Chapter 6: Natural Convection

Section 6.2: Natural Convection Correlations

6.2-1 (6-1 in text) A pipe that is 3 m long has an outer diameter D_{out} = 0.1 m and is bent in the center to form an "L" shape, as shown in Figure P6.2-1. One leg is vertical and the other leg is horizontal. The pipe is made of thin-walled copper and saturated steam at atmospheric pressure is circulating through the pipe. The pipe is in a large room and the air temperature far from the pipe is T_∞ = 30°C at atmospheric pressure. The conduction resistance associated with the pipe wall and the convection resistance associated with steam can be neglected.

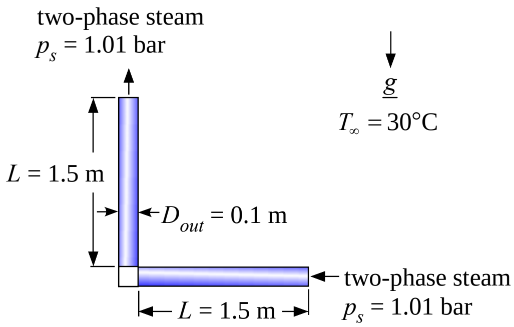

Figure P6.2-1: An "L"-shaped pipe.

a) Determine the Grashof, Rayleigh, and Nusselt numbers and the corresponding average heat transfer coefficient for the horizontal section of the pipe.
b) Determine the Grashof, Rayleigh, and Nusselt numbers and the corresponding average heat transfer coefficient for the vertical section of the pipe.
c) Calculate the total rate of heat transfer to the air.

6.2-2 Clean engine oil enters a L = 50 m long thin-walled pipe having an outer diameter D_{out} = 30 mm. The mass flow rate is $\dot{m}$ = 0.25 kg/s and the inlet temperature is T_{in} = 150°C. The pipe is suspended in a large room in which the air temperature is T_{∞} = 20°C. Estimate the heat transfer rate from the oil to the room and the outlet oil temperature.

6.2-3 (6-2 in text) A resistance temperature detector (RTD) is inserted into a methane pipeline to measure the gas temperature. The sensor is spherical with a diameter, D = 5.0 mm, and is exposed to methane at p_f = 10 atm with a fluid temperature of T_f = 20°C. The resistance of the sensor is related to its temperature; the resistance is measured by passing a known current through the resistor and measuring the associated voltage drop. The current causes an ohmic dissipation of $\dot{q}$ = 5.0 milliW. You have been asked to estimate the associated self-heating error as a function of the velocity of the methane in the pipe, V_f. Focus on the very low velocity operation (e.g., 0 to 0.1 m/s) where self-heating might be large. The self-heating error is the amount that the temperature sensor surface must rise relative to the surrounding fluid in order to transfer the heat associated with ohmic dissipation. You may neglect radiation for this problem.

a.) Assume that only forced convection is important and prepare a plot showing the self-heating error as a function of the methane velocity for velocities ranging from 0 to 0.1 m/s.

b.) Assume that only natural convection is important and determine the self-heating error in this limit. Overlay this value on your plot from (a).

c.) Prepare a plot that shows your prediction for the self heating error as a function of velocity considering both natural convection and forced convection effects. Assume that the pipe is mounted horizontally.

6.2-4 (6-3 in text) Figure P6.2-4 illustrates a flat plate solar collector that is mounted at an angle of τ = 45 degrees on the roof of a house. The collector is used to heat water; a series of tubes are soldered to the back-side of a black plate. The collector plate is contained in a case with a glass cover.

