measurements, made a position x = 0.7 m from the leading edge of the plate (as shown in Figure P4.9-6), are summarized in the table below.



(a)

y (m)	T (K)
0	350
0.0012	362
0.0045	400
0.008	425
0.012	440
0.015	447
0.020	450

(b)

Figure 4.9-6 Schematic of experiment and experimental measurements

a) Estimate the thermal boundary layer thickness from the experimental data.
b) What is the thermal boundary layer thickness according to accepted correlations?
c) Estimate the local heat transfer coefficient at x = 0.7 m using the experimental data.
d) What it the local heat transfer coefficient according to accepted correlations?

4.9-7 (4-19 in text) Molten metal droplets must be injected into a plasma for an extreme ultraviolet radiation source, as shown in Figure P4.9-7.

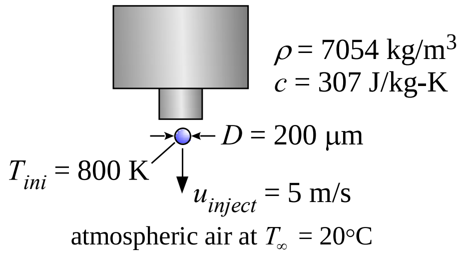


Figure P4.9-7: Injection of molten metal droplets.

The fuel droplets have a diameter of D = 200 μm and are injected at a velocity u_{inject} = 5 m/s with temperature T_{ini} = 800 K. The density of the droplet ρ = 7054 kg/m^3 and the specific heat capacity is c = 307 J/kg-K. You may assume that the droplet can be treated as a lumped capacitance. The droplet is exposed to still air at T_∞ =20ºC.

a.) Develop a numerical model in EES using the Integral command that can keep track of the velocity, temperature, and position of the droplet as a function of time.

b.) Plot the velocity as a function of time and the temperature as a function of time.

c.) Plot the temperature as a function of position. If the temperature of the droplet must be greater than 500 K when it reaches the plasma then what is the maximum distance that can separate the plasma from the injector?

4.9-8 Figure 4.9-8 illustrates a series of plates inserted into a stream of water in order to provide cooling to a power electronics system.

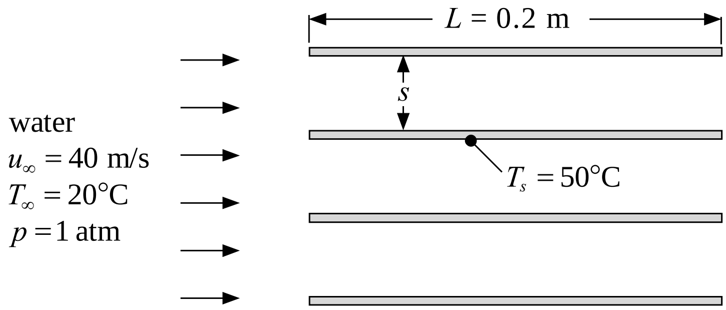


Figure P4.9-8: Plates in a stream of water.

The plates are L = 0.2 m long and W = 0.05 m wide (into the page). The plates are spaced far enough apart that the boundary layers from adjacent plates do not meet, therefore, the flow over the plates can be treated as an external flow rather than an internal flow. The free stream velocity is u_∞ = 40 m/s, the free stream temperature and pressure are T_∞ = 20°C and p = 1 atm, respectively. The surface of the plates are maintained at T_s = 50°C. Assume that the plates are smooth.

a.) What is the total rate of heat transfer from each plate.

b.) Estimate the minimum spacing (s) that can be used before the boundary layers from adjacent plates meet and the problem becomes an internal flow rather than an external flow problem?

c.) Surface roughness begins to affect a turbulent flow when the size of the roughness elements (e) are comparable to the size of the viscous sublayer. Estimate the maximum size of the roughness elements that can be present on the plate before the plate roughness will begin to affect your answer from (a).

4.9-9 (4-20 in text) Figure 4-34 in your text illustrates the drag coefficient for a cylinder as a function of Reynolds number.

a.) Using Figure 4-34 of your text, discuss briefly (1-2 sentences) why it might make sense to add dimples to a baseball bat.

b.) Using Figure 4-34 of your text, estimate how fast you would have to be able to swing a bat in order for it to make sense to think about adding dimples (the estimate can be rough but should be explained well). Assume that a bat has diameter D = 0.04 m and air has properties ρ = 1 kg/m^3 and μ = 0.00002 Pa-s.

4.9-10 Figure P4.9-10 illustrates a very simple heat exchanger.

