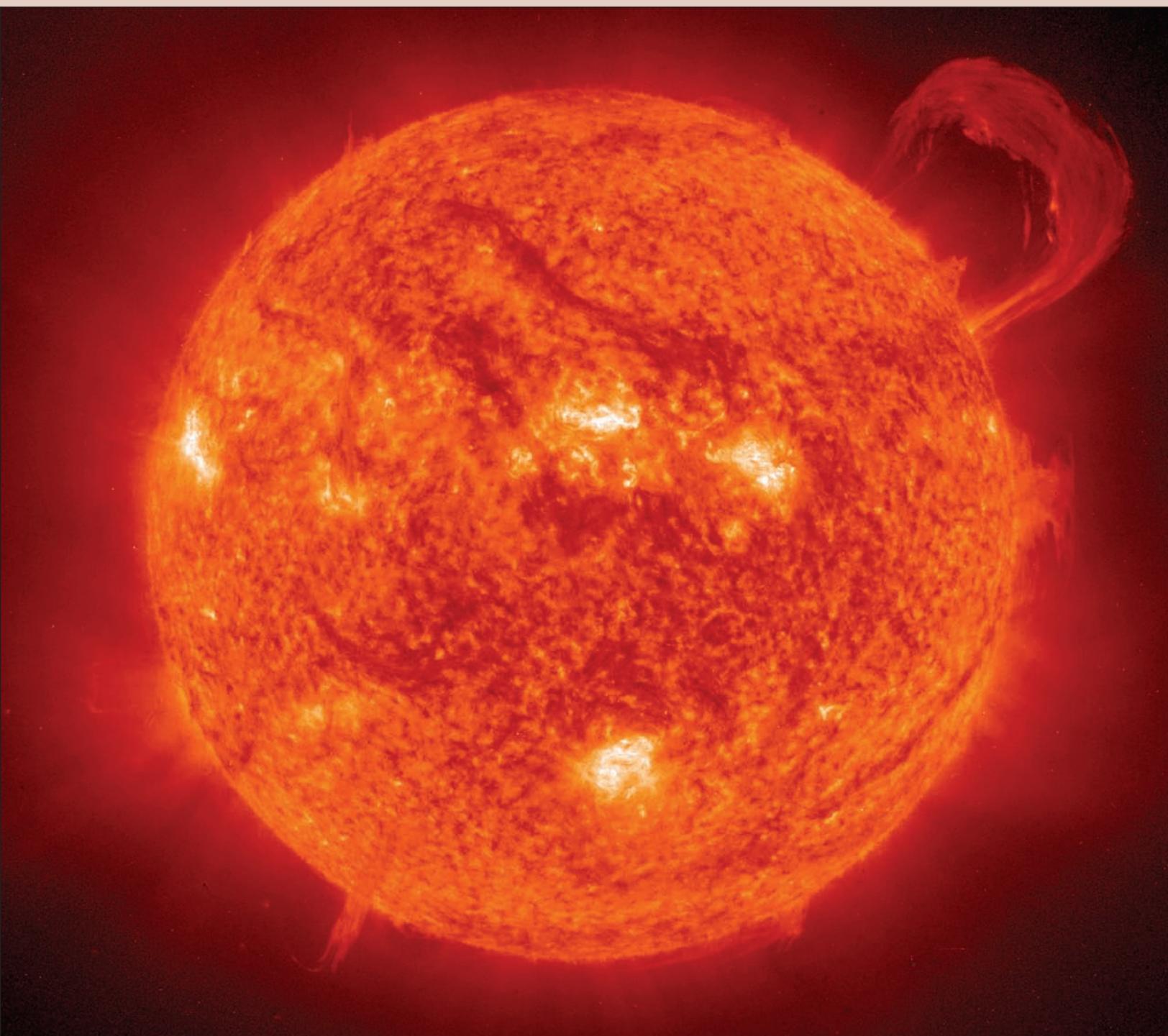


# ENERGY AND ENERGY TRANSFER



***After studying Chapter 4, you should:***

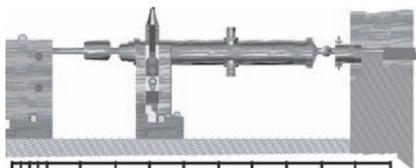
- *Understand the differences between bulk (macroscopic) and internal (microscopic) energies possessed by systems and control volumes.*
- *Understand the formal definitions of both heat and work and be able to state these precisely.*
- *Be able to identify various forms of work in practical situations and be able to calculate the following knowing their state variables: compression and/or expansion work, viscous stress work, shaft work, electrical work, and flow work.*
- *Be able to identify the three modes of heat transfer in practical situations.*
- *Understand and be able to use the concept of heat flux.*
- *Be able to write and explain Fourier's law of conduction for 1-D Cartesian, 1-D cylindrical, and 1-D spherical systems.*
- *Be able to explain the physical meaning of the temperature gradient,  $dT/dx$ , and, more generally,  $\nabla T$ .*
- *Be able to write rate expressions for convection heat transfer (both  $\dot{Q}_{\text{conv}}$  and  $\dot{Q}''_{\text{conv}}$ ) and to define all of the factors therein.*
- *Understand the following radiation concepts and definitions: blackbody properties, Stefan–Boltzmann law, emissive power, gray-body properties, and emissivity.*
- *Be able to write and explain the simplified rate law for radiation exchange between two surfaces, recognizing that many restrictions apply.*

# Chapter 4 Overview

IN THIS CHAPTER, we review the concept of energy and the various ways in which a system or control volume can possess energy at both microscopic (molecular) and macroscopic levels. We also carefully define heat and work, which are boundary interactions and, therefore, not properties of a system or control volume. The chapter concludes with an examination of the rate laws that govern heat transfer. We begin with a brief historical overview of our subject matter.



Benjamin Thompson (1753–1814)



Thompson's canon boring experiments connected the concepts of *heat* and *work*.

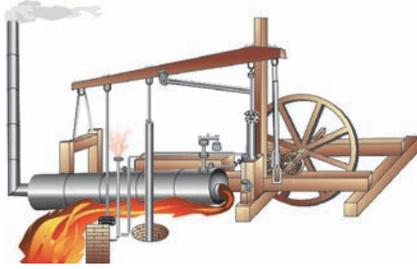
## 4.1 HISTORICAL CONTEXT

With the word *energy* being commonplace, you may find it difficult to believe that the scientific underpinning of this concept as we know it today is just 200 years old. In 1801, **Thomas Young** (1773–1829) (of Young's modulus fame) presented the idea that *the energy of a system is the capacity to do work* [1]. In the early 1800s, various forms of energy had yet to be defined in useful ways. For example, the concept of heat was a muddle of the concepts that we now distinguish as temperature, internal energy, and heat. The discovery that energy is conserved—a premier conservation principle of classical physics—had to wait until **Robert Mayer's** (1814–1878) statement of the theory of conservation of energy in 1842. One of the keys to the recognition of this law was that work could be converted to heat and vice versa. **Benjamin Thompson** (1753–1814), a traitor to the colonists in the Revolutionary War between Great Britain and her American colonies, discovered in 1798, by a series of carefully planned and conducted experiments, that friction produces an inexhaustible supply of heat. Exactly what heat is, however, was not clear in 1798.

The invention and development of the steam engine spurred theoretical development of the thermal sciences at the end of the eighteenth and beginning of the nineteenth centuries. Ironically, the steam engine's very low thermal efficiency (a few percent) may have prevented **Sadi Carnot** (1796–1832), the discoverer of the **second law of thermodynamics**, from also discovering the **first law of thermodynamics** (i.e., the conservation of energy principle). With early steam engines converting such a small percentage of the heat supplied to work, it appeared to Carnot that there was no conversion of heat to work, but merely a heat addition at high temperature and an equal heat rejection at a low temperature. For the interested reader, a timeline of important lives and events in the history of the thermal sciences is presented in Appendix A.

## 4.2 SYSTEM AND CONTROL-VOLUME ENERGY

A system or a control volume can possess energy, first, as a consequence of its bulk motion or position within a force field, such as gravity, and second, as a consequence of the motion of its constituent molecules and their



Oliver Evan's high-pressure *Columbian* steam engine patented in 1790. Drawing courtesy of Library of Congress.

interactions (i.e., the internal energy). We can therefore express the total energy as the sum of these two energy components:

$$E_{\text{sys}} = E_{\text{bulk}} + U_{\text{internal}} \quad (4.1a)$$

The same relationship holds for a control volume:

$$E_{\text{cv}} = E_{\text{bulk,cv}} + U_{\text{internal,cv}} \quad (4.1b)$$

We now examine each of the terms on the right-hand side of Eq. 4.1.

## 4.2a Energy Associated with System or Control Volume as a Whole

A system or control volume can possess energy by virtue of its bulk motion. The most common forms of this bulk energy in engineering applications are kinetic energy  $KE$  and potential energy  $PE$ . We can then write

$$E_{\text{bulk}} = (KE)_{\text{bulk}} + (PE)_{\text{bulk}} \quad (4.2)$$

Assuming a rigid system with no relative motion of subsystem elements, we may easily relate the kinetic and potential energies to the linear and angular velocities of the system and its position as follows:

$$E_{\text{bulk}} = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 + Mg(z - z_{\text{ref}}) \quad (4.3)$$

Here we assume that the only contribution to the potential energy is the action of gravity. The first two terms on the right-hand side of Eq. 4.3 are the translational and rotational contributions to the system kinetic energy, where  $V$  is the magnitude of the bulk translational velocity of the system center of mass,  $I$  is the system rotational moment of inertia,  $\omega$  is the system angular velocity, and  $z_{\text{ref}}$  is some reference elevation. Implicitly, the reference kinetic energies are zero (i.e.,  $V_{\text{ref}} = 0$  and  $\omega_{\text{ref}} = 0$ ). Equation 4.3 should be familiar to you from your previous study of physics and rigid-body mechanics. If other external fields exist in addition to gravity (e.g., magnetic or electrostatic fields), additional contributions to the potential energy can result depending on the nature of the matter within the system. Discussion of these effects is beyond the scope of this book. Additional information can be found in more advanced texts such as Ref. [2].

The specification of a single translational velocity and a single angular velocity in Eq. 4.3 is possible only for a rigid system. In a control volume, the bulk velocity is likely to vary with position; thus, the kinetic energies are obtained by integrating over the control volume:

$$(KE)_{\text{bulk}} = \frac{1}{2} \int_{M_{\text{cv}}} V^2 dM, \quad (4.4a)$$

or

$$(KE)_{\text{bulk}} = \frac{1}{2} \int_{\text{CV}} \rho V^2 dV, \quad (4.4b)$$



**FIGURE 4.1**

*Examples of systems and a control volume having macroscopic energies: (a) a baseball thrown with spin imparted, (b) a bicycle rolling down a hill, and (c) a solid-fuel rocket.*

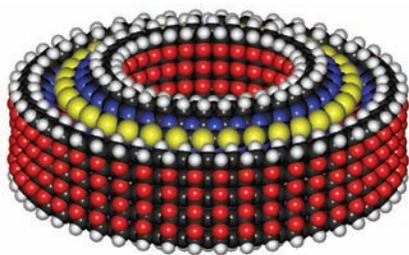
where the differential mass  $dM = \rho dV$ . The application of Eq. 4.4 to a rigid translating and rotating system captures the two kinetic energy terms of Eq. 4.3; thus, Eq. 4.4 applies equally well to a control volume or a system.

Figure 4.1 shows several examples that illustrate the energies associated with bulk motions. The spinning baseball in Fig. 4.1a is a system of fixed mass. At any instant in time, the baseball possesses a particular translational velocity  $V$ , a particular angular velocity  $\omega$ , and a particular elevation relative to a datum,  $z - z_{\text{ref}}$ . Knowing the mass of the ball and its mass moment of inertia, we could easily calculate the instantaneous values of the terms contributing to  $E_{\text{bulk}}$ .

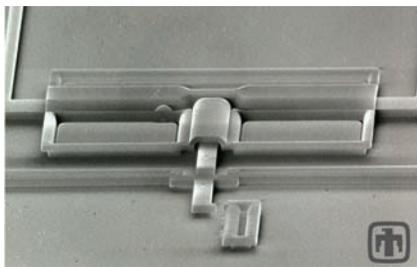
A similar, but more complex, example is the system defined by the bicycle (Fig. 4.1b), where we have deliberately excluded the rider from our system. In this case, the translational kinetic energy and the potential energy are easy to determine. Since the entire system does not rotate with a single velocity about a common axis, the rotational kinetic energy would have to be determined by subdividing the system to treat each rotating part separately. For example, each wheel makes a separate contribution to the system rotational kinetic energy.

A third example (Fig. 4.1c) is a control volume containing a solid-fuel rocket. We assume that the rocket contains no internal macroscopic moving parts, such as pumps, as would be found in a liquid-fuel rocket. With this assumption, evaluating the control-volume kinetic energy is simplified. Nevertheless, evaluating the integral in Eq. 4.4 remains a daunting task. To do this requires knowledge of the local velocity and density at every point within the complex flow created by the burning of the solid propellant. Evaluating the potential energy is much easier. If the control-volume center of mass does not change as the propellant is consumed, or if we ignore any small change in its location, the potential energy is simply  $M_{\text{cv}}g(z - z_{\text{ref}})$ .

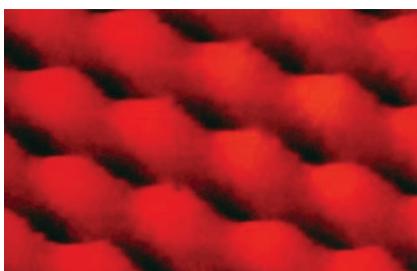
The three examples of Fig. 4.1 were deliberately chosen to illustrate situations in which macroscopic system energies can be important. In many



Molecular nanotechnology seeks to build engineering devices and components, like this bearing and sleeve, using sequences of chemical reactions. Drawing courtesy of Zyvex

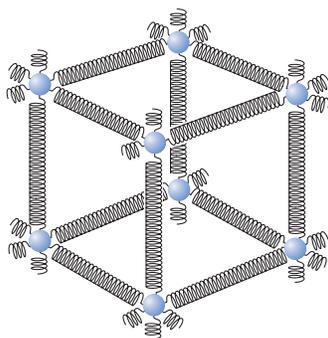


In this single-piston microsteam engine, electric current heats and vaporizes water within the cylinder pushing the piston out. Capillary forces then retract the piston once current is removed. Courtesy Sandia National Laboratories, SUMMiT™ Technologies, [www.mems.sandia.gov](http://www.mems.sandia.gov).