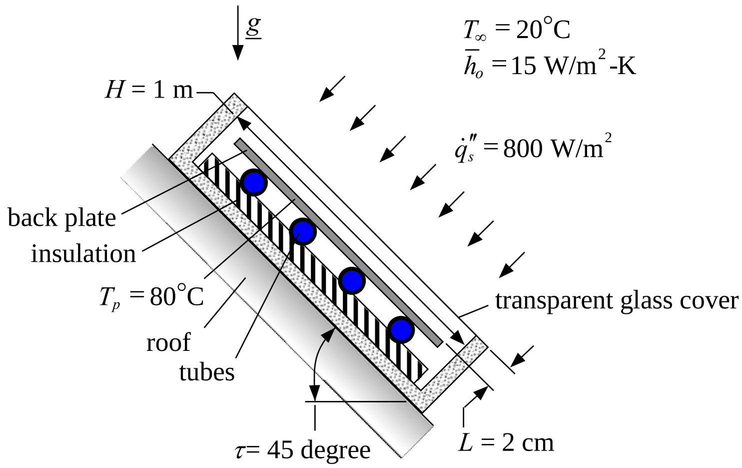

Figure P6.2-4: Flat plate solar collector

Assume that the solar collector is H = 1 m wide by W = 1 m long (into the page) and the distance between the heated plate and the glass covering is L = 2 cm. The collector receives a solar flux $\dot{q}''_s$ =800 W/m^2 and the collector plate can be assumed to absorb all of the solar energy. The collected energy is either transferred to the water in the pipe (in which case the energy is used to provide useful water heating) or lost due to heat transfer with the environment (either by radiation, which will be neglected in this problem, or convection). The collector plate temperature is T_p = 80°C and the ambient temperature is T_∞ = 20°C. The heat transfer coefficient on the external surface of the glass is due to forced convection (there is a slight breeze) and equal to $\overline{h}_o$ = 15 W/m^2-K. The glass is thin and can be neglected from the standpoint of providing any thermal resistance between the plate and ambient.

a.) Determine the rate of heat loss from plate due to convection; you may assume that the insulation on the back of the tubes is perfect so no heat is conducted to the roof and that radiation from the plate is negligible.

b.) What is the efficiency of the solar collector, $\eta_{collector}$, defined as the ratio of the energy delivered to the water to the energy received from the sun?

c.) Prepare a plot showing the collector efficiency as a function of the plate to glass spacing, L. Explain the shape of the plot.

6.2-5 (6-4 in text) Figure P6.2-5 illustrates a single-paned glass window that is L = 6 ft high and W = 4 ft wide; the glass is th_g = 0.25 inch thick and has conductivity k_g = 1.4 W/m-K.

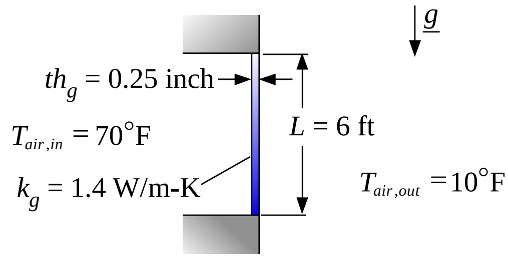

Figure P6.2-5: Single-paned glass window.

On a typical winter day, the outdoor temperature is $T_{air,out}$ =10°F and you keep the indoor temperature at $T_{air,in}$ = 70°F.

a.) On a still winter day, estimate the rate of heat loss from the window.

b.) Winter lasts t_{winter}= 90 days and you are heating with electrical resistance heaters. Electricity costs e_{cost} = \$0.12/kW-hr. How much does the heat loss through the window cost you over the course of 1 winter?

c.) Assume that 50% of your heat loss in your house is through your windows and that you have N_{window} = 10 single paned windows in your house. Prepare a plot showing the cost of heating your house as a function of the thermostat set point (i.e., the indoor air temperature).

6.2-6 (6-5 in text) The single-glazed window in Problem 6.2-5 (6-4 in text) is replaced with a double-glazed window. Both glass panes are 0.25 inch thick and the gap between the panes is 0.5 inch. The gap contains dry air at atmospheric pressure. All other information is the same as in Problem 6.2-5 (6-4). Neglect heat transfer by radiation.

a.) Repeat the calculations requested in parts (a) and (b) of Problem 6.2-5 (6-4).

b.) Summarize and explain the benefits of the double-glazed window.

6.2-7 (6-6 in text) You have seen an advertisement for argon-filled windows. These windows are similar in construction to the window described in Problem 6.2-7 (6-5 in text), except that argon, rather than air, is contained in the gap. Neglect heat transfer by radiation.

a.) Repeat Problem 6.2-7 (6-5) assuming that the gap contains argon.
b.) Are the claims that argon reduces heat loss valid? If so, why does this behavior occur?
c.) Would nitrogen (which is cheaper) work as well? Why or why not? Can you suggest another gas that would work better than argon?

6.2-8 (6-7 in text) You are involved in a project to design a solar collector for heating air. Two competing designs are shown in Figure P6.2-8.