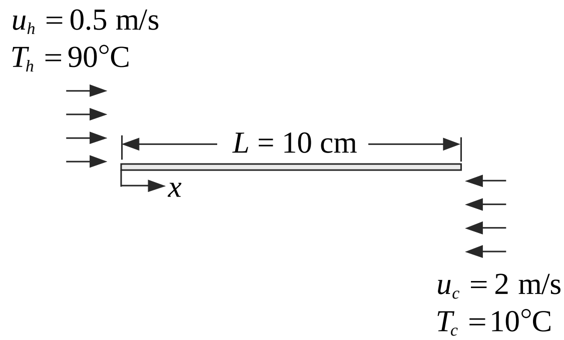


Figure P4.9-10: Simple heat exchanger.

Hot water at T_h = 90ºC flows on one side of a thin metallic plate with free stream velocity u_h = 0.5 m/s. Cold water at T_c = 10ºC flows on the other side in the opposite direction with free stream velocity u_c = 2 m/s. The plate is L = 10 cm long in the direction of flow and W = 20 cm long in the direction perpendicular to the flow. Assume that the plate comes to a uniform temperature. The pressure of the water on both sides is atmospheric.

a.) Determine the rate of heat transfer from the hot fluid to the cold fluid.

b.) What is the net force on the plate in the x-direction (i.e., in the direction of the hot fluid flow)?

c.) Plot the heat transfer rate and net force in the x-direction as a function of the velocity of the hot fluid. Explain any interesting characteristics on your plot.

d.) Do you expect the metal plate to be cooler at the leading edge (x = 0) or trailing edge (x = L)? Explain your answer.

4.9-11 Figure P4.9-11 illustrates the wall of a house. When the wind is blowing hard in the middle of winter, it feels like it is colder in your house. This problem examines whether the amount of heat loss from your house is really strongly affected by the outdoor wind velocity.

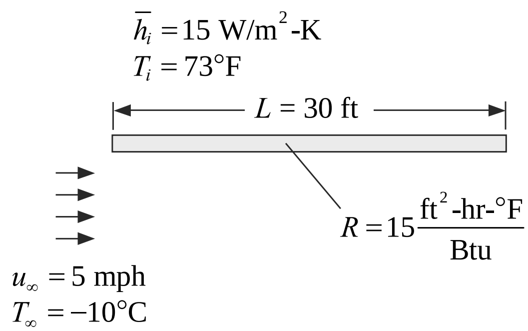


Figure P4.9-11: A wall of your house.

The height of the wall is H = 10 ft and the length is L = 30 ft. The velocity of the wind is u_∞ = 5 mph and the temperature of the air outside is T_∞ = -10ºC. The indoor air temperature is T_i = 73ºF and the natural convection heat transfer coefficient on the inside of the wall is $\bar{h}_i$ = 15 W/m^2-K. The R-value of the wall is R = 15 ft^2-hr-F/Btu; this is the area-specific thermal resistance to conduction through the wall.

a.) What is the rate of energy transfer from the wall of the house?

b.) Plot the rate of heat loss as a function of the wind velocity. Comment on any interesting features of your plot. Also comment on whether you feel that the heat loss predicted as u_∞ approaches zero is accurate.

4.9-12 A hot wire anemometer is used to measure the velocity of an air flow. The device consists of a wire with length L = 1 cm and diameter D = 0.5 mm. The nominal operating conditions are at an air velocity u = 50 ft/s with ambient temperature T_∞ = 20°C and atmospheric pressure. The wire is energized with a current in order to maintain the temperature of the wire ΔT = 5 K above the ambient temperature. The ohmic dissipation, $\dot{g}$, in the wire is related to the velocity of the air.

a.) What is the ohmic dissipation in the wire at the nominal conditions?

b.) Plot the air velocity as a function of ohmic dissipation - this is your calibration curve for the device, you will measure the generation and use this plot to infer the air velocity.

c.) Estimate the error in your measurement related to uncertainty in the measurement of generation if you can measure the ohmic dissipation to within $\delta\dot{g}$ = 2 mW.

d.) Estimate the error in your measurement related to uncertainty in the measurement of the temperature difference if you can measure ΔT to within δT = 0.2 K.

e.) You neglected radiation in your calculation - estimate the error that might be induced by radiation from the hot wire.

f.) Can your hot wire be used to measure the velocity of oxygen rather than air? Estimate the error associated with using your correlation from (b) with oxygen.

4.1-2 (4-1 in text) Water at atmospheric pressure, free stream velocity u_∞ = 1.0 m/s and temperature T_∞ = 25°C flows over a flat plate with a surface temperature T_s = 90°C. The plate is L = 0.15 m long. Assume that the flow is laminar over the entire length of the plate.

a.) Estimate, using your knowledge of how boundary layers grow, the size of the momentum and thermal boundary layers at the trailing edge of the plate (i.e., at $x = L$). Do not use a correlation from your book, instead use the approximate model for boundary layer growth.

b.) Use your answer from (a) to estimate the shear stress at the trailing edge of the plate and the heat transfer coefficient at the trailing edge of the plate.

c.) You measure a shear stress of $\tau_{s,meas}$ = 1.0 Pa at the trailing edge of the plate; use the Modified Reynolds Analogy to predict the heat transfer coefficient at this location.