False-color scanning tunneling microscope (STM) image of the surface of pyrolytic graphite reveals the regular pattern of individual carbon atoms.

Rereading the section on internal energy in Chapter 2 is useful at this point.



**FIGURE 4.2**  
*Energy storage in the vibrating lattice of a solid.*

thermal science applications, however, these macroscopic system energies can be neglected because they, or their changes, are small compared to heat and work exchanges at boundaries and to changes in internal (molecular) energy during a process. Alternatively, we may select system or control-volume boundaries to exclude some bulk energies. Nevertheless, being able to identify all of the various energies and energy exchanges is crucial in analyzing thermal systems. In our future theoretical developments and examples, we will be careful to point out when we neglect the macroscopic energies. Such practice should help you develop critical thinking in your approach to thermal science problems.

## 4.2b Energy Associated with Matter at a Microscopic Level

As defined in Chapter 2, **internal energy** is the energy associated with the motion of the microscopic particles (atoms, molecules, electrons, etc.) comprising a system or control volume. Seeing how macroscopic properties relate to the microscopic structure of matter can be intellectually satisfying, although an understanding of microscopic behavior is not necessary to solve most engineering problems in the thermal sciences. (Recall that the subjects of classical thermodynamics and heat transfer predate modern ideas concerning the microscopic nature of matter.) Interestingly, many of the challenges associated with microminiaturization of engineering devices (a hot topic in mechanical engineering at the time of this book's writing) require a detailed understanding of microscopic behavior. See, for example, Refs. [3, 4].

In Chapter 2, we saw that gas molecules possess energy in ways analogous to our macroscopic systems: translational kinetic energy, rotational kinetic energy, and vibrational kinetic and potential energies. As the temperature is increased, not only is more energy associated with some storage modes (e.g., translational kinetic energy) but new states become accessible to the molecule (e.g., vibrational kinetic and potential energies). Figure 2.4 illustrated these degrees of freedom and energy storage modes; their impact on specific heats was shown in Fig. 2.6.

For solids, the internal energy is associated with lattice vibrations that give rise to vibrational kinetic and potential energies. Figure 4.2 illustrates in cartoon fashion the basic structure of a solid being composed of masses (molecular centers) connected by springs (intermolecular forces). The internal energy associated with liquids has its origin in the relatively close range interactions among the molecules making up the liquid. A simplified view is to consider the structure of a liquid lying between the extremes of a disorganized gas and a well-ordered crystalline solid.

## 4.3 ENERGY TRANSFER ACROSS BOUNDARIES

### 4.3a Heat

#### *Definition*

The recognition that heat is not a property of a system—not something that the system possesses—but, rather, an exchange of energy from one system to another, or to the surroundings, was a breakthrough in thermodynamics. Our

formal definition of heat [2] is the following:

**Heat is energy transferred, without transfer of mass, across the boundary of a system (or across a control surface) because of a temperature difference between the system and the surroundings or a temperature gradient at the boundary.**

Because of the importance of this definition, let us elaborate some of the important implications: First, heat occurs only at the boundary of a system; that is, it is a boundary phenomenon. As a consequence, a system cannot *contain* heat. The addition of heat to a system with all else held constant, however, increases the energy of a system, and, conversely, heat removal from a system decreases the energy of a system. Another important element in this definition of heat is that the “driving force” for this energy exchange is a temperature difference or temperature gradient. Other energy transfers across a system boundary may occur, but only heat is controlled solely by a temperature difference. We can gain some physical insight into this boundary energy exchange by examining the molecular processes involved. The energy exchange proper is carried out by the collision of molecules in which the higher kinetic energy molecules, in general, impart some of their energy to the lower kinetic energy molecules. Because the higher kinetic energy molecules are at a higher temperature than the lower energy molecules, the direction of the energy exchange is from high temperature to low temperature. A more rigorous treatment of this process would involve the subject of irreversible thermodynamics [5–7] and is beyond the scope of this book.



### Semantics

In spite of our attempts here to be very precise about the definition of heat, some semantic problems arise out of traditions and nomenclature developed prior to the advent of modern thermodynamic principles. Specifically, in the common term *heat transfer*, the word *transfer* is redundant. The science of heat transfer was developed in the early 1800s [8], prior to our understanding that heat is not possessed by a body and to the development of energy conservation.<sup>1</sup>

Rates of heat transfer and rates per unit area are also of importance and need to be distinguished. We adopt the following symbols to denote these heat interactions:

$$Q = \text{heat (or heat transfer)} \quad [=] \text{ J,}$$

$$\dot{Q} = \text{rate of heat transfer} \quad [=] \text{ J/s or W,}$$

$$\dot{Q}'' = \text{heat flux} \quad [=] \text{ W/m}^2.$$

We also use the word **adiabatic** to describe a process in which there is no heat transfer. Later in this chapter we will elaborate on the principles of heat transfer.

<sup>1</sup> When Fourier published his theory of heat transfer (1811–1822), the common wisdom was that *caloric* (or heat) was a material substance, despite the fact that Benjamin Thompson in 1798 had shown that friction produces an inexhaustible supply of heat. Regardless of the fundamental nature of heat, Fourier’s mathematical analyses describing temperature distributions in solids are accurate descriptions and stand yet today as the foundation of heat-transfer theory.

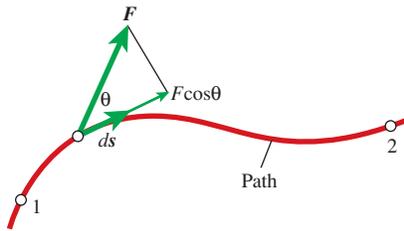


Heat transfer is important in food preparation and body temperature regulation.

### 4.3b Work

#### Definition

Another fundamental transfer of energy across a system boundary is work. All forms of work, regardless of their origin, are fundamentally expressions of a force acting through a distance,



$$\delta W = \mathbf{F} \cdot d\mathbf{s}; \quad (4.5)$$

that is, work is the scalar product of a vector force and the displacement vector  $d\mathbf{s} = \hat{i}dx + \hat{j}dy + \hat{k}dz$ , where  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  are the unit vectors in a Cartesian coordinate system. We adopt the notation  $\delta W$  to indicate the incremental quantity of work done along the differential path associated with the tangent of  $ds$ . For a particular process, Eq. 4.5 can be integrated following the process path from position 1 to position 2 to obtain the total work done:

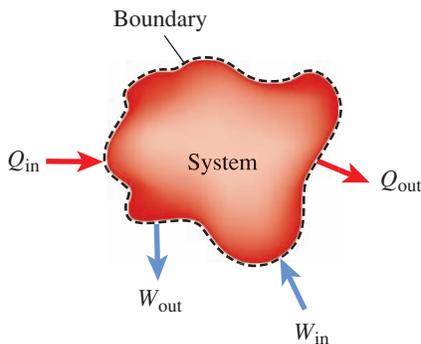
$${}_1W_2 \equiv \oint \mathbf{F} \cdot d\mathbf{s}, \quad (4.6)$$

where the  $\oint$  symbol indicates a path integral. Equations 4.5 and 4.6 should be familiar to you from your previous study of physics.

We adopt the particular notation  ${}_1W_2$  to emphasize that this quantity is the work done in going from point 1 to point 2, which cannot be represented as a difference. In contrast, energy changes associated with a system undergoing a process are expressed as differences. For example, the change in system energy for a process that takes the system from state 1 to state 2 is expressed  $\Delta E = E_2 - E_1$ . To write a similar expression for work would be nonsense, as there is no such thing as  $W_1$  or  $W_2$ .

The force appearing in Eqs. 4.5 and 4.6 may be a purely mechanical one, or it may have other origins, such as the force acting on a charge moving through an electric field or the force on a particle with a magnetic moment moving through a magnetic field. In mechanical engineering applications, work is most commonly associated with mechanical and electrical forces. In this book, we deal only with work arising from these two forces, although we define a mechanical force fairly broadly.

In our study of thermal sciences, it is important to emphasize that work occurs *only* at the boundary of a system or a control volume. Like heat, work is not possessed by a thermodynamic system or a control volume but is just the name of a particular form of energy transfer from a system to the surroundings, or vice versa. For this reason we draw arrows representing work or heat that start or stop at the system or control volume boundary without crossing. In this context, we offer the following formal definition of work:



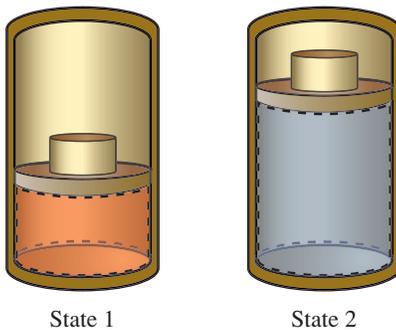
**Work is the transfer of energy across a system or control-volume boundary, exclusive of energy carried across the boundary by a flow, and not the result of a temperature gradient at the boundary or a difference in temperature between the system and the surroundings.**

Before presenting examples of work, it is useful to convert Eq. 4.6 to a form expressing the rate at which work is done. The time rate of doing work

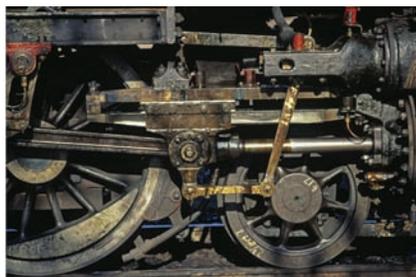
Table 4.1 Common Types of Work

Type	Expression for $W$	Expression for $\dot{W}$ or $\mathcal{P}^*$
Expansion or compression work	$\delta W = Pd\mathcal{V}$ ${}_1W_2 = \int_1^2 Pd\mathcal{V}$	$\dot{W} = P \frac{d\mathcal{V}}{dt}$
Viscous work	${}_1W_2 = \int_{t_1}^{t_2} \tau_{\text{visc}} AV dt$	$\dot{W} = \tau_{\text{visc}} AV$
Shaft work	${}_1W_2 = \int_{\Omega_1}^{\Omega_2} \mathcal{T} d\Omega$	$\dot{W} = \mathcal{T} \omega$
Electrical work	${}_1W_2 = \int_{t_1}^{t_2} i \Delta \mathcal{V} dt$	$\dot{W} = i \Delta \mathcal{V}$
Flow work	${}_1W_2 = \int_{t_1}^{t_2} \dot{m} P \mathcal{V} dt$	$\dot{W} = \dot{m} P \mathcal{V}$

\*Rate of work,  $\dot{W}$  and power  $\mathcal{P}$  are used synonymously throughout this book.



**FIGURE 4.3**  
Work is done as a gas expands and pushes back the surroundings in a piston–cylinder assembly.



is called **power**, defined as

$$\mathcal{P} \equiv \dot{W} = \lim_{\Delta t \rightarrow 0} \frac{\delta W}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\mathbf{F} \cdot d\mathbf{s}}{\Delta t} = \mathbf{F} \cdot \frac{d\mathbf{s}}{dt}, \quad (4.7a)$$

or

$$\mathcal{P} = \mathbf{F} \cdot \mathbf{V}, \quad (4.7b)$$

where we recognize that  $ds/dt$  is the velocity vector  $\mathbf{V}$ . From this definition, we see that power enters or exits a system or control volume wherever a component of a force is aligned with the velocity at the boundary.

### Types

Some common types of work are listed in Table 4.1. In many situations, the power, or rate of working, is the important quantify; therefore, expressions to evaluate the power are also shown.

**Expansion (or Compression) Work** In systems or control volumes where a boundary moves, work is performed *by* the system if it expands, whereas work is done *on* the system if the system is compressed. Concomitantly, work is done *on* the surroundings by an expanding system, and work is done *by* the surroundings when the system contracts. As an example of this type of work, consider the expansion of a gas contained in a piston–cylinder assembly as shown in Fig. 4.3. We use this specific example to illustrate application of the

fundamental definition of work and some of the subtleties that need to be considered. Examining the entire boundary of the system, we see that work can only be done at the portion of the boundary that is in contact with the piston, as this is the only part of the boundary where there is motion. To have a force act over a distance (Eq. 4.5), the boundary must move. In our examination of the boundary, we also note that the only force present is that due to the pressure of the gas in the system, where we have ignored any possible viscous forces created by friction between the cylinder wall and the moving gas. For our simple geometry, the magnitude of the force exerted by the gas on the piston is given by

$$F = PA,$$

where  $A$  is the cross-sectional area of the piston and  $P$  is the pressure. The pressure force acts vertically upward in the same direction as the piston motion; thus, the dot product  $\mathbf{F} \cdot d\mathbf{s}$  in our definition reduces to the product of the magnitude of the force and the vertical displacement (i.e.,  $Fdx$ ). The incremental work done is then

$$\delta W = PAdx.$$

For our simple cylindrical geometry, we immediately recognize that  $Adx$  is the volume displaced; that is,

$$d\mathcal{V} = Adx.$$

The incremental work then is

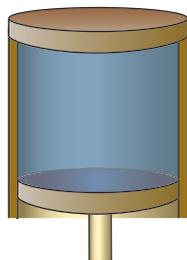
$$\delta W = Pd\mathcal{V}, \quad (4.8a)$$

and so the total work done in going from state 1 to state 2 is

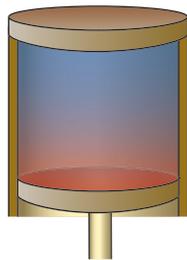
$${}_1W_2 = \int_1^2 Pd\mathcal{V}. \quad (4.8b)$$

We can also express the instantaneous power produced as

$$\mathcal{P} = \dot{W} = P \frac{d\mathcal{V}}{dt}. \quad (4.8c)$$



Slow compression



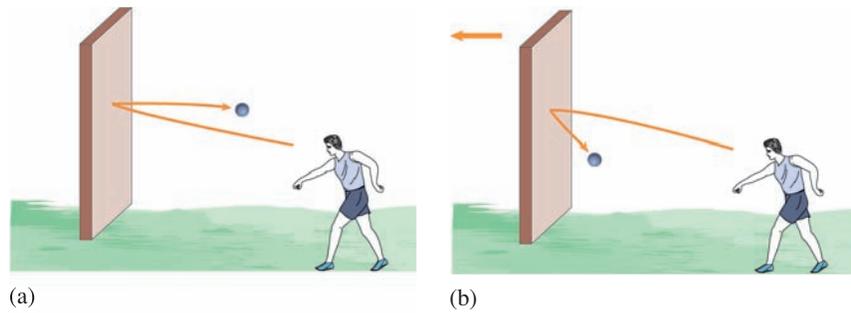
Rapid compression

These expressions for work and power (Eqs. 4.8a–4.8c) represent the single reversible work mode associated with a simple compressible substance.