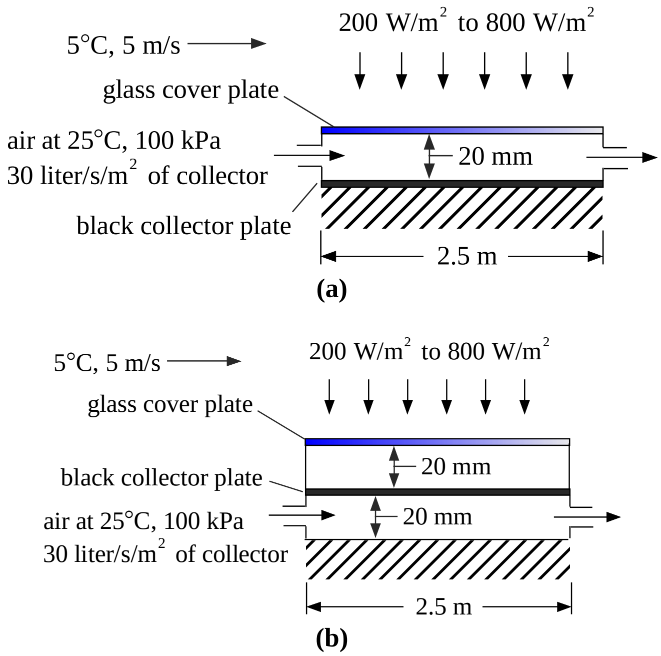

Figure P6.2-8: Air heating solar collector with (a) air flowing above the collector plate and (b) air flowing below the collector plate.

Both designs employ a transparent glass cover plate and a thin metal opaque black collector plate upon which solar radiation is completely absorbed. The glazing is standard safety glass with a thickness of 6 mm. In the first design, shown in Figure P6.2-8(a), air is blown through the gap between the cover and plate. In the second design, shown in Figure P6.2-8(b), the air flows in a second gap that is below the collector plate and free convection occurs between the collector plate and the glass cover plate. The collector is 1 meter wide (into the page) and 2.5 m long (in the air flow direction) and oriented horizontally. In both designs, the gaps are 20 mm wide. Air at 25°C, 100 kPa enters the flow passage at a flow rate of 30 liters/sec per square meter of collector area (area exposed to solar radiation) in both cases. The outdoor temperature (above the glass cover plate) is 5°C and there is a wind that may be represented as a forced convective flow with a free stream velocity of 5 m/s in the flow direction. Calculate and plot the efficiency of the two collector designs as a function of the solar radiation absorbed on the plate for values between 200 and 800 W/m^2. Assume that the insulation is adiabatic and neglect radiation in these calculations.

6.2-9 Figure P6.2-9 illustrates a device that is used to measure the thermal conductivity of a sample of material.

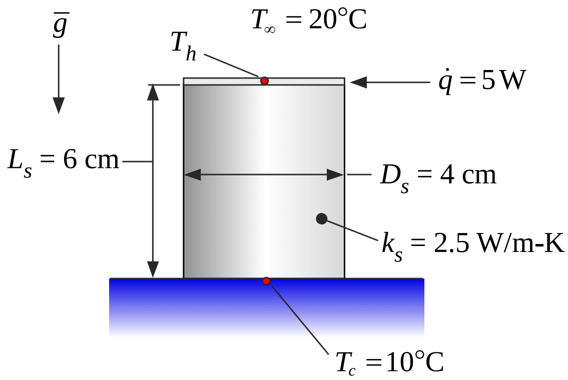

Figure P6.2-9: Thermal conductivity measurement device.

The sample has diameter D_s = 4 cm and length L_s = 6 cm. The actual conductivity of the sample is k_s = 2.5 W/m-K. A heat transfer rate of $\dot{q}$ = 5 W is applied to a thin plate that is attached to the top of the sample and the temperature at the top of the sample, T_h, is measured. The temperature of the bottom of the sample is maintained at T_c = 10ºC. The conductivity of the sample is computed according to:

$$k_{s,meas} = \frac{\dot{q}\, L_s\, 4}{\pi\, D_s^2 \left(T_h - T_c\right)} \tag{1}$$

Equation (1) is derived by assuming that all of the applied heat passes through the sample. You have been asked to assess the error in the sample conductivity measurement due to heat loss from the top of the sample by natural convection.

a.) Determine the hot end temperature, T_h. You may neglect the resistance to conduction through the heater, radiation from the top surface, convection from the sides of the sample, and contact resistance at the interfaces. Assume that the problem is steady-state.