4.1-6 (4-2 in text) Figure P4.1-6 illustrates the flow of a fluid with T_∞ = 0ºC, u_∞ = 1 m/s over a flat plate.

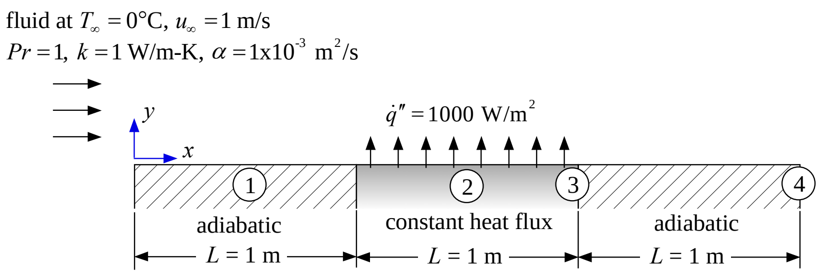


Figure P4.1-6: Flow over a flat plate.

The flat plate is made up of three sections, each with length L = 1 m. The first and last sections are insulated and the middle section is exposed to a constant heat flux, $\dot{q}''$ = 1000 W/m^2. The properties of the fluid are Prandtl number Pr = 1, conductivity k = 1 W/m-K, and thermal diffusivity α = 1x10^{-3} m^2/s. Assume that the flow is laminar over the entire surface.

a.) Sketch the momentum and thermal boundary layers as a function of position, x. Do not worry about the qualitative characteristics of your sketch - get the quantitative characteristics correct.

b.) Sketch the temperature distribution (the temperature as a function of distance from the plate y) at the 4 locations indicated in Figure P4.1-6. Location 1 is half-way through the first adiabatic region, Location 2 is half-way through the heated region, Location 3 is at the trailing edge of the heated region (in the heated region), and Location 4 is at the trailing edge of the final adiabatic region. Again, focus on getting as many of the qualitative characteristics of your sketch correct as you can.

c.) Sketch the temperature of the surface of the plate as a function of position, x. Get the qualitative features of your sketch correct.

d.) Predict, approximately, the temperature of the surface at locations 1, 2, 3, and 4 in Figure P4.1-6. Do not use a correlation. Instead, use your conceptual understanding of how boundary layers behave to come up with very approximate estimates of these temperatures.

4.3-2 (4-3 in text) You have fabricated a 1000x scale model of a microscale feature that is to be used in a microchip. The device itself is only 1 μm in size and is therefore too small to test accurately. However, you'd like to know the heat transfer coefficient between the device and an air flow that has a velocity of 10 m/s.

a.) What velocity should you use for the test and how will the measured heat transfer coefficient be related to the actual one?

4.3-3 (4-4 in text) Your company has come up with a randomly packed fibrous material that could be used as a regenerator packing. Currently there are no correlations available that would allow the prediction of the heat transfer coefficient for the packing. Therefore, you have carried out a series of tests to measure the heat transfer coefficient. A D_{bed} = 2 cm diameter bed is filled with these fibers with diameter d_{fiber} = 200 μm. The nominal temperature and pressure of the testing is T_{nom} = 20ºC and p_{nom} = 1 atm, respectively. The mass flow rate of the test fluid, $\dot{m}$, is varied and the heat transfer coefficient is measured. Several fluids, including air, water, and ethanol, are used for testing. The data are shown in Table P4.3-3; the data can be downloaded from the resource website (www.cambridge.org/nellis&klein) as EES lookup tables (P4-4_air.lkt, P4-4_ethanol.lkt, and P4-4_water.lkt).

Table P4.3-3: Heat transfer data.

Air		Water		Ethanol	
Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)	Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)	Mass flow rate (kg/s)	Heat transfer coefficient (W/m^2-K)
0.0001454	170.7	0.00787	8464	0.009124	4162
0.0004073	311.9	0.02204	15470	0.02555	7607
0.0006691	413.7	0.0362	20515	0.04197	10088
0.0009309	491.8	0.05037	24391	0.05839	11993
0.001193	572.7	0.06454	28399	0.07481	13964
0.001454	631.1	0.0787	31296	0.09124	15388

a.) Plot the heat transfer coefficient as a function of mass flow rate for the three different different test fluids.

b.) Plot the Nusselt number as a function of the Reynolds number for the three different test fluids. Use the fiber diameter as the characteristic length and the free-flow velocity (i.e., the velocity in the bed if it were empty) as the characteristic velocity.

c.) Correlate the data for all of the fluids using a function of the form: $Nu = a\,Re^b\,Pr^c$. Note that you will want to transform the results using a natural logarithm and use the Linear Regression option from the Tables menu to determine *a*, *b*, and *c*.

d.) Use your correlation to estimate the heat transfer coefficient for 20 kg/s of oil passing through a 50 cm diameter bed composed of fibers with 2 mm diameter. The oil has density 875 kg/m^3, viscosity 0.018 Pa-s, conductivity 0.14 W/m-K, and Prandtl number 20.