At this juncture it is important to ask, What assumptions are built into Eqs. 4.8 that might restrict their use? First, as suggested in our development, we assume that the only force acting at the system boundary is that resulting from pressure. Second, we require that the pressure be a meaningful thermodynamic property of the system as a whole. For this to be true, the motion of the piston must be sufficiently slow so that there is enough time for a sufficient number of molecular collisions to cause the pressure to be uniform within the gas volume. A characteristic time to achieve mechanical (pressure) equilibrium is of the order of the height of the volume divided by the speed of sound in the gas. As an example, consider room-temperature air and a volume height of 150 mm ( $\sim 6$  in). For this situation, approximately 0.4 ms are required for the change at the moving boundary to be communicated to the gas

**FIGURE 4.4**

(a) A ball thrown at a stationary wall rebounds with the same speed that it strikes the wall. (b) If the wall is moving away from the incoming ball, the rebound velocity is less than the approach velocity.



Equilibrium and quasi-equilibrium processes are discussed in Chapter 1. You may find a review of that material useful here.

molecules at the bottom of the cylinder. Thus, our theoretical restriction is that the process must be quasi-static, where the practical meaning of quasi-static is determined by the time scale  $t_c \equiv L_c/a$ , where  $L_c$  is the characteristic length and  $a$  is the speed of sound.

Our piston–cylinder example is also useful to illustrate that motion is required to produce work, that is, to transfer energy from the gas molecules to the surroundings in the form of work. Consider throwing a ball against a stationary wall in which the ball rebounds in a perfectly elastic manner as suggested in Fig. 4.4. If, say, you threw the ball at 100 m/s, the ball would rebound back at 100 m/s. The kinetic energy of the ball,  $MV^2/2$ , is thus the same before and after the collision. The ball experiences no loss of energy. We now allow the wall to move. What then happens to the magnitude of the rebound velocity? In this case, the ball will return with a velocity less than its incoming velocity. For example, when the wall moves at 25 m/s, the rebound velocity is 50 m/s, a value substantially less than the incoming velocity of 100 m/s. The kinetic energy of the ball undergoes a considerable reduction after the collision with the moving wall. Analogous to our ball-throwing example, gas molecules lose energy in their collisions with a receding boundary. If the boundary is advancing, the molecules, of course, gain energy. This example highlights the fundamental idea that work is an energy transfer across a boundary.

### Example 4.1



Consider a piston–cylinder arrangement containing  $5.057 \times 10^{-4}$  kg of dry air. For the following two quasi-static processes, determine the quantity of work performed by or on the air in the cylinder:

- Constant-pressure heat-addition process
- Isothermal expansion process

Also completely define the final state (i.e.,  $P_2$ ,  $V_2$ , and  $T_2$ ) and sketch the process on  $P$ – $V$  coordinates. For both processes, the following conditions apply:

**At initial state**

$$V_1 = 2.54 \times 10^{-4} \text{ m}^3$$

$$T_1 = 350 \text{ K}$$

**At final state**

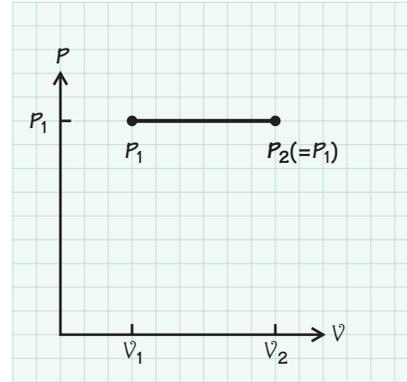
$$V_2 = 5.72 \times 10^{-4} \text{ m}^3$$

#### **Solution (Part A)**

*Known* Constant-pressure process,  $M$ ,  $V_1$ ,  $T_1$ ,  $V_2$

*Find*  ${}_1W_2$ ,  $P_2$ ,  $T_2$

## Sketch



## Assumptions

- i. Quasi-static process (given)
- ii. Ideal-gas behavior

*Analysis* Because the process is quasi-static, we can apply Eq. 4.8b, which is easily integrated since the pressure is constant, so

$${}_1W_2 = \int_1^2 P d\mathcal{V} = P \int_1^2 d\mathcal{V} = P(\mathcal{V}_2 - \mathcal{V}_1).$$

To evaluate this equation, we only need to find the pressure, as both  $\mathcal{V}_1$  and  $\mathcal{V}_2$  are given. To find  $P$ , we apply the ideal-gas equation of state (Eq. 2.28c) at state 1:

$$P_1 \mathcal{V}_1 = MRT_1,$$

or

$$P_1 = \frac{MRT_1}{\mathcal{V}_1},$$

where  $R$  ( $\equiv R_u/\mathcal{M} = 287.0 \text{ J/kg} \cdot \text{K}$ ) is the gas constant for air (Appendix C). Substituting numerical values, we get

$$P_1 = \frac{5.057 \times 10^{-4} (287.0) 350}{2.54 \times 10^{-4}} = 200 \times 10^3$$

$$[=] \frac{\text{kg}(\text{J/kg} \cdot \text{K}) \text{K}}{\text{m}^3} \left[ \frac{1 \text{ N} \cdot \text{m}}{\text{J}} \right] \left[ \frac{1 \text{ Pa}}{\text{N/m}^2} \right] = \text{Pa}.$$

The work is then

$${}_1W_2 = 200 \times 10^3 (5.72 \times 10^{-4} - 2.54 \times 10^{-4}) = 63.6$$

$$[=] \text{Pa}(\text{m}^3) \left[ \frac{1 \text{ N/m}^2}{\text{Pa}} \right] \left[ \frac{1 \text{ J}}{\text{N} \cdot \text{m}} \right] = \text{J}.$$

We now define the final state. If we know two independent, intensive properties, the state principle tells us that all other properties can be found. Since  $P$  is constant, we know  $P_2 = P_1 = 200 \text{ kPa}$ , and  $\mathcal{V}_2$  and  $M$  are given. With this information, we again apply the ideal-gas equation of state

(Eq. 2.28c), this time to find  $T_2$ :

$$\begin{aligned} T_2 &= \frac{P_2 \mathcal{V}_2}{MR} \\ &= \frac{200 \times 10^3 (5.72 \times 10^{-4})}{5.057 \times 10^{-4} (287.0)} = 788.2 \\ [=] & \frac{\text{Pa}(\text{m}^3)}{\text{kg}(\text{J}/\text{kg} \cdot \text{K})} \left[ \frac{1 \text{ N}/\text{m}^2}{\text{Pa}} \right] \left[ \frac{1 \text{ J}}{\text{N} \cdot \text{m}} \right] = \text{K}. \end{aligned}$$

*Comment (Part A)* Knowing that the pressure is constant made the calculation of the work quite easy. Note also the importance of the ideal-gas equation of state to determine properties at both state 1 and state 2.

### Solution (Part B)

*Known* Isothermal process,  $M$ ,  $\mathcal{V}_1$ ,  $T_1$ ,  $\mathcal{V}_2$

*Find*  ${}_1W_2$ ,  $P_2$ ,  $T_2$ .

*Assumptions*

- i. Quasi-static process
- ii. Ideal-gas behavior

*Analysis* We delay drawing a  $P$ - $\mathcal{V}$  sketch until the appropriate mathematic relationship between  $P$  and  $\mathcal{V}$  is determined. We appeal again to the ideal-gas law to do this:

$$P = MRT \left( \frac{1}{\mathcal{V}} \right).$$

Here we recognize that 1.  $MRT$  is a constant, since an isothermal process is one carried out at constant temperature, and 2. the  $P$ - $\mathcal{V}$  relation is hyperbolic ( $P \sim \mathcal{V}^{-1}$ ). The work can now be found from Eq. 4.8b as

$$\begin{aligned} {}_1W_2 &= \int_1^2 P d\mathcal{V} = MRT_1 \int_1^2 \frac{d\mathcal{V}}{\mathcal{V}} \\ &= MRT_1 [\ln \mathcal{V}]_1^2 = MRT_1 (\ln \mathcal{V}_2 - \ln \mathcal{V}_1) = MRT_1 \ln \frac{\mathcal{V}_2}{\mathcal{V}_1}. \end{aligned}$$

Substituting numerical values, we obtain

$$\begin{aligned} {}_1W_2 &= 5.057 \times 10^{-4} (287.0) (350) \ln \left[ \frac{5.72 \times 10^{-4}}{2.54 \times 10^{-4}} \right] = 41.2 \\ [=] & \text{kg}(\text{J}/\text{kg} \cdot \text{K})\text{K} = \text{J}. \end{aligned}$$

To completely define state 2, we now need  $P_2$ . Following the same procedure as in Part A, we apply the ideal-gas equation of state (Eq. 2.28c):

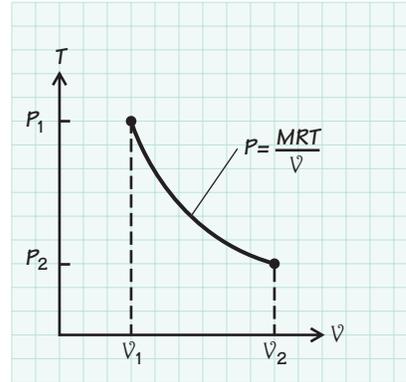
$$P_2 = \frac{MRT_2}{\mathcal{V}_2}.$$

Since  $T_2 = T_1 = 350$  K,

$$P_2 = \frac{5.057 \times 10^{-4} (287.0)(350)}{5.72 \times 10^{-4}} = 88.8 \times 10^3$$

$$[=] \frac{\text{kg}(\text{J}/\text{kg} \cdot \text{K})\text{K}}{\text{m}^3} \left[ \frac{1 \text{ N} \cdot \text{m}}{\text{J}} \right] \left[ \frac{1 \text{ Pa}}{\text{N}/\text{m}^2} \right] = \text{Pa}.$$

We can now plot this process on  $P$ - $\mathcal{V}$  coordinates:



*Comment (Part B)* Note that the area under this curve is the work. Comparing this graph with that of Part A, we immediately see that less work is performed in the isothermal process, which is consistent with our calculations. Being able to sketch a process on  $P$ - $\mathcal{V}$  coordinates is particularly useful in dealing with thermodynamic systems. Such sketches immediately show the work (area under the curve), provided the process is carried out quasi-statically, a requirement for  ${}_1W_2 \equiv \int P d\mathcal{V}$ . Note also that in the solutions to both Parts A and B, we employed only fundamental definitions and the state principle.

### Self Test 4.1



Determine whether the work in Example 4.1 is performed on or by the system and on or by the surroundings.

*(Answer: Since the numerical values for the work for the system are positive, the work is being performed by the system. A commensurate quantity of work is performed on the surroundings.)*

### Example 4.2 SI Engine Application

Chapter 6 shows the importance of isentropic processes in defining ideal efficiencies for power producing devices. ➤

The compression and expansion processes associated with spark-ignition engines can be modeled crudely as quasi-static, adiabatic (no heat transfer), isentropic (constant-entropy) processes. For compression and expansion processes described in this way, neither the pressure nor the temperature will remain constant during the process (cf. Example 4.1). Furthermore, we assume that the working fluid is dry air, rather than a mixture of fuel and air (compression) or combustion products (expansion).<sup>2</sup> With these assumptions,

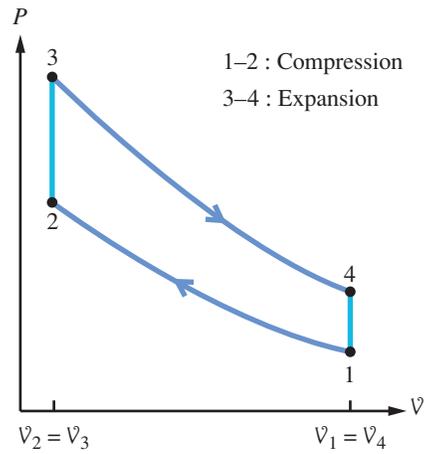
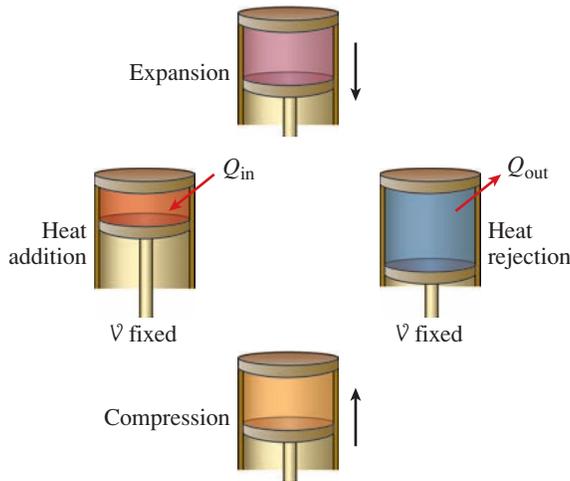
<sup>2</sup> These assumptions of adiabatic, isentropic processes with air form the basis for the *air-standard Otto cycle*, aspects of which we will explore at appropriate locations throughout the book.