b.) Use your answer from (a) to compute the measured conductivity and the fractional error in the conductivity measurement (i.e., the error normalized by the actual conductivity) that can be attributed to natural convection from the top of the device.

c.) If you can measure the temperature difference with an uncertainty of δT = 1 K then the fractional uncertainty in the conductivity measurement associated with the temperature measurement is given by:

$$err_{k,T} = \frac{\delta T}{\left(T_h - T_c\right)} \tag{2}$$

Determine the fractional uncertainty in the conductivity measurement due to the temperature measurement uncertainty.

d.) Plot the fractional error in the conductivity measurement due to natural convection (your answer from b), the fractional error in the conductivity measurement due to uncertainty in the temperature measurement (your answer from c), and the total fractional error in the conductivity measurement as a function of $\dot{q}$. Explain the shape of each of your plots.

6.2-10 Figure P6.2-10 illustrates the bottom of a freezer chamber placed in a laboratory environment.

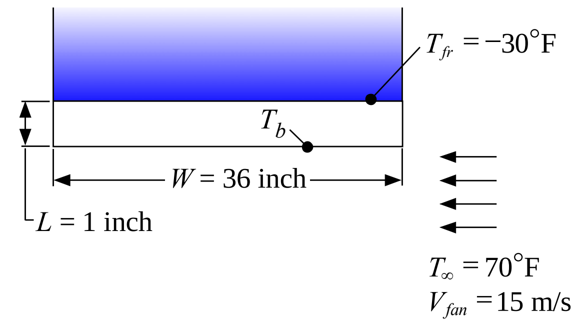

Figure P6.2-10: Air gap installed on the bottom surface of a freezer.

The freezer surface is maintained at T_{fr} = -30°F. It is necessary to prevent condensation and freezing from the water in the room air at T_∞ = 70°F and also to insulate the freezer. Therefore, an air gap is installed on the bottom surface. The bottom of the freezer is square with side width W = 36 inch and the width of the air gap is L= 1 inch. The outer surface of the air gap is exposed to flow driven by a fan with velocity V_{fan}= 15 m/s. You may neglect the conduction resistance of the wall as well as radiation.

a.) Determine the bottom temperature of the freezer (T_b in Figure P6.2-10) and the rate of heat transfer to the freezer.

b.) Plot the bottom surface temperature as a function of the fan velocity. Explain the shape of your plot.

c.) Plot the rate of heat transfer to the freezer as a function of the air gap spacing, L. Explain the shape of your plot. Assume that V_{fan} = 15 m/s for this problem.

6.2-11 A horizontal pipe carrying chilled water passes through a room filled with stagnant air at T_∞ = 75°F. The surface of the pipe is at T_s = 40°F. The pipe has an outer diameter of D = 0.5 inch and length L = 12 ft. Determine the rate of heat transfer to the pipe.

Section 6.3: Self-Similar Solutions

6.3-1 (6-8 in text) Reconsider Problem 6.2-5 (6-4 in text). In Problem 6.2-5 (6-4), the glass was assumed to be isothermal and the correlations for the average heat transfer coefficient were used. In this problem, account for the variation of the local heat transfer coefficient on either side of the window using the self-similar solution. Neglect conduction along the length of the glass but allow the glass temperature to vary with position due to the variation of the heat transfer coefficient with position.

a.) Determine the total rate of heat transfer through the window. Compare your answer with the solution for Problem 6.2-5 (6-4).

b.) Plot the inner and outer temperature of the glass as a function of position.

c.) If the relative humidity of the indoor air is 35% then will condensate form on the window? If so, at what location will the condensate end?

6.3-2 (6-9 in text) A self-similar solution can be obtained for the free convection problem where a heated vertical plate has a surface temperature (T_s) that varies with position according to: $T_s - T_\infty = A\,x^n$ where x is measured from the bottom of the plate.

a.) Transform the governing partial differential equations for momentum conservation in the x-direction and thermal energy conservation into ordinary differential equations for f and $\tilde{\theta}$.

b.) Transform the boundary conditions for u, v, and T into boundary conditions for f and $\tilde{\theta}$.

c.) Develop a numerical solution for this problem.

d.) Plot the dimensionless temperature and velocity ($\tilde{\theta}$ and $\frac{df}{d\eta}$) as a function of dimensionless position (η) for the case where Pr = 1 and n = 0.5.

e.) Plot the product of the local Nusselt number and the Grashof number based on the local plate temperature to the $-^1/_4$ power as a function of Pr for various values of n.

f.) Plot the product of the average Nusselt number and the Grashof number based on the average plate temperature to the $-^1/_4$ power as a function of Pr for various values of n.

g.) Plot the average Nusselt number as a function of the Grashof number based on the average plate temperature to the $-^1/_4$ power for a plate with a constant heat flux for Pr = 0.7.