4.3-6 (4-5 in text) Your company makes an extrusion that can be used as a lightweight structural member; the extrusion is long and thin and has an odd cross-sectional shape that is optimized for structural performance. This product has been used primarily in the air-craft industry; however, your company wants to use the extrusion in an application where it will experience cross-flow of water rather than air. There is some concern that the drag force experienced by the extrusion will be larger than it can handle. Because the cross-section of the extrusion is not simple (e.g., circular or square) you cannot go look up a correlation for the drag coefficient in the same way that you could for a cylinder. However, because of the extensive use of the extrusion in the air-craft industry you have an extensive amount of data relating the drag force on the extrusion to velocity when it is exposed to a cross-flow of air. These data have been collated and are shown graphically in Figure P4.3-6.

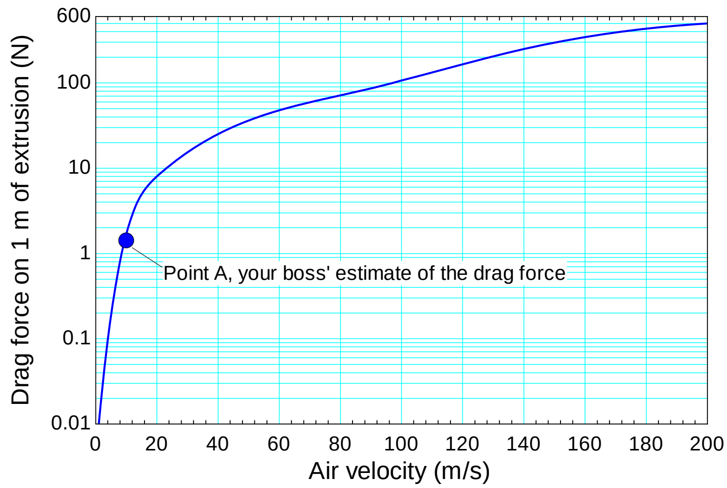


Figure P4.3-6: Drag force as a function of velocity for the extrusion when it is exposed to a cross-flow of air.

Your boss insists that the drag force for the extrusion exposed to water can be obtained by looking at Figure P4.3-6 and picking off the data at the point where V = 10 m/s (Pt. A in Figure P4.3-6); this corresponds to a drag force of about 1.7 N/m of extrusion.

a.) Is your boss correct? Explain why or why not.

b.) If you think that your boss is not correct, then explain how you could use the data shown in Figure P4.3-6 to estimate the drag force that will be experienced by the extrusion for a water cross-flow velocity of 10 m/s.

c.) Use the data in Figure P4.3-6 to estimate the drag force that will be experienced by the extrusion for a water cross-flow velocity of 10 m/s.

Sections 4.4: The Self-Similar Solution for Laminar Flow over a Flat Plate

4.4-1 (4-6 in text) The momentum and thermal boundary layer can be substantially affected by either injecting or removing fluid at the plate surface. For example, Figure P4.4-1 shows the surface of a turbine blade exposed to the free stream flow of a hot combustion gas with velocity u_∞ and temperature T_∞. The surface of the blade is protected by blowing gas through pores in the surface in a process called transpiration cooling.

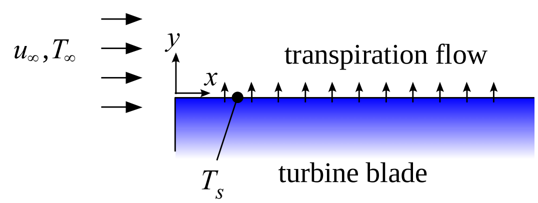


Figure P4.4-1: Transpiration cooled turbine blade.

The velocity of the injected gas is a function of x: $v_{y=0} = C\sqrt{\frac{u_\infty \upsilon}{x}}$, where C is a dimensionless constant and υ is the kinematic viscosity of the fluid. The gas is injected at the same temperature as the surface of the plate, T_s. The Prandtl number of the combustion gas is $Pr = 0.7$.

a.) Develop a self-similar solution to the momentum equation for this problem using a Crank-Nicolson numerical integration implemented in EES.

b.) Plot the dimensionless velocity (u/u_∞) as a function of the similarity parameter, η, for various values of C.

c.) The boundary layer will "blow-off" of the plate at the point where the shear stress at the plate surface becomes zero. What is the maximum value of C that can be tolerated before the boundary layer becomes unstable?

d.) Plot the ratio of the friction factor experienced by the plate with transpiration to the friction factor experienced by a plate without transpiration as a function of the parameter C.

e.) Develop a self-similar solution to the thermal energy equation for this problem using a Crank-Nicolson numerical integration implemented in EES.

f.) Plot the dimensionless temperature difference, $(T - T_s)/(T_\infty - T_s)$, as a function of the similarity parameter, η, for various values of C.

g.) Plot the ratio of the Nusselt number experienced by the plate with transpiration to the Nusselt number experienced by a plate without transpiration as a function of the parameter C.