Table 2.5 in Chapter 2 presents ideal-gas property relationships for isentropic processes. The relationship used here is Eq. 2.43.

the compression/expansion processes obey

$$P\mathcal{V}^\gamma = \text{constant},$$

where  $\gamma (\equiv c_p/c_v)$  is the ratio of specific heats and has a value of 1.4 for air over a wide range of temperatures. Using this model, determine the compression work  ${}_1W_2$ , the expansion work  ${}_3W_4$ , and the net work associated with the cycle defined by the following processes:



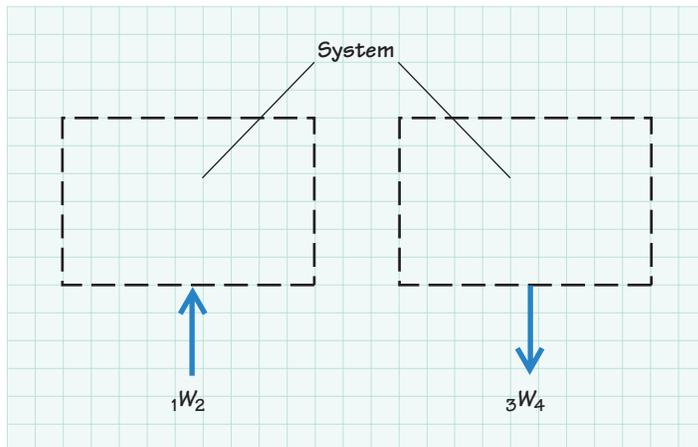
State	1	2	3	4
$P$ (kPa)	100	—	—	—
$T$ (K)	300	—	2800	—
$\mathcal{V}$ (m <sup>3</sup> )	$6.543 \times 10^{-4}$	$0.8179 \times 10^{-4}$	$0.8179 \times 10^{-4}$	$6.543 \times 10^{-4}$

**Solution**

*Known* Adiabatic isentropic processes ( $P\mathcal{V}^\gamma = \text{constant}$ ); selected properties at states 1, 2, 3, and 4; working fluid is air

*Find*  ${}_1W_2$ ,  ${}_3W_4$ , net work

*Sketch*



*Assumptions*

- i. Quasi-static processes
- ii. Ideal-gas behavior

*Analysis* First we recognize that we are dealing with a thermodynamic system, the air trapped in the cylinder, and not a control volume. This fact, combined with the assumption of quasi-static compression and/or expansion, allows us to use Eq. 4.8b to evaluate the work  ${}_1W_2$  and  ${}_3W_4$ , so

$${}_1W_2 = \int_1^2 Pd\mathcal{V}$$

and

$${}_3W_4 = \int_3^4 Pd\mathcal{V}.$$

The functional relationship between  $P$  and  $\mathcal{V}$  is given by  $P\mathcal{V}^\gamma = \text{constant}$ . Knowing both  $P$  and  $\mathcal{V}$  at state 1, we express the process from state 1 to state 2 as

$$P\mathcal{V}^\gamma = P_1\mathcal{V}_1^\gamma,$$

or

$$P = \frac{(P_1\mathcal{V}_1^\gamma)}{\mathcal{V}^\gamma}.$$

Substituting this into Eq. 4.8b and integrating yield

$$\begin{aligned} {}_1W_2 &= \int_1^2 Pd\mathcal{V} = (P_1\mathcal{V}_1^\gamma) \int_1^2 \frac{d\mathcal{V}}{\mathcal{V}^\gamma} \\ &= (P_1\mathcal{V}_1^\gamma) \left[ \frac{\mathcal{V}^{1-\gamma}}{1-\gamma} \right]_{\mathcal{V}_1}^{\mathcal{V}_2} \\ &= \frac{P_1\mathcal{V}_1^\gamma}{1-\gamma} [\mathcal{V}_2^{1-\gamma} - \mathcal{V}_1^{1-\gamma}]. \end{aligned}$$

Since all of the quantities on the right-hand side are known, we numerically evaluate  ${}_1W_2$  as follows:

$$\begin{aligned} {}_1W_2 &= \frac{100 \times 10^3 (6.543 \times 10^{-4})^{1.4}}{-0.4} [(0.8179 \times 10^{-4})^{-0.4} \\ &\quad - (6.543 \times 10^{-4})^{-0.4}] \\ &= -212.2 \\ &[=] \text{Pa(m)}^3 \left[ \frac{1 \text{ N/m}^2}{\text{Pa}} \right] \left[ \frac{1 \text{ J}}{\text{N} \cdot \text{m}} \right] = \text{J}. \end{aligned}$$

Note that the sign of  ${}_1W_2$  is negative since work is done *on* the air.

Our analysis of the expansion process is similar; however, it is complicated by not knowing the pressure at state 3. We need this to evaluate the constant associated with  $P\mathcal{V}^\gamma = \text{constant} = P_3\mathcal{V}_3^\gamma$ . To find  $P_3$  we recognize that, by definition of a system,  $M_1 = M_2 = M_3 = M_4 = M$ . We thus apply the ideal-gas equation of state (Eq. 2.28) twice, once to find  $M$ , using state 1 properties, and a second time to obtain  $P_3$ . We express these operations

mathematically as

$$M = \frac{P_1 \mathcal{V}_1}{RT_1}$$

and

$$P_3 = M \frac{RT_3}{\mathcal{V}_3} = \left( \frac{P_1 \mathcal{V}_1}{RT_1} \right) \frac{RT_3}{\mathcal{V}_3},$$

which simplifies to

$$P_3 = P_1 \left( \frac{\mathcal{V}_1}{\mathcal{V}_3} \right) \left( \frac{T_3}{T_1} \right).$$

Substituting numerical values, we obtain

$$P_3 = 100 \times 10^3 \left( \frac{6.543 \times 10^{-4}}{0.8179 \times 10^{-4}} \right) \left( \frac{2800}{300} \right) \text{Pa} = 7.466 \times 10^6 \text{Pa}.$$

Following the same procedures as for the compression process, we integrate Eq. 4.8b to obtain

$${}_3W_4 = \frac{P_3 \mathcal{V}_3^\gamma}{1 - \gamma} [\mathcal{V}_4^{1-\gamma} - \mathcal{V}_3^{1-\gamma}],$$

which is numerically evaluated as

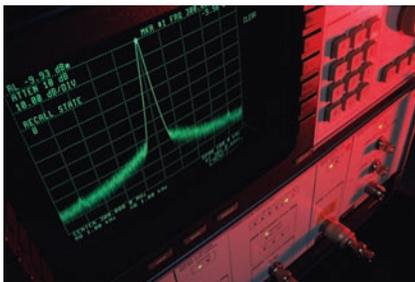
$$\begin{aligned} {}_3W_4 &= \frac{7.466 \times 10^6 (0.8179 \times 10^{-4})^{1.4}}{-0.4} [(6.543 \times 10^{-4})^{-0.4} \\ &\quad - (0.8179 \times 10^{-4})^{-0.4}] \text{ J} \\ &= +862.1 \text{ J}. \end{aligned}$$

The plus sign here emphasizes that work is done *by* the air during the expansion process.

Since there is no volume change for both processes 2–3 and 4–1, no work is done in either process; thus, the net work for the cycle 1–2–3–4–1 is

$$\begin{aligned} W_{\text{net}} &= {}_1W_2 + {}_3W_4 = -212.2 + 862.1 \text{ J} \\ &= +649.9 \text{ J}. \end{aligned}$$

*Comments* We first note that the net work done is positive, which meets our expectations that engines produce work. It is also important to point out how our crude model differs from the actual processes in a real spark-ignition engine: First, heat transfer is present in the real engine; in particular, there is a substantial heat loss from the hot gases to the cylinder walls during the expansion process. Also affecting the actual net work is the timing of the combustion process, which begins before the piston reaches top center and ends somewhat after the piston begins its descent during the expansion. Both of these factors affect the  $P$ – $\mathcal{V}$  relationship; nevertheless, if we were to measure  $P$  versus  $\mathcal{V}$  and apply Eq. 4.8b, this would yield a close approximation to the work performed. In fact, experimental  $P$ – $\mathcal{V}$  data are used in just this way in engine research. Heat losses and a finite combustion time result in the actual work being less than predicted by our crude model.



Pressure transducers mounted within the combustion chamber of an engine provide a record of pressure versus time.

### Self Test 4.2



A piston–cylinder device contains 5 kg of saturated liquid water at a pressure of 100 kPa. Heat is added until a saturated vapor state exists. Determine the work performed and whether it is done by or on the system.

(Answer: 846.4 kJ, done by the system)

**Table 4.2 System Expansion or Compression Work: Special Cases for Ideal Gases**

Process	$({}_1W_2)_{out}$	
Constant pressure	$P(V_2 - V_1)$	T4.2a
Constant temperature	$P_1 V_1 \ln \frac{V_2}{V_1}$	T4.2b
	or	
	$MRT_1 \ln \frac{V_2}{V_1}$	T4.2c
Constant entropy*	$\frac{P_2 V_2 - P_1 V_1}{1 - \gamma}$	T4.2d
	or	
	$\frac{MR(T_2 - T_1)}{1 - \gamma}$	T4.2e
Polytropic	$\frac{P_2 V_2 - P_1 V_1}{1 - n}$	T4.2f
	or	
	$\frac{MR(T_2 - T_1)}{1 - n}$	T4.2g

\*Assumes constant specific heats

We summarize and generalize the results of Examples 4.1 and 4.2 in Table 4.2. Note that the expressions in Table 4.2 are restricted to ideal gases and assume all processes are internally reversible.



Pressure and viscous forces are both important in a simple journal bearing where a rotating shaft is supported by a thin film of oil.

**Viscous Work** A moving fluid can exert a shear force at a solid surface or against neighboring fluid elements. Therefore, for control volumes with boundaries that expose these viscous forces, work may or may not be done depending on the velocity at the boundary. The viscous shear force acting on a differential area  $dA$  is given by

$$d\mathbf{F}_{\text{visc}} = \boldsymbol{\tau}_{\text{visc}} dA,$$

where  $\boldsymbol{\tau}_{\text{visc}}$  is the viscous shear stress.

Since a viscous shear force can only result as a consequence of fluid motion, we consider only the rate of work. If we assume that the viscous shear stress is the same everywhere over the area  $A$ , then

$$\mathbf{F}_{\text{visc}} = \boldsymbol{\tau}_{\text{visc}} A.$$

Furthermore, if the velocity is also uniform over  $A$ , then the rate of doing work is

$$\dot{W}_{\text{visc}} = \mathbf{F}_{\text{visc}} \cdot \mathbf{V},$$

or

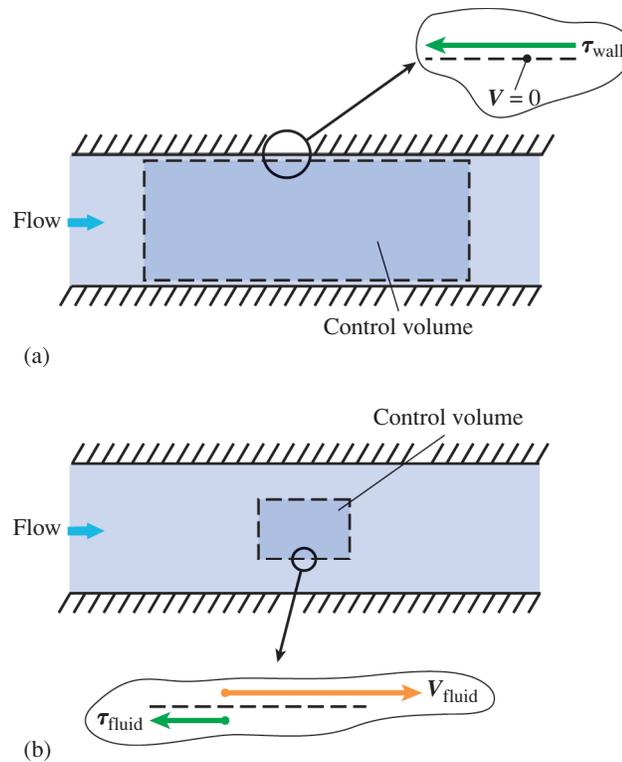
$$\dot{W}_{\text{visc}} = (\boldsymbol{\tau}_{\text{visc}} \cdot \mathbf{V})A.$$

In general, both  $\boldsymbol{\tau}_{\text{visc}}$  and  $\mathbf{V}$  vary over a control surface; thus, we need to integrate over this surface to obtain the rate of working, that is,

$$\dot{W}_{\text{visc}} = \int_{\text{CS}} (\boldsymbol{\tau}_{\text{visc}} \cdot \mathbf{V}) dA. \quad (4.9)$$

**FIGURE 4.5**

Viscous shear forces are associated with moving fluids. At a solid surface (a), although a shear force exists, the velocity is zero. Away from the surface (b), a shear force acts at the control surface opposite to the flow velocity. In (a), no viscous work is done, whereas in (b) work is done by the surroundings on the control volume.



In analyses of macroscopic (integral) control volumes, we may be able to eliminate viscous work from consideration by a judicious choice of boundaries. To illustrate this, we consider the two cases shown in Fig. 4.5. In Fig. 4.5a, we have chosen the control-volume boundary to be the interface between the fluid and the solid wall. At this boundary, a shear stress exists whose magnitude is proportional to the fluid viscosity and the velocity gradient at the wall in the fluid.<sup>3</sup> The velocity, however, is zero at the wall because of the no-slip condition. Since the velocity  $V$  is zero, the dot product  $\tau_{\text{visc}} \cdot V$  and, consequently,  $\dot{W}_{\text{visc}}$  are zero. No work is done at the wall. This conclusion is important to remember when choosing control-volume boundaries, as a wise choice can avoid dealing with  $\dot{W}_{\text{visc}}$ .

Figure 4.5b shows a situation in which viscous work is present. Assuming a velocity gradient exists to produce a viscous shear stress, we have all of the conditions necessary for  $\dot{W}_{\text{visc}}$  to exist: aligned components of both  $\tau_{\text{visc}}$  and  $V$ . This is clearly the case for the horizontal portion of the control surface selected for this example. A different situation arises, however, for the portions of the control surface that are perpendicular to the flow, that is, the flow entrance and exit. Here any shear stresses acting over the control surface are now *perpendicular* to the velocity so that  $\tau_{\text{visc}} \cdot V = 0$ ; that is, the shear direction is vertical, whereas the velocity direction is horizontal.<sup>4</sup> Pressure forces here will, however, perform work. We consider this in a subsequent section.

The no-slip condition is illustrated in Examples 3.6 and 3.7 and is discussed in the associated text in Chapter 3.

<sup>3</sup> Fluid mechanics textbooks (e.g., Refs. [9, 10]) deal with viscous forces in considerable detail. For our purposes here, it is sufficient to know that such forces result whenever a velocity gradient exists in a fluid.

<sup>4</sup> In general, determining all of the viscous forces that act on a control surface is a complex process lying beyond the scope of this book. For most engineering applications, however, ignoring viscous shear forces when the flow is perpendicular to the control surface is reasonable. Furthermore, viscous normal stresses are frequently small and can be neglected.

**Example 4.3**

In a grinding and polishing operation, water at 300 K is supplied at a flow rate of  $4.264 \times 10^{-3}$  kg/s through a long, straight tube having an inside diameter of 6.35 mm. Assuming the flow within the tube is laminar and exhibits a parabolic velocity profile (Table 3.1),

$$v_x(r) = v_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right],$$

determine the viscous shear work, per meter length, for cylindrical control volumes having radii of  $r = R/8$ ,  $R/2$ , and  $R$ , where  $R$  is the tube radius. The viscous shear stress for this flow is expressed as

$$\tau_{\text{visc}}(r) = \mu \frac{dv_x(r)}{dr},$$

where  $\mu$  is the viscosity.