Chapter 6: Natural Convection

Section 6.2: Natural Convection Correlations

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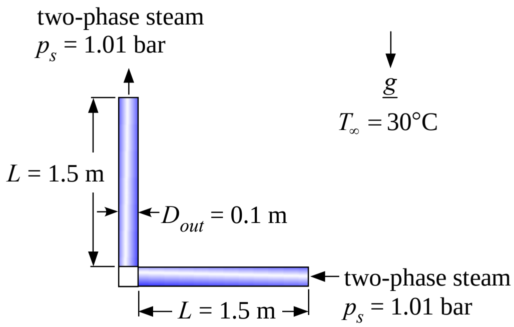

Figure P6.2-1: An "L"-shaped pipe.

a) Determine the Grashof, Rayleigh, and Nusselt numbers and the corresponding average heat transfer coefficient for the horizontal section of the pipe.
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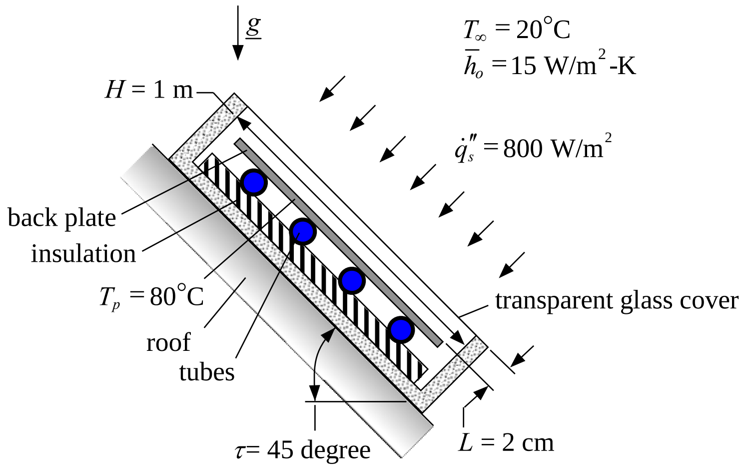

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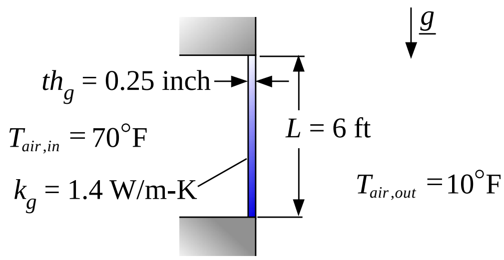

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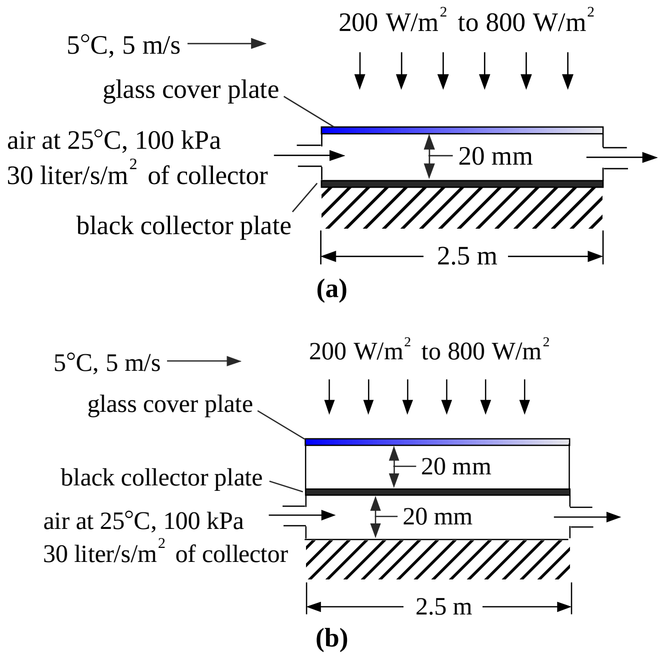

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