4.4-2 (4-7 in text) Develop a self-similar solution for the flow over a flat plate that includes viscous dissipation. The ordinary differential equation governing the dimensionless temperature difference should include an additional term that is related to the Eckert number.

a.) Plot the dimensionless temperature difference, $(T - T_s)/(T_\infty - T_s)$, as a function of the similarity parameter, η, for various values of Ec with $Pr = 10$.

b.) Plot the ratio of the Nusselt number to the Nusselt number neglecting viscous dissipation (i.e., with $Ec = 0$) as a function of the Eckert number for various values of the Prandtl number.

4.7-1 (4-8 in text) Use the Spalding model to obtain a velocity and temperature law of the wall (your temperature law of the wall should be obtained numerically using the EES Integral command). Compare your result with the Prandtl-Taylor model. Use a molecular Prandtl number of Pr = 0.7 and a turbulent Prandtl number of Pr_{turb} = 0.9.

4.7-2 (4-9 in text) Use the van Driest model to obtain a velocity and temperature law of the wall. (Both of these results should be obtained numerically using the EES Integral command). Compare your result with the Prandtl-Taylor model. Use a molecular Prandtl number of Pr = 0.7 and a turbulent Prandtl number of Pr_{turb} = 0.9.

4.7-3 (4-10 in text) In Section 4.5, a conceptual model of a turbulent flow was justified based on the fact that the thermal resistance of the viscous sublayer (δ_{vs}/k) is larger than the thermal resistance of the turbulent boundary layer (δ_{turb}/k_{turb}). Estimate the magnitude of each of these terms for a flow of water over a smooth flat plate and evaluate the validity of this simplification. The free stream velocity is u_∞ = 10 m/s and the plate is L = 1 m long. The water is at 20°C and 1 atm.

4.8-4 (4-11 in text) Figure P4.8-4 illustrates a flat plate that has an unheated starting length (ε); the hydrodynamic boundary layer grows from the leading edge of the plate while the thermal boundary layer grows from $x > L_{uh}$. Assume that the plate has a constant surface temperature, T_s, for $x > L_{uh}$.

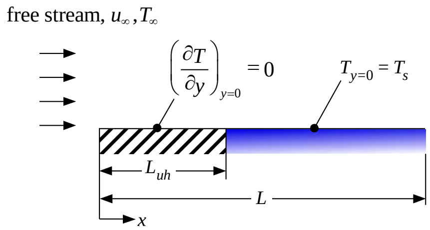


Figure P4.8-4: Plate with an unheated starting length.

Determine a correlation for the local Nusselt number in this situation using the integral technique. Use a third order velocity and temperature distribution. Neglect viscous dissipation. You may find it useful to solve for the ratio of the thermal to momentum boundary layer thickness.

4.8-5 (4-12 in text) A flow of a liquid metal over a flat plate, shown in Figure 4.8-5, is being considered during the design of an advanced nuclear reactor.

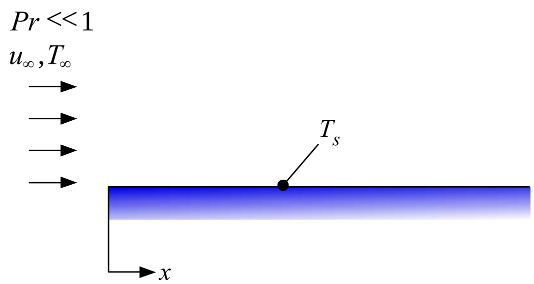


Figure P4.8-5: Flow of a low Prandtl number liquid metal over a flat plate.

You are to develop a solution to this problem using an integral technique. Because the Prandtl number is much less than one, it is appropriate to assume that $\delta_m << \delta_t$ and therefore the velocity is constant and equal to u_∞ throughout the thermal boundary layer. Use a 2nd order temperature distribution and neglect viscous dissipation.

a.) If the temperature of the plate is constant and equal to T_s then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

b.) Plot your result from (a) as a function of *Re* for *Pr* = 0.001. Overlay on your plot the correlation for the local number for flow over a constant temperature flat plate found in Section 4.9.

c.) Plot your result from (a) and the correlation from Section 4.9 as a function of *Pr* for *Re* = $1\text{x}10^4$. Your Prandtl number range should be from 0.001 to 100 and it should be clear from your plot that the solutions begin to diverge as the Prandtl number approaches unity.

d.) If the heat flux at the surface of the plate is constant and equal to q_s'' then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

e.) If the heat flux at the surface of the plate varies linearly with position and is zero at the leading edge of the plate, then derive a solution for the local Nusselt number on the plate surface as a function of the Reynolds number and the Prandtl number.

f.) Plot the solutions from (a), (d), and (e) as a function of Reynolds number for *Pr* = 0.001.