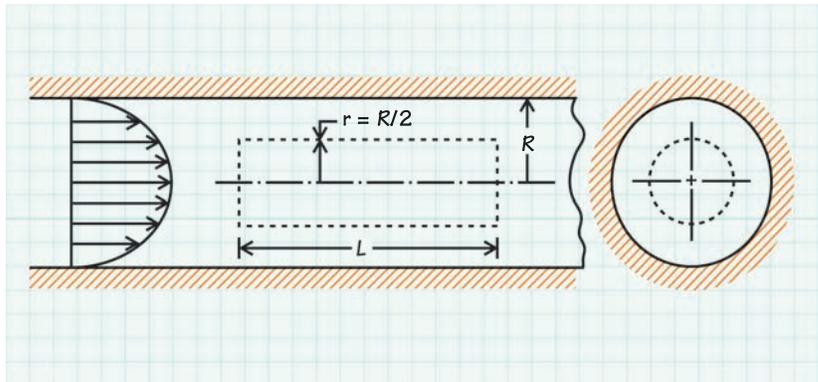
**Solution**

*Known* Velocity distribution, shear stress expression,  $\dot{m}$ ,  $T$ ,  $R (= d/2)$

*Find*  $\dot{W}_{\text{visc}}$  per unit length

*Sketch*

For  $r = R/2$ :

*Assumptions*

- i. Steady-state, steady flow
- ii. Laminar flow with parabolic velocity profile
- iii. Newtonian fluid (i.e., the viscosity is constant and  $\tau_{\text{visc}}$  is directly proportional to the velocity gradient  $dv_x/dr$ )
- iv. Density and viscosity at unknown pressure approximately equal those of a saturated liquid at the given temperature

*Analysis* We start with Eq. 4.9, recognizing that both the viscous shear stress and velocity are uniform over the circumferential area of the control volume; thus,

$$\dot{W}_{\text{visc}} = \tau_{\text{visc}}(r) v_x(r) A(r),$$

where  $v_x(r)$  is given and  $A(r) = 2\pi rL$ . We evaluate the viscous shear stress from the given expression:

$$\tau_{\text{visc}} = \mu \frac{dv_x(r)}{dr} = \mu \frac{d}{dr} \left[ v_{\text{max}} \left( 1 - \left( \frac{r}{R} \right)^2 \right) \right] = -\mu v_{\text{max}} 2r/R^2.$$

We can relate  $v_{\text{max}}$  to the given mass flow rate  $\dot{m}$  by recognizing that  $v_{\text{avg}} = v_{\text{max}}/2$  and by applying the definition of a flow rate (Eq. 3.15):

$$\dot{m} = \rho v_{\text{avg}} A_{x\text{-sec}} = \rho v_{\text{max}} A_{x\text{-sec}}/2,$$

or

$$v_{\text{max}} = \frac{2\dot{m}}{\rho A_{x\text{-sec}}} = \frac{2\dot{m}}{\rho \pi R^2}.$$

Returning to our original expression (definition) for  $\dot{W}_{\text{visc}}$  and substituting the detailed expressions for  $\tau_{\text{visc}}(r)$ ,  $v_x(r)$ , and  $A(r)$ , we write

$$\dot{W}_{\text{visc}} = 4\pi\mu v_{\text{max}}^2 L \left( \frac{r}{R} \right)^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right].$$

From the NIST database or Appendix G, we obtain

$$\rho(300 \text{ K}) = 997 \text{ kg/m}^3,$$

$$\mu(300 \text{ K}) = 855 \times 10^{-6} \text{ N}\cdot\text{s/m}^2.$$

Before numerically evaluating  $\dot{W}_{\text{visc}}$ , we determine  $v_{\text{max}}$ :

$$v_{\text{max}} = \frac{2(4.264 \times 10^{-3})}{997\pi(0.00635/2)^2} = 0.270$$

$$[=] \frac{\text{kg/s}}{(\text{kg/m}^3)\text{m}^2} = \text{m/s}.$$

Evaluating  $\dot{W}_{\text{visc}}$ , we find

$$\begin{aligned} \dot{W}_{\text{visc}} &= 4\pi(855 \times 10^{-6})(0.270)^2 L \left( \frac{r}{R} \right)^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \\ &= 0.000783L \left[ \left( \frac{r}{R} \right)^2 - \left( \frac{r}{R} \right)^4 \right] \\ [ &= ] (\text{N}\cdot\text{s/m}^2)(\text{m/s})^2 \text{m} \left[ \frac{1 \text{ J}}{\text{N}\cdot\text{m}} \right] \left[ \frac{1 \text{ W}}{\text{J/s}} \right] = \text{W}. \end{aligned}$$

The following table shows our final results for the three values of  $r/R$ :

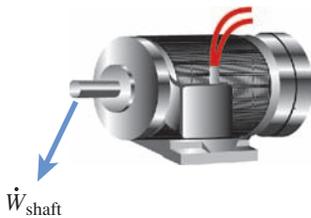
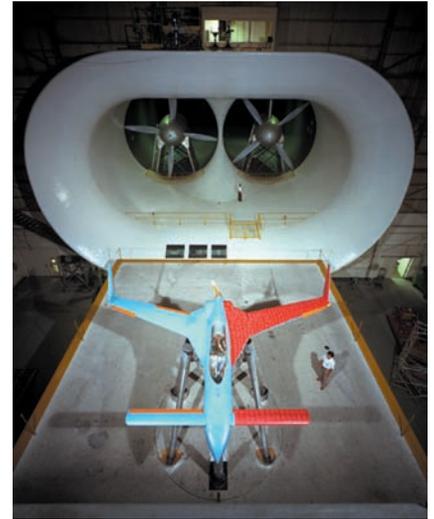
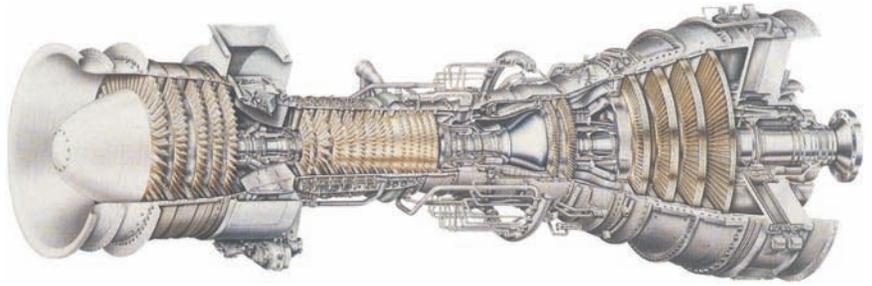
$r/R$	$\left( \frac{r}{R} \right)^2 - \left( \frac{r}{R} \right)^4$	$\dot{W}_{\text{visc}}/L$
1/8	0.01538	$1.20 \times 10^{-5} \text{ W/m}$
1/2	0.1875	$1.47 \times 10^{-4} \text{ W/m}$
1	0	0

*Comments* We see that this viscous work is quite small. In many practical situations, the viscous work is neglected in the application of the conservation of energy principle because this work is so small compared to other terms.

See Table 3.1 in Chapter 3 for relationships between maximum and average velocities for various internal flows.

**FIGURE 4.6**

Examples of devices in which shaft work or power is important. Stationary gas-turbine engine (top), diesel engine (left), wind tunnel fans, and propeller-driven aircraft (right). Drawings and photographs courtesy of General Electric Co., Scania, and NASA, respectively.

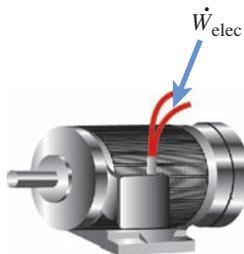


**Shaft Work**<sup>5</sup> In many practical devices, power is transmitted across a control surface via a rotating shaft. Figure 4.6 shows gas-turbine and diesel engines, fans, and a propeller-driven aircraft, all of which rely on shaft power for their operation. A control surface that cuts through a shaft exposes a force acting over a distance. From a formal analysis of the forces within the shaft and the application of Eq. 4.7, the work rate or shaft power is expressed as

$$\dot{W}_{\text{shaft}} = \mathcal{P}_{\text{shaft}} = \mathcal{T}\omega, \quad (4.10)$$

where  $\mathcal{T}$  is the torque and  $\omega$  is the angular velocity of the shaft. In many of the applications in this book,  $\dot{W}_{\text{shaft}}$  (or  $\mathcal{P}_{\text{shaft}}$ ) will be a given quantity or a quantity derived from, usually, a conservation of energy expression; thus, we seldom refer to either the torque or the angular velocity.

**Electrical Work** The flow of an electrical current across the boundary of either a system or a control volume results in a flow of energy. This transfer of energy is work or power. That this is true can be seen from a careful application of our definition of work (Eq. 4.5) to the electrical forces and the motion of electrons through a conductor. For our purposes, it is sufficient to know how the electrical work and power relate to voltage and current.<sup>6</sup>



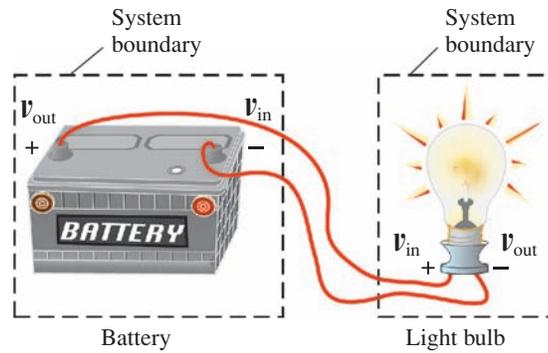
$$W_{\text{elec}} = \int_{t_1}^{t_2} i \Delta \mathbf{V} dt \quad (4.11a)$$

<sup>5</sup> The rate of work  $\dot{W}$ , or power  $\mathcal{P}$ , is frequently implied by the use of the word *work*, as is done here. The context usually makes clear whether the reference is to  $W$  or  $\dot{W}$  (i.e.,  $\mathcal{P}$ ).

<sup>6</sup> Implicit in our discussion of electrical work, current, and voltage is that we are dealing with DC circuits or with AC circuits containing only resistance elements. Treatment of AC circuits, nonresistive loads, and power factors is beyond the scope of this book.

**FIGURE 4.7**

Power exits the system defined as the battery, whereas power enters the system defined as the light bulb.



Electrical power drives a motor; the shaft power from the motor, in turn, drives a pump.

and

$$\dot{W}_{\text{elec}} = \mathcal{P}_{\text{elec}} = i\Delta\mathcal{V}. \quad (4.11b)$$

Note that electrical power flows into the system when  $\Delta\mathcal{V}$  ( $= \mathcal{V}_{\text{in}} - \mathcal{V}_{\text{out}}$ ) is positive and, conversely, flows out when  $\Delta\mathcal{V}$  ( $= \mathcal{V}_{\text{in}} - \mathcal{V}_{\text{out}}$ ) is negative.

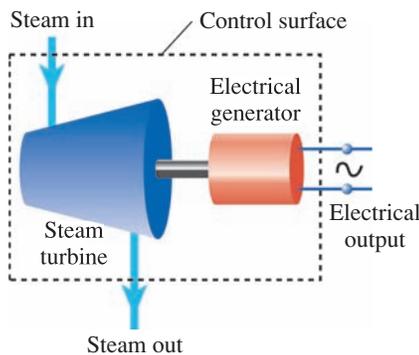
As a practical note,  $\dot{W}_{\text{elec}}$  can be related to the electrical current  $i$  and the system electrical resistance  $R_e$  by

$$\dot{W}_{\text{elec}} = i^2 R_e. \quad (4.11c)$$

Equation 4.11c expresses the concept of **Joule heating**, named in honor of its discoverer James Prescott Joule [11].

Figure 4.7 shows examples of electrical work. Choosing a boundary to select just the battery as a thermodynamic system, we see that electrical power is delivered across the boundary from the system to the surroundings, since  $\mathcal{V}_{\text{out}} > \mathcal{V}_{\text{in}}$ . In contrast, selecting the light bulb to be a system, we see that electrical power is now delivered in the opposite direction (i.e., from the surroundings to the system). Here  $\mathcal{V}_{\text{in}}$  is greater than  $\mathcal{V}_{\text{out}}$ . If we were to choose a system boundary that enclosed *both* the battery and the light bulb, no work interaction would exist.

As another example, consider the steam power plant, one of our integrating applications. The choice of control volumes in Fig. 4.8 shows electrical work crossing to the surroundings, while high-pressure, high-temperature steam enters the control volume and low-pressure, low-temperature steam exits the control volume.

**FIGURE 4.8**

The control volume shown includes a steam turbine and an electrical generator. Electrical work exits the control volume (left); 500-MW turbine generator (middle); electrical generator windings (right).



**Flow Work** The work associated with moving a fluid into and out of a control volume is called **flow work**. Pressure forces acting over flow inlets and exits produce this work. To evaluate the flow work, we appeal to our fundamental definition, Eq. 4.7b, which requires the identification of the appropriate forces.

When ascertaining what forces act *on* a control volume, our point of view is always from a position *outside* of the control volume. Forces due to pressure always act perpendicular to a control surface and are directed to the interior of the control volume. As an example, consider the flow in a pipe illustrated in Fig. 4.9. We focus our attention on the inlet and exit areas designated as stations 1 and 2. Assuming a uniform pressure distribution at stations 1 and 2, we can calculate the pressure forces as simply the products of the respective pressures and areas as indicated.

Having identified the forces at the inlet and exit, we now can evaluate the flow work done *on* the fluid at the inlet surface designated 1. Since the velocity over the inlet area  $A_1$  has the same direction as  $F_{P,1}$ , the dot product in Eq. 4.7b is simply

$$\dot{W}_{\text{flow},1} = P_1 A_1 V_1,$$

where, for the time being, we have assumed that  $V_1$  is uniform over  $A_1$ . The simple mathematical manipulation of multiplying and dividing by the density  $\rho$  yields

$$\dot{W}_{\text{flow},1} = \rho_1 A_1 V_1 \frac{P_1}{\rho_1}.$$

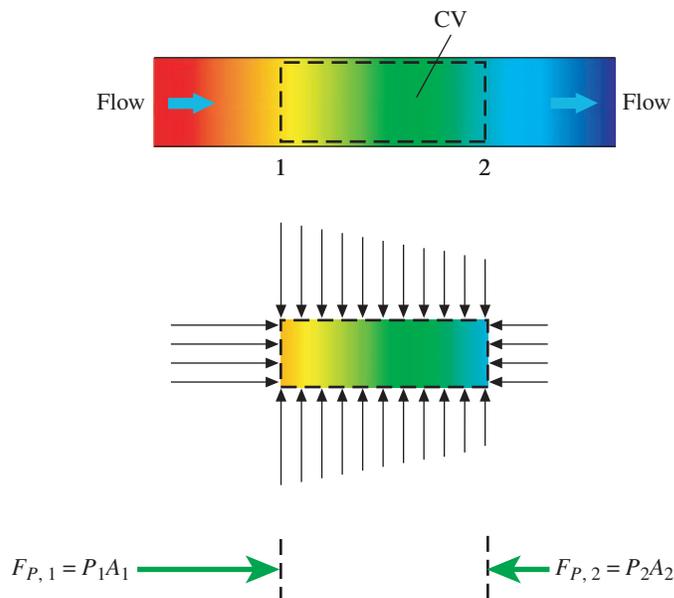
In this equation, we recognize that  $\dot{m}_1 = \rho_1 A_1 V_1$  and that  $1/\rho_1$  is just  $v_1$ , the specific volume. Thus,

$$\dot{W}_{\text{flow},1} = \dot{m}_1 P_1 v_1, \quad (4.12a)$$

where we emphasize that this quantity is the rate of work done *on* the control volume by the surroundings. It is a simple matter to show that Eq. 4.12 applies even if the velocity distribution is not uniform; of course, both  $P_1$  and  $v_1$  must be uniform over  $A_1$ .