4.8-6 (4-13 in text) Determine the local friction coefficient as a function of Reynolds number for laminar flow over a flat plate using the momentum integral technique. Assume a velocity distribution of the form: $u/u_{\infty} = a\sin\left(b\,y/\delta_m + c\right)$ where a, b, and c are undetermined constants. Compare your answer to the Blasius solution from Section 4.4.

4.8-7 (4-14 in text) A flat plate that is L = 0.2 m long experiences a heat flux given by:

$$\dot{q}_s'' = \dot{q}_{max}'' \cos\left(\frac{\pi\, x}{4\,L}\right)$$

where $\dot{q}_{max}''$ = 2x10^4 W/m^2. The free stream velocity is u_∞ = 20 m/s and the free stream temperature is T_∞ = 20°C. The fluid passing over the plate has thermal diffusivity α = 1x10^{-4} m^2/s, conductivity k = 0.5 W/m-K, and Prandtl number Pr = 2.0.

a.) Use a linear temperature distribution and a linear velocity distribution in order to obtain an ordinary differential equation for the thermal boundary layer thickness.

b.) Solve the ordinary differential equation from (a) numerically. At the leading edge of the plate there is a singularity; obtain an analytical solution in this region and start your numerical solution at the x position where the analytical solution is no longer valid.

c.) Plot the surface temperature of the plate as a function of axial position.

d.) Overlay on your plot from (c) the surface temperature calculated using the correlation for the local heat transfer coefficient on a flat plate.

4.9-2 (4-15 in text) You and your friend are looking for an apartment in a high-rise building. You have your choice of 4 different south-facing units (units #2 through #5) in a city where the wind is predominately from west-to-east, as shown in Figure P4.9-2. You are responsible for paying the heating bill for your apartment and you have noticed that the exterior wall is pretty cheaply built. Your friend has taken heat transfer and therefore is convinced that you should take unit #5 in order to minimize the cost of heating the unit because the boundary layer will be thickest and heat transfer coefficient smallest for the exterior wall of that unit. Prepare an analysis that can predict the cost of heating each of the 4 units so that you can (a) decide whether the difference is worth considering, and (b) if it is, choose the optimal unit. Assume that the heating season is *time* = 4 months long (120 days) and the average outdoor air temperature during that time is T_∞ = 0°C and average wind velocity is u_∞ = 5 mph. The dimensions of the external walls are provided in Figure P4.9-2; assume that no heat loss occurs except through the external walls. Further, assume that the walls have a total thermal resistance on a unit area basis (not including convection) of R''_w = 1 K-m^2/W. The internal heat transfer coefficient is $\bar{h}_{in}$ = 10 W/m^2-K. You like to keep your apartment at T_{in} = 22°C and use electric heating at a cost of ec = 0.15$/kW-hr. You may use the properties of air at T_∞ for your analysis and neglect the effect of any windows.

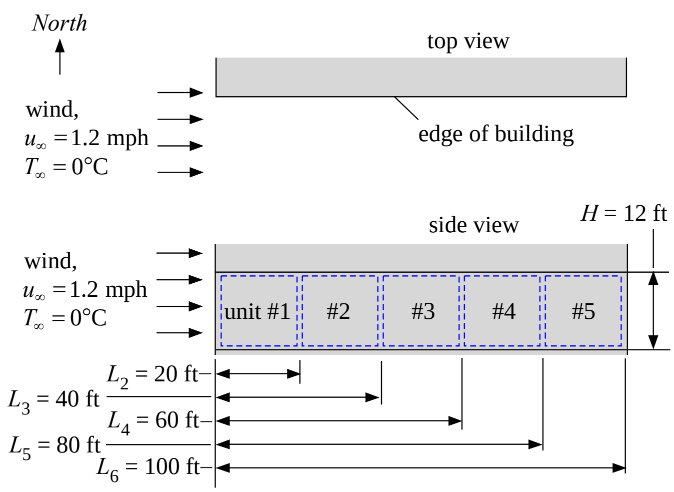


Figure P4.9-2: Location of the external wall of units 2 through 5 relative to the wind direction.

a.) Determine the average yearly heating cost for each of the 4 units and discuss which apartment is best and why.

b.) Prepare a plot showing the heating cost for unit #2 and unit #5 as a function of the wind velocity for the range 0.5 mph to 5.0 mph. Explain any interesting characteristics that you observe.