A similar derivation can be applied to obtain the flow work at the control volume exit (i.e., at station 2). In this case, however, the pressure force is directed

**FIGURE 4.9**  
Pressure forces associated with a control volume inside of a pipe with a flowing fluid.



opposite to the velocity; thus, the dot product  $\mathbf{F}_{P,2} \cdot \mathbf{V}_2 = F_{P,2} V_2 \cos(180^\circ) = -F_{P,2} V_2$ . The rate of flow work performed on the fluid is then

$$\dot{W}_{\text{flow},2} = -\dot{m}_2 P_2 v_2. \quad (4.12b)$$

We end this development by noting that flow work only occurs when a fluid crosses a boundary. If there is no flow, there cannot be any component of the velocity aligned with the pressure force.

To review enthalpy, see Eq. 2.17 and the associated discussion in Chapter 2.

Foreshadowing the development of the conservation of energy principle in Chapter 5, we point out that the  $Pv$  product in the flow work is frequently grouped with the internal energy of the entering or exiting fluid (i.e.,  $u + Pv$ ). You may recall that this particular grouping of thermodynamic properties is termed the enthalpy.

## 4.4 SIGN CONVENTIONS AND UNITS

In the next chapter, we will examine the principle of energy conservation and the many ways that this principle can be expressed. Since writing a conservation of energy expression is analogous to maintaining an accountant's ledger, we need to know whether various energy terms are credits or debits to our energy account. In this brief section, we present a consistent set of sign conventions for heat and work interactions.

Heat transfer and its time rate are *positive* when the direction of the energy exchange is *from the surroundings to a system or control volume*. Conversely, heat transfer from a system to the surroundings is a negative quantity. Work and power are defined to be *positive* when they are delivered *from a system or control volume to the surroundings* and negative when the converse is true. Thus, the work associated with an expanding gas is positive, whereas the work associated with compression is negative, as we saw in Example 4.2 for the spark-ignition engine. In a steam turbine, the steam expands and produces positive power, as suggested by Fig. 4.8. In contrast, a water pump requires a power input to operate.

The SI unit associated with energy, heat, and work is the joule, which is abbreviated as J. The joule is derived from the definition of work and relates to the fundamental units as follows:

$$\text{joule} = \text{newton} \times \text{meter} = \frac{\text{kilogram} \cdot \text{meter}}{\text{second}^2} \times \text{meter},$$

or

$$\text{J} = \text{kg} \cdot \text{m}^2/\text{s}^2.$$

Power, the time rate of doing work, is expressed in watts (W), as is the heat transfer rate,  $\dot{Q}$ . The watt is expressed in terms of the fundamental units as

$$\text{watt} = \frac{\text{joule}}{\text{second}},$$

or

$$\text{W} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}.$$

In the United States, other units also are used for heat work and power. The British thermal unit (Btu) is frequently used for energy, heat, and work; and horsepower is used for mechanical power. Conversions to SI units are as follows:

$$1 \text{ British thermal unit (Btu)} = 1055.056 \text{ joules (J)},$$

$$1 \text{ horsepower (hp)} = 745.7 \text{ watts (W)}.$$

Other units are used as well. An extensive list of unit conversions is provided on the inside covers of this book. Unfortunately, a wide variety of units are commonly used in commerce and industry. You should be comfortable and proficient in converting units. Some end-of-chapter problems are designed for you to practice these conversions.

### Example 4.4 Steam Power Plant Application



Consider the simple steam power plant cycle shown in Fig. 4.10. Using the indicated control volumes, identify all of the energy transfers (i.e., heat and work interactions) for each of the components.

#### Solution

We start at the water (feedwater) pump and proceed around the loop.

**Water Pump** We have redrawn the control volume for the pump in Fig. 4.11a. Here we identify a work input from an electric motor (i.e., shaft work in) and flow work in and out. We assume that the casing of the pump



Feedwater pump. Photograph courtesy of Flowsolve Corporation.

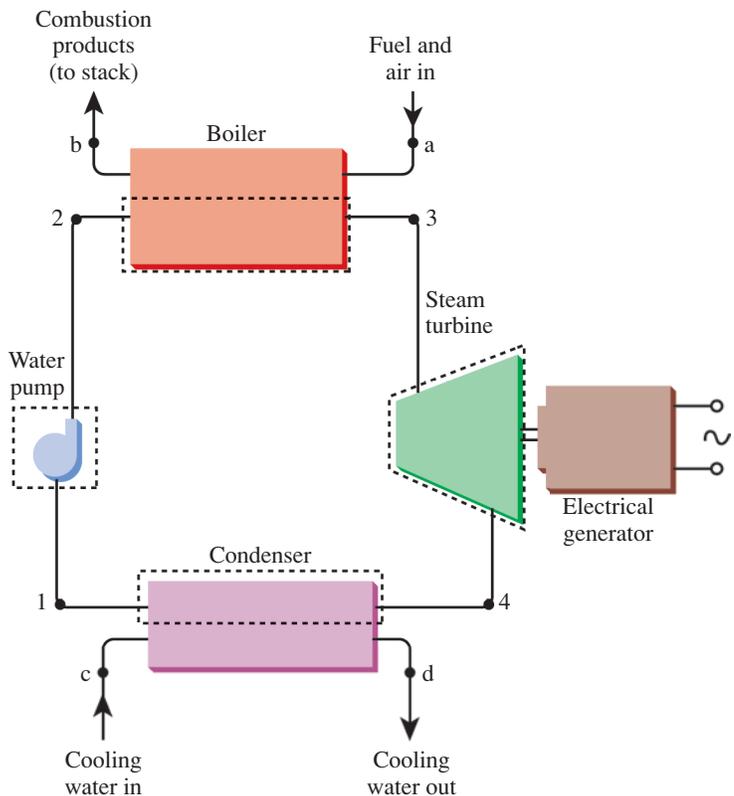
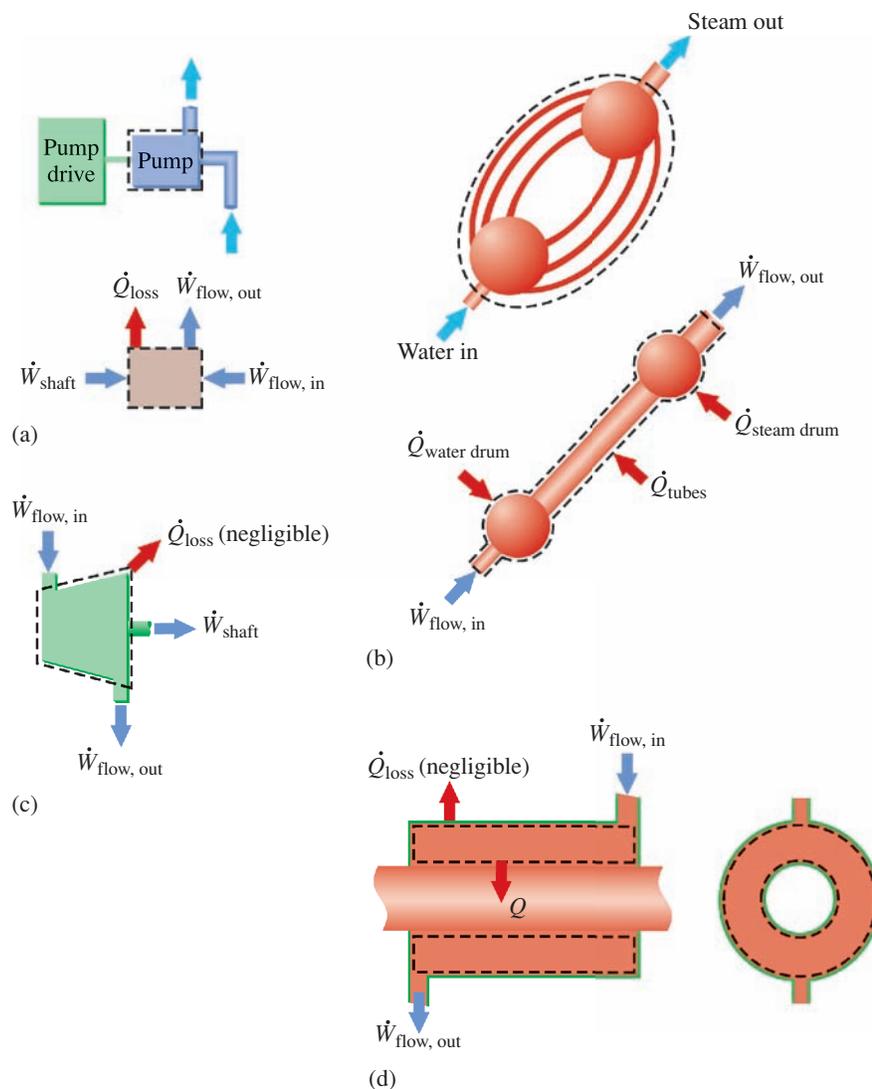


FIGURE 4.10

Rankine cycle schematic for Example 4.4 with individual control volumes for each component.

FIGURE 4.11

Control volumes showing heat and work interactions for (a) feedwater pump, (b) boiler, (c) steam turbine, and (d) condenser.



Steam is generated in these boilers by the combustion of residues from lumber waste and pulp paper (biomass). Photograph courtesy of NREL.



Steam turbine rotor.

will be hotter than the surroundings, so a heat loss is also indicated. We will see in Chapter 5 that this heat loss is negligible compared to the other energy transfers.

**Boiler** In the boiler, water enters the water drum and steam exits the steam drum. Water is converted to steam in a series of tubes connecting the water and steam drums. The outside surfaces of these tubes are exposed to hot products of combustion. Figure 1.2 in Chapter 1 shows a cutaway view of such a boiler. Our control volume (Fig. 4.11b) includes the drums and tubes, along with the water and the steam that they contain, and is represented by a single tube in Fig. 4.11b. The hot combustion products are external to our control volume. Because the tube and drum geometries are significantly different, we have arbitrarily divided the heat transfer from the combustion products to our control volume into three components. Again, flow work exists where the control surface cuts through the flowing fluid. Note that we could have just as easily chosen a control volume that contains only the water and steam. Sometimes an analysis is simplified by either including or excluding the hardware surrounding the working fluid.

**Steam Turbine** We now consider the steam turbine. Our control surface here (Fig. 4.11c) cuts through the inlet steam line, exposing the flow work at the inlet, and similarly at the outlet. The control surface also cuts through the



Condensers and cooling towers at The Geysers power plant in California. Photograph courtesy of NREL.

shaft connecting the turbine to the electrical generator and, thus, shaft work occurs at this location. Since the outer casing of the turbine is likely to be hotter than the surroundings, there will be some heat transfer from the control volume. Although this heat loss is shown in Fig. 4.11c for completeness, the loss is quite small in comparison to all of the other energy transfers and is usually neglected in thermodynamic analyses.

**Condenser** In the condenser, the entering high-quality steam condenses to all water. Figure 4.11d schematically shows the condenser in which the steam is the shell-side fluid occupying the annular control volume, while cold water flows through the tube. The single tube in this schematic represents all of the tubes in the real condenser (see Fig. 1.5). The primary heat transfer is from the condensing steam to the cold water. Again, there is a small, and usually negligible, heat loss to the surroundings. Flow work again is present as the fluid must be pushed into and out of the control volume.

*Comment* Note that we have identified the small heat losses that occur in all of these real devices. Although these are usually neglected in applying the conservation of energy principle to these devices, it is important that you become skillful in identifying *all* heat and work interactions. You can always discard a term as you proceed, but if you missed an important term at the beginning of an analysis, there is no later recourse.

It is also important to point out that in conservation of energy analyses the flow work terms identified in all of these devices are conventionally grouped with the rate of internal energy flowing into or out of the control volume as the rate of enthalpy flow (i.e.,  $\dot{m}u + \dot{W}_{\text{flow}} = \dot{m}u + \dot{m}Pv = \dot{m}h$ ). We will deal with this at some length in the next chapter.

*Self Test*  
4.3

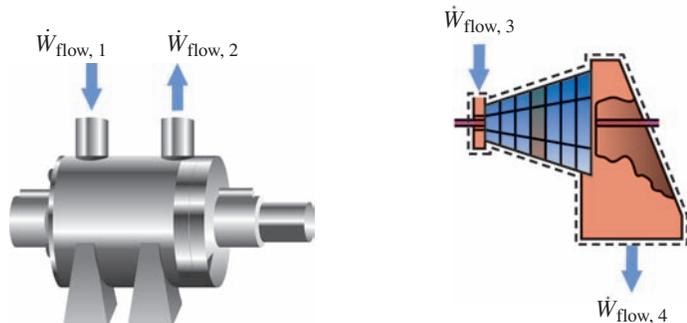


Write an expression for the net work and net heat transfer for the steam power plant of Fig. 4.10.

(Answer:  $\dot{W}_{\text{net}} = {}_1\dot{W}_2 + {}_3\dot{W}_4 = \dot{W}_{\text{turb}} - \dot{W}_{\text{pump}}$ ,  $\dot{Q}_{\text{net}} = {}_2\dot{Q}_3 + {}_4\dot{Q}_1 = \dot{Q}_{\text{boil}} - \dot{Q}_{\text{cond}}$ . Note that the directions of work and heat transfer are explicitly defined in Fig. 4.11.)

**Example 4.5 Steam Power Plant Application**

For the Rankine cycle illustrated in Fig. 4.10, calculate and compare the net flow work associated with the feedwater pump and the steam turbine for the following conditions:



Working fluid (water/steam) flow rate: 2.36 kg/s.