4.9-3 (4-16 in text) A solar photovoltaic panel is mounted on a mobile traffic sign in order to provide power without being connected to the grid. The panel is W = 0.75 m wide by L = 0.5 m long. The wind blows across the panel with velocity u_∞ = 5 miles/hr and temperature T_∞ = 90°F, as shown in Figure P4.9-3. The back side of the panel is insulated. The panel surface has an emissivity of ε = 1.0 and radiates to surroundings at T_∞. The PV panel receives a solar flux of $\dot{q}_s''$ = 490 W/m^2. The panel produces electricity with an efficiency η (that is, the amount of electrical energy produced by the panel is the product of the efficiency, the solar flux, and the panel area). The efficiency of the panel is a function of surface temperature; at 20°C the efficiency is 15% and the efficiency drops by 0.25%/K as the surface temperature increases (i.e., if the panel surface is at 40°C then the efficiency has been reduced to 10%). All of the solar radiation absorbed by the panel and not transformed into electrical energy must be either radiated or convected to its surroundings.

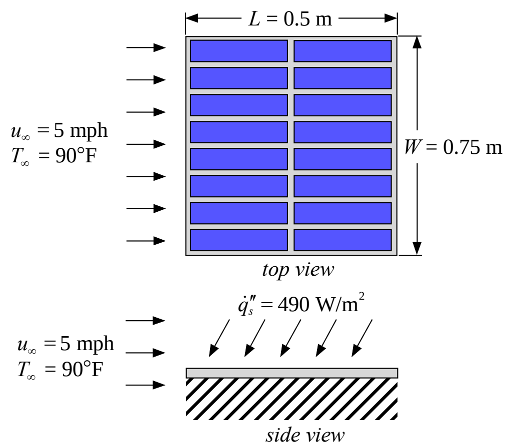


Figure P4.9-3: Solar panel for providing power to a remote traffic sign.

a.) Determine the panel surface temperature, T_s, and the amount of electrical energy generated by the panel.

b.) Prepare a plot of the electrical energy generated by the panel as a function of the solar flux for $\dot{q}_s''$ ranging from 100 W/m^2 to 700 W/m^2. Your plot should show that there is an optimal value for the solar flux – explain this result.

c.) Prepare a plot of the electrical energy generated by the panel as a function of the wind velocity (with $\dot{q}_s''$ = 490 W/m^2) for u_∞ ranging from 5 mph to 50 mph – explain any interesting aspects of your plot.

d.) Prepare a plot of the shear force experienced by the panel due to the wind as a function of wind velocity for u_∞ ranging from 5 mph to 50 mph – explain any interesting aspects of your plot.

4.9-4 (4-17 in text) The wind chill temperature is loosely defined as the temperature that it "feels like" outside when the wind is blowing. More precisely, the wind chill temperature is the temperature of still air that would produce the same bare skin temperature that you experience on a windy day. If you are alive then you are always transferring thermal energy (at rate $\dot{q}$) from your skin (at temperature T_{skin}). On a windy day, this heat loss is resisted by a convection resistance where the heat transfer coefficient is related to forced convection ($R_{conv,fc}$), as shown in Figure P4.9-4(a). The skin temperature is therefore greater than the air temperature (T_{air}). On a still day, this heat loss is resisted by a lower convection resistance because the heat transfer coefficient is related to natural convection ($R_{conv,nc}$), as shown in Fig. P4.9-4(b). For a given heat loss, air temperature, and wind velocity, the wind chill temperature (T_{WC}) is the temperature of still air that produces the same skin temperature.

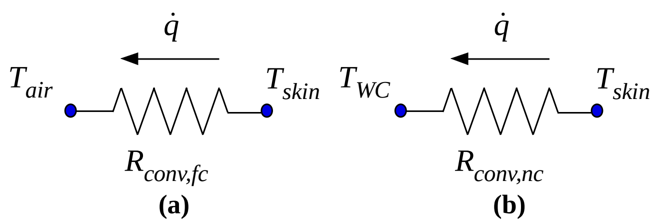


Figure P4.9-4: Resistance network for a body losing heat on (a) a windy and (b) a still day.

It is surprisingly complicated to compute the wind chill temperature because it requires that you know the rate at which the body is losing heat and the heat transfer coefficient between a body and air on both a windy and still day. At the same time, the wind chill temperature is important and controversial because it affects winter tourism in many places. The military and other government agencies that deploy personnel in extreme climates are also very interested in the wind chill temperature in order to establish allowable exposure limits. This problem looks at the wind chill temperature using your heat transfer background. It has been shown that the heat transfer coefficient for most animals can be obtained by treating them as if they were spherical with an equivalent volume.

a.) What is the diameter of a sphere that has the same volume as a man weighing M = 170 lb_m (assume that the density of human flesh is ρ_f = 64 lb_m/ft^3)?
b.) Assuming that the man can be treated as a sphere, compute the skin temperature for the man on a day when the wind blows at V = 10 mph and the air temperature is T_{air} = 0°F. Assume that the metabolic heat generation for the man is $\dot{q}$ = 150 W.
c.) Assume that the natural convection heat transfer coefficient that would occur on a day with no wind is h_{nc} = 8.0 W/m^2-K. What is the wind chill temperature?