State 1	State 2	State 3	State 4
Saturated liquid	Compressed liquid	Saturated vapor	Wet mixture
$P_1 = 5 \text{ kPa}$	$P_2 = 1 \text{ MPa}$	$P_3 = 1 \text{ MPa}$	$P_4 = 5 \text{ kPa}$
			$x_4 = 0.9$

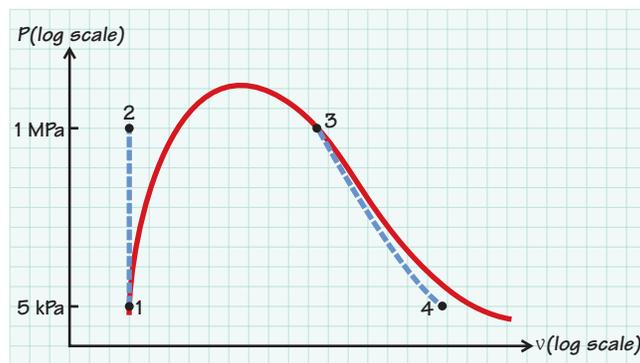
These conditions are typical for an oil-fired industrial power plant producing approximately 1000 kW electrical power [12].

### Solution

*Known*  $\dot{m}$ , physical states,  $P_1, P_2, P_3, P_4, x_4$

*Find*  $\dot{W}_{\text{flow, net}}$  for pump and turbine

### Sketch



### Assumptions

- The specific volume of the compressed liquid at state 2 is approximately equal to that of the saturated liquid at state 1.
- Properties are uniform over the inlet and outlet stations.

*Analysis* We calculate the flow work from the straightforward application of Eqs. 4.12a and 4.12b, recognizing that the net work is the sum of the flow work in and flow work out with careful attention being paid to signs. Since all of the required pressures are given, we need only determine the specific volumes. Starting with the feedwater pump, we find the inlet specific volume from the saturated steam tables (NIST database or Appendix D):

$$v_1 = v_f(P_{\text{sat}} = 5 \text{ kPa}) = 0.0010053 \text{ m}^3/\text{kg}.$$

The specific volume at state 2 can be approximated as being the same as that at state 1. If we knew another property at state 2, we could use tabulated (or computer-based) data in the compressed liquid region to obtain a precise value. Applying the definition of flow work (Eq. 4.12), we calculate

$$\begin{aligned} \dot{W}_{\text{flow, 1}} &= \dot{m}P_1v_1 \\ &= 2.36(5 \times 10^3)0.0010053 \\ &= 11.9 \\ &[=](\text{kg/s})(\text{N/m}^2)(\text{m}^3/\text{kg}) \left[ \frac{1 \text{ J}}{\text{N} \cdot \text{m}} \right] \left[ \frac{1 \text{ W}}{\text{J/s}} \right] = \text{W}, \end{aligned}$$

$$\begin{aligned} \dot{W}_{\text{flow, 2}} &= -\dot{m}P_2v_2 \\ &= -2.36(1 \times 10^6)0.0010053 \text{ W} \\ &= -2372.5 \text{ W}, \end{aligned}$$

and

$$\begin{aligned}\dot{W}_{\text{flow, pump net}} &= \dot{W}_{\text{flow, 1}} + \dot{W}_{\text{flow, 2}} \\ &= 11.9 - 2372.5 \text{ W} \\ &= -2360.6 \text{ W}.\end{aligned}$$

For the steam turbine, the inlet specific volume is found from the NIST database or Appendix D to be

$$v_3 = v_g(P_{\text{sat}} = 1 \text{ MPa}) = 0.19436 \text{ m}^3/\text{kg}.$$

Since the outlet condition lies in the wet region, we use the quality ( $x_4$ ) to determine  $v_4$ , that is,

$$v_4 = (1 - x_4)v_f + x_4v_g,$$

where  $v_f$  and  $v_g$  are the specific volumes for the saturated liquid and vapor at  $P_4$  ( $= 5 \text{ kPa}$ ), respectively. Using values for  $v_f$  and  $v_g$  from the NIST database or Appendix D, we calculate

$$\begin{aligned}v_4 &= 0.1(0.0010053) + 0.9(28.185) \text{ m}^3/\text{kg} \\ &= 25.37 \text{ m}^3/\text{kg}.\end{aligned}$$

The flow work can now be calculated:

$$\begin{aligned}\dot{W}_{\text{flow, 3}} &= \dot{m}P_3v_3 \\ &= 2.36(1 \times 10^6)0.19436 \text{ W} \\ &= 458,690 \text{ W}\end{aligned}$$

and

$$\begin{aligned}\dot{W}_{\text{flow, 4}} &= -\dot{m}P_4v_4 \\ &= 2.36(5 \times 10^3)25.37 \text{ W} \\ &= 299,370 \text{ W}.\end{aligned}$$

Thus, the net turbine flow work is

$$\begin{aligned}\dot{W}_{\text{flow, turbine net}} &= \dot{W}_{\text{flow, 3}} + \dot{W}_{\text{flow, 4}} \\ &= 458,690 - 299,370 \text{ W} \\ &= 159,320 \text{ W}.\end{aligned}$$

*Comment* In comparing the net flow work for the pump and turbine, we note, first, that the magnitude of the turbine flow work is about 70 times that of the pump, and, second, that the signs differ between the pump and turbine flow work. That the turbine flow work is so large results from the specific volume of the steam being much larger than that of the liquid water. In the next chapter, we will return to the sign issue in our first-law (conservation of energy) analysis of pumps and turbines.

### Self Test 4.4



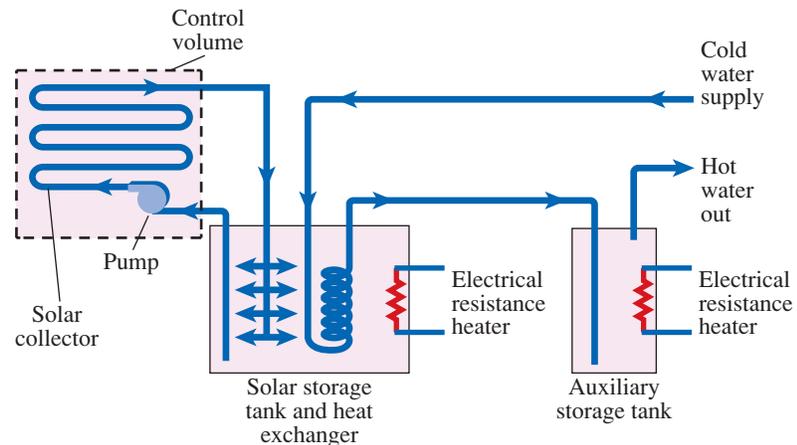
Calculate the change in enthalpy of the working fluid for the turbine of Example 4.5 by using (a) the definition of enthalpy (i.e.,  $h = u + Pv$ ) and (b) tabulated values for  $h$ .

(Answer: (a)  $-454.32 \text{ kJ/kg}$ , (b)  $-459.32 \text{ kJ/kg}$ . Note: The relatively large discrepancy (1%) results from interpolation for properties at 5 kPa. Using NIST data at 5 kPa without interpolation yields no discrepancy.)

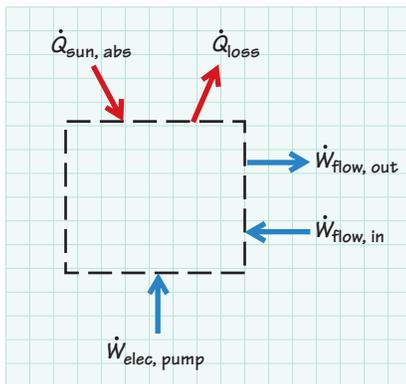
## Example 4.6



Solar-heated domestic hot water is provided using the system shown schematically in Fig. 4.12 [13]. The solar collector consists of a 292.6-m length of pliable black plastic tubing (EPDM) through which water is continuously pumped. An electric motor drives the pump. The solar-heated water is returned to the solar storage tank. The high-pressure ( $\sim 400$  kPa) domestic water is physically separated from the solar-heated water, allowing the solar circuit to operate close to atmospheric pressure. For the control volume indicated by the dashed line in Fig. 4.12, identify all of the heat and work interactions.



**FIGURE 4.12**  
System for solar heating of domestic hot water [13] considered in Example 4.6.

**Solution**

We begin by redrawing the control volume. We identify a work (power) input associated with the pump, and flow work in and out where the control surface cuts through the tubes. The heat transfer is a bit more complicated, and we will define two components rather than just considering a single net rate. A portion of the radiant energy from the sun is absorbed by the control volume, which we designate as  $\dot{Q}_{\text{sun,abs}}$ . Since the collector will be at a higher temperature than the surroundings (air, ground, etc.), there will be heat transferred to the surroundings,  $\dot{Q}_{\text{loss}}$ . These two heat-transfer components are shown as arrows pointing in the known directions.

*Comments* The details of the heat-transfer processes depend critically on the specific geometry, the characteristics of the solar radiation, and the properties of the surroundings (e.g., ambient temperature and wind speed). We will investigate some of these factors later in this chapter.

**Self Test**  
4.5


Consider the solar storage tank and heat exchanger in Fig. 4.12. Identify the energy transfers associated with the cold-water coil in the tank.

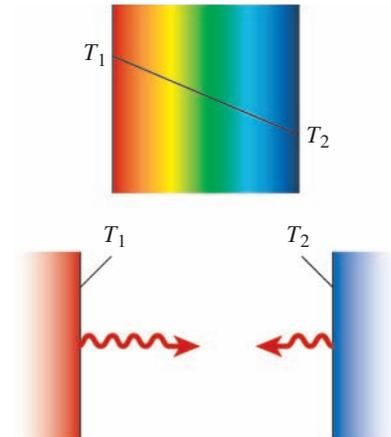
(Answer: Flow work in, flow work out, and heat transfer in)

## 4.5 RATE LAWS FOR HEAT TRANSFER

Recall our definition of heat (or heat transfer) as the transfer of energy across a system or control-volume boundary resulting from a difference in temperature or a temperature gradient. In this section, we present the basic relationships that allow us to calculate the heat transfer, knowing either temperature differences or temperature gradients.

There are two physical mechanisms for heat transfer: 1. the transfer of energy resulting from molecular collisions, lattice vibrations, and unbound electron flow, that is, **conduction**, and 2. the net exchange of electromagnetic radiation, that is, **radiation**. Since conduction depends on interactions among neighboring particles (i.e., a local exchange of energy), the process is said to be **diffusional** in nature and is driven by **temperature gradients**. In contrast, no medium is required for the transfer of energy by electromagnetic radiation and **temperature differences** control. Actually, the driving potential for radiation is a difference of the fourth power of the absolute temperature [i.e.,  $\Delta(T^4)$ ]. In the case of flowing fluids, **convection** is treated as a third mode of heat transfer; however, conduction is still the only fundamental mechanism for energy exchange at the boundary in a convection problem.

Examples of heat transfer abound in both our natural and synthetic environments. A typical home in the United States is chocked full of devices that involve heat transfer: light bulbs, furnaces, space heaters, toasters, hair dryers, computers, stoves, ovens, air conditioners, heat pumps, etc. Keeping your body at an appropriate temperature is a close-to-home and very practical example of heat transfer.



A temperature gradient within a medium drives energy transfer by conduction (top), whereas no medium is required for energy transfer by radiation (bottom).

### 4.5a Conduction<sup>7</sup>

The fundamental rate law governing conduction heat transfer in solids and stagnant fluids is **Fourier's law**, which is expressed as

$$\dot{Q}_{\text{cond}} = -kA \frac{dT}{dx} \quad (4.13)$$

for a one-dimensional Cartesian system. In Eq. 4.13,  $k$  is the **thermal conductivity** of the conducting medium,  $A$  is the area perpendicular to the direction of heat transfer (i.e., perpendicular to the  $x$ -direction for this one-dimensional case), and  $dT/dx$  is the temperature gradient. The negative sign appearing in Eq. 4.13 is the result of the fact that heat transfer has an associated direction. Heat transfer is directed in the positive  $x$ -direction when the temperature gradient is negative, as illustrated in Fig. 4.13. That heat transfer has a natural direction from a high-temperature region to a low-temperature region is a consequence of the second law of thermodynamics, which we will state in Chapter 6. In general, thermal conductivity values depend on temperature; however, problems can frequently be simplified by using an appropriate average value. You can find thermal conductivity values for a number of common substances in the appendices to this book. You can also evaluate thermal conductivities for various fluids using the NIST database.

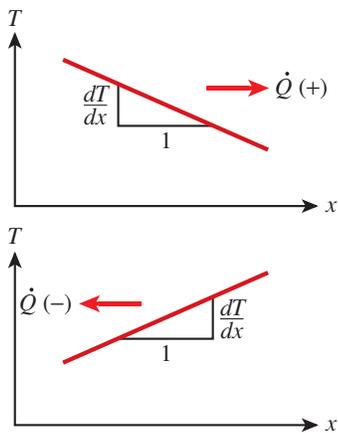
Fourier's law can be more generally expressed as a vector quantity as follows:

$$\dot{Q}_{\text{cond}}'' = -k\nabla T, \quad (4.14)$$



**Fourier's law is a key relationship in the study of conduction. Equation 4.13 is a specific form, whereas Eq. 4.14 expresses the general case.**

<sup>7</sup> This section may be skipped without loss of continuity.



**FIGURE 4.13**

The temperature gradient  $dT/dx$  determines the direction of heat transfer. A negative  $dT/dx$  (top) results in a positive heat transfer (i.e., from left to right), whereas a positive  $dT/dx$  (bottom) results in a negative heat transfer (i.e., from right to left).

where we define the **heat flux** vector  $\dot{Q}''$  to be the heat transfer rate per unit area:

$$\dot{Q}''_{\text{cond}} \equiv \dot{Q}_{\text{cond}}/A. \quad (4.15)$$

The area in Eq. 4.14 is perpendicular to the direction of heat flow. The temperature gradient  $\nabla T$  for a Cartesian system is simply

$$\nabla T = \hat{i} \frac{dT}{dx} + \hat{j} \frac{dT}{dy} + \hat{k} \frac{dT}{dz},$$

where  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are the directional unit vectors. We will restrict our discussion to one-dimensional geometries: 1-D Cartesian, 1-D cylindrical, and 1-D spherical systems. These geometries are illustrated in Fig. 4.14. The corresponding expression of Fourier's law in cylindrical and spherical systems is

$$\dot{Q}_{\text{cond}}(r) = -kA(r) \frac{dT}{dr}, \quad (4.16a)$$

where

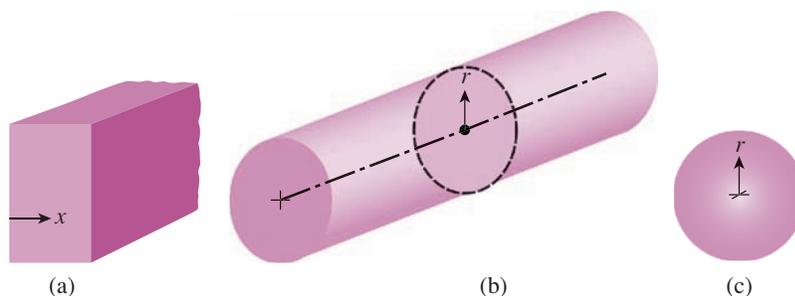
$$A(r) = 2\pi rL \quad (1\text{-D cylindrical}), \quad (4.16b)$$

$$A(r) = 4\pi r^2 \quad (1\text{-D spherical}). \quad (4.16c)$$

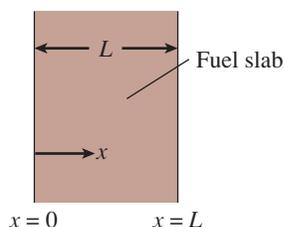
In Eq. 4.16b,  $L$  is an arbitrary length of the cylindrical domain. Many real-world systems can be approximated as being one dimensional. For example, a plane wall whose height and width are much greater than its thickness may be treated as a 1-D Cartesian system; long tubes, pipes, and wires may be considered to be 1-D cylindrical systems. The following examples illustrate the application of Fourier's law to determine heat-transfer rates.

**FIGURE 4.14**

One-dimensional geometries: (a) planar Cartesian system, (b) radial cylindrical system, and (c) radial spherical system.



### Example 4.7



Consider a plane slab of nuclear fuel having a thickness  $L$  of 0.02 m and a thermal conductivity  $k$  of 55 W/m  $\cdot$   $^{\circ}\text{C}$ , as shown in the sketch. For a particular steady-state operating condition, the following temperature distribution within the fuel exists:  $T = T(x) = 312 + 2000x + (2 \times 10^5)x^2 - (1.2 \times 10^7)x^3$  for  $0 \leq x \leq L$ , where  $T$  is in  $^{\circ}\text{C}$  and  $x$  in meters. Water flows over the surfaces at  $x = 0$  and  $x = L$ .

Determine the heat flux at the left and right faces of the fuel slab. Is heat being transferred into or out of the slab at these locations?

**Solution**

*Known*  $L, k, T(x)$ , steady state

*Find*  $\dot{Q}''(0), \dot{Q}''(L)$

*Assumptions*

- i. 1-D conduction
- ii. Constant  $k$

*Analysis* We apply Fourier's law (Eq. 4.13) and the definition of heat flux (Eq. 4.15) to obtain the desired quantities; hence we have

$$\dot{Q}''(x) \equiv \frac{\dot{Q}(x)}{A} = -k \frac{dT(x)}{dx}.$$

The temperature gradient is obtained by differentiating the given temperature distribution:

$$\frac{dT(x)}{dx} = 2000 + (4 \times 10^5)x - (3.6 \times 10^7)x^2.$$

Evaluating this at  $x = 0$  and  $x = 0.02$  m yields

$$\frac{dT(0)}{dx} = 2000 + 0 + 0 \text{ °C/m} = 2000 \text{ °C/m},$$

$$\frac{dT(L)}{dx} = 2000 + 8000 - 14,400 \text{ °C/m} = -4400 \text{ °C/m}.$$

The heat flux at the left face is then

$$\begin{aligned} \dot{Q}''(0) &= -55(2000) = -110,000 \\ &[=] (\text{W/m} \cdot \text{°C})(\text{°C/m}) = \text{W/m}^2. \end{aligned}$$

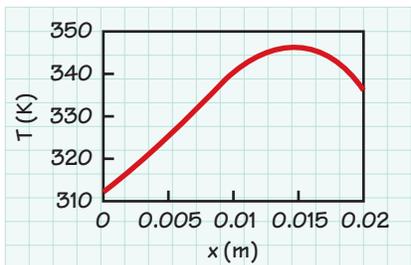
The minus sign here indicates that the heat flow is directed in the negative  $x$ -direction, that is, *out* of the slab at the left face. At the right face ( $x = L$ ),

$$\begin{aligned} \dot{Q}''(L) &= -55(-4400) \text{ W/m}^2 \\ &= +242,000 \text{ W/m}^2. \end{aligned}$$

The plus sign emphasizes that the heat transfer is in the positive  $x$ -direction and is also *out* of the slab.

*Comments* This example emphasizes the importance of the temperature gradient in determining the heat transfer rate. Note that the temperature distribution is highly nonlinear (see sketch) and any attempt to evaluate  $dT/dx$  as  $\Delta T/\Delta x$  using temperatures at  $x = 0$  and  $x = L$  is doomed to failure, as you can easily verify.

We also note the potential confusion associated with sign conventions for heat transfer. Fourier's law provides a sign convention that is tied to the coordinate system, but inspection is required to determine whether the heat transfer is *to* or *from* the system of interest.


**Self Test**  
4.6


**Redo Example 4.7 with a linear temperature profile of  $T(x) = 312 + 1000x$ . Additionally, determine the heat flux at the center of the slab. Comment on the direction of heat flow through the slab.**

(Answer:  $\dot{Q}(x = 0) = -55 \text{ kW}$ ,  $\dot{Q}(x = L) = -55 \text{ kW}$ ,  $\dot{Q}(x = L/2) = -55 \text{ kW}$ . The heat flux is constant through the wall and flows into the right face and out the left face. For this condition, the fuel slab generates no thermal energy.)

## Example 4.8

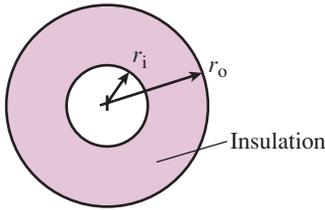


Hot water flows through a copper tube. To minimize the heat loss from hot water, the copper tube is covered with an 18-mm-thick layer of cellular glass insulation. The inside diameter of the insulation is 19 mm, and the thermal conductivity of the insulation is  $0.067 \text{ W/m} \cdot \text{K}$ . The steady-state temperature distribution through the insulation is given by

$$T(r) = \frac{T_i - T_o}{\ln(r_i/r_o)} \ln \frac{r}{r_o} + T_o,$$

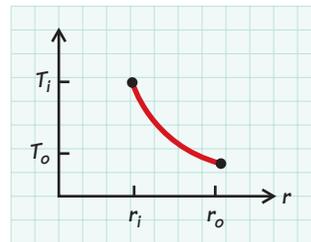
where the subscripts *i* and *o* refer to the inner and outer radii of the insulation, respectively, as shown in the sketch. The corresponding temperatures are  $T_i = 360 \text{ K}$  and  $T_o = 315 \text{ K}$ .

Calculate the heat-transfer rates (per unit length) and heat fluxes at  $r = r_i$  and  $r = r_o$ .

**Solution**

*Known*  $r_i, r_o, T_i, T_o, K_{\text{ins}}$ , steady state, logarithmic temperature profile

*Find*  $\dot{Q}(r_i)/L, \dot{Q}(r_o)/L, \dot{Q}''(r_i), \dot{Q}''(r_o)$

*Sketch**Assumptions*

- i. The system is one dimensional and radial.
- ii. Thermal conductivity is constant.
- iii. Fourier's law applies to cellular glass insulation even though material is inhomogeneous.

*Analysis* The heat-transfer rates can be determined directly from Fourier's law for a 1-D cylindrical coordinate system (Eq. 4.16a) and the heat fluxes from their definition (Eq. 4.15). Starting with Eq. 4.16a, we write

$$\dot{Q}(r) = -kA(r) \frac{dT(r)}{dr},$$

noting that both the area and the temperature gradient  $dT(r)/dr$  depend on  $r$ . Differentiating the given temperature distribution yields

$$\frac{dT(r)}{dr} = \frac{d}{dr} \left[ \frac{T_i - T_o}{\ln(r_i/r_o)} \ln \frac{r}{r_o} + T_o \right] = \frac{T_i - T_o}{\ln(r_i/r_o)} \frac{1}{r}.$$

The area in Eq. 4.16a is the area *perpendicular* to the heat flow, that is, the circumferential area, and is given by

$$A(r) = 2\pi rL, \quad (4.16b)$$

where  $L$  is an arbitrary length of the insulation. Substituting this result and the expression for  $dT(r)/dr$  into Eq. 4.16a gives

$$\dot{Q}(r) = -k(2\pi rL) \left[ \frac{T_i - T_o}{\ln(r_i/r_o)} \frac{1}{r} \right],$$

where we note that the  $r$  dependence cancels. Thus  $\dot{Q}(r_i) = \dot{Q}(r_o) = \dot{Q}$ . Expressing the heat-transfer rate per unit length and recognizing that  $-\ln(r_i/r_o) = +\ln(r_o/r_i)$ , we write

$$\frac{\dot{Q}}{L} = \frac{2\pi k(T_i - T_o)}{\ln(r_o/r_i)}.$$

To evaluate numerically, we first calculate  $r_i$  and  $r_o$ :

$$\begin{aligned} r_i &= 0.019/2 \text{ m} = 0.0095 \text{ m}, \\ r_o &= 0.0095 + 0.018 \text{ m} = 0.0275 \text{ m}. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\dot{Q}}{L} &= \frac{2\pi(0.067)(360 - 315)}{\ln\left(\frac{0.0275}{0.0095}\right)} = 17.8 \\ & [=](\text{W/m} \cdot \text{K})\text{K} = \text{W/m}. \end{aligned}$$

To find the heat flux, we divide the heat-transfer rate by the local area (Eq. 4.15):

$$\dot{Q}''(r) = \frac{\dot{Q}}{A(r)} = \frac{\dot{Q}}{2\pi rL} = \frac{\dot{Q}/L}{2\pi r}.$$

The heat flux at  $r_i$  is then

$$\dot{Q}''(r_i) = \frac{17.8}{2\pi(0.0095)} \text{ W/m}^2 = 298 \text{ W/m}^2,$$

and at  $r_o$ ,

$$\dot{Q}''(r_o) = \frac{17.8}{2\pi(0.0275)} \text{ W/m}^2 = 103 \text{ W/m}^2.$$

*Comment* This example illustrates the importance differences between the heat-transfer rate  $\dot{Q}$  and the heat flux  $\dot{Q}''$ . For a radial system with a constant heat-transfer rate, the heat flux is not constant because of the variation of area in the direction of the heat flow.

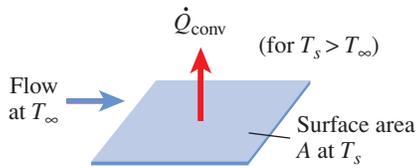
Equation 4.15 expresses a key definition. ➤

### Self Test 4.7



The pure copper tube of Example 4.8 has a wall thickness of 0.070 in. If the outer tube wall temperature is 360 K, determine the temperature at the inner wall. Comment on your result. *Hint:* The heat flux out of the copper tube is equal to the heat flux into the insulation at the pipe–insulation interface.

(Answer:  $T = 360.0015 \text{ K}$ . Since copper has such a large thermal conductivity, heat flows through it with very little temperature difference.)

**FIGURE 4.15**

Convection heat transfer occurs when a fluid flows over a surface. When the surface temperature  $T_s$  is greater than the fluid temperature  $T_\infty$ , the direction of the heat transfer is from the surface to the fluid.

## 4.5b Convection<sup>8</sup>

Convection heat transfer is defined as the heat-transfer process that occurs at the interface between a solid surface and a flowing fluid and is not associated with radiation (Fig. 4.15). As discussed in Chapter 3, the fluid velocity at a fluid–solid interface is zero. As a result of this no-slip condition, Fourier’s law applies within the fluid immediately adjacent to the wall. We can explicitly write Eq. 4.13 for the fluid at the wall as

$$\dot{Q}''_{\text{conv}} = -k_f \left. \frac{dT}{dy} \right|_{\text{wall}}, \quad (4.17)$$

where  $k_f$  is the fluid thermal conductivity and  $dT/dy|_{\text{wall}}$  is the temperature gradient in the fluid. What complicates the evaluation of this simple relation is having to know the temperature gradient in the fluid at the wall. The motion of the fluid away from the wall determines the temperature distribution throughout the flow, including the near-wall region, and, hence, the temperature gradient at the wall. In some relatively simple situations, Eq. 4.17 can be evaluated from theory [14, 15].



(a)



(b)



(c)



(d)



(e)

**FIGURE 4.16**

Examples of forced (a, b) and free convection (c–e): (a) A fan blows air over a heating coil in a hair dryer. (b) An automobile radiator relies on a pump to move the coolant through the internal flow passages and motion of the car to force the air through the external passages. (c) A steam radiator transfers energy to the environment by both free convection and radiation. (d) Fins passively cool power transistors in many electronic devices. (e) A free-convection, thermal plume rises from a standing person. The schlieren technique allows density gradients to be made visible. Photographs (a,c,d) by Paul Ruby, (b) by Sibtosch Pal, and (e) by Gary Settles.

<sup>8</sup> This section may be skipped without any loss in continuity.



Calculated temperature gradients for a flow over a heated, horizontal plate. The largest gradients (red) are near the leading edge of the plate; thus, the heat flux is greatest near the leading edge (see Eq. 4.17). Flow is from left to right with the vertical coordinate expanded by a factor of 2.

Equations 4.18 and 4.19 define average and local heat-transfer coefficients, respectively.



Rising air currents (thermals) are created when the surface of the earth, warmed by the sun, heats the air at ground level (top). Fans and pumps frequently drive forced convection (bottom).

Table 4.3 Typical Values of Convective Heat-Transfer Coefficients [16]

Process	Heat-Transfer Coefficient, $h_{\text{conv}}$ ( $\text{W}/\text{m}^2 \cdot \text{K}$ or $^{\circ}\text{C}$ )
<b>Free convection</b>	
Gases	2–25
Liquids	50–1000
<b>Forced convection</b>	
Gases	25–250
Liquids	50–20,000
<b>Convection with phase change</b>	
Boiling or condensation	2500–100,000

Because of the complexity associated with convection heat transfer, an empirical approach is frequently adopted. Such empiricism has its origins with Sir Isaac Newton but has been greatly refined in modern times. A simple empirical rate law is given by

$$\dot{Q}_{\text{conv}} = \bar{h}_{\text{conv}} A (T_s - T_{\infty}), \quad (4.18)$$

where  $\bar{h}_{\text{conv}}$  is the **heat-transfer coefficient** averaged over the surface area  $A$ ,  