According to the National Weather Service (http://www.weather.gov/om/windchill/), the wind chill temperature can be computed according to:

$$T_{WC} = 35.74 + 0.6215 T_{air} - 35.75 V^{0.16} + 0.4275 T_{air} V^{0.16}$$

where T_{air} is the air temperature in °F and V is the wind velocity in mph.

d.) Use the National Weather Service equation to compute T_{WC} on a day when T_{air} = 0°F and V = 10 mph.

e.) Plot the wind chill temperature on a day with T_{air} = 0°F as a function of the wind velocity; show the value predicted by your model and by the National Weather Service equation for wind velocities ranging from 5 to 30 mph.

4.9-5 (4-18 in text) A soldering iron tip can be approximated as a cylinder of metal with radius r_{out} = 5.0 mm and length L = 20 mm. The metal is carbon steel; assume that the steel has constant density ρ = 7854 kg/m^3 and constant conductivity k = 50.5 W/m-K, but a specific heat capacity that varies with temperature according to:

$$c = 374.9\left[\frac{\text{J}}{\text{kg-K}}\right] + 0.0992\left[\frac{\text{J}}{\text{kg-K}^2}\right]T + 3.596\text{x}10^{-4}\left[\frac{\text{J}}{\text{kg-K}^3}\right]T^2$$

The surface of the iron radiates and convects to surroundings that have temperature T_{amb} = 20°C. Radiation and convection occur from the sides of the cylinder (the top and bottom are insulated). The soldering iron is exposed to an air flow (across the cylinder) with a velocity V = 3.5 m/s at T_{amb} and P_{amb} = 1 atm. The surface of the iron has an emissivity ε = 1.0. The iron is heated electrically by ohmic dissipation; the rate at which electrical energy is added to the iron is $\dot{g}$ = 35 W.

a.) Assume that the soldering iron tip can be treated as a lumped capacitance. Develop a numerical model using the Euler technique that can predict the temperature of the soldering iron as a function of time after it is activated. Assume that it is activated when the tip is at ambient temperature. Be sure to account for the fact that the heat transfer coefficient, the radiation resistance, and the heat capacity of the soldering iron tip are all a function of the temperature of the tip.

b.) Plot the temperature of the soldering iron as a function of time. Make sure that your plot covers sufficient time that your soldering iron has reached steady state.

c.) Verify that the soldering iron tip can be treated as a lumped capacitance.

4.9-7 (4-19 in text) Molten metal droplets must be injected into a plasma for an extreme ultraviolet radiation source, as shown in Figure P4.9-7.

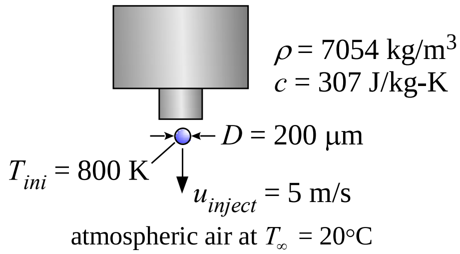


Figure P4.9-7: Injection of molten metal droplets.

The fuel droplets have a diameter of D = 200 μm and are injected at a velocity u_{inject} = 5 m/s with temperature T_{ini} = 800 K. The density of the droplet ρ = 7054 kg/m^3 and the specific heat capacity is c = 307 J/kg-K. You may assume that the droplet can be treated as a lumped capacitance. The droplet is exposed to still air at T_∞ =20ºC.

a.) Develop a numerical model in EES using the Integral command that can keep track of the velocity, temperature, and position of the droplet as a function of time.

b.) Plot the velocity as a function of time and the temperature as a function of time.

c.) Plot the temperature as a function of position. If the temperature of the droplet must be greater than 500 K when it reaches the plasma then what is the maximum distance that can separate the plasma from the injector?

4.9-9 (4-20 in text) Figure 4-34 in your text illustrates the drag coefficient for a cylinder as a function of Reynolds number.

a.) Using Figure 4-34 of your text, discuss briefly (1-2 sentences) why it might make sense to add dimples to a baseball bat.

b.) Using Figure 4-34 of your text, estimate how fast you would have to be able to swing a bat in order for it to make sense to think about adding dimples (the estimate can be rough but should be explained well). Assume that a bat has diameter D = 0.04 m and air has properties ρ = 1 kg/m^3 and μ = 0.00002 Pa-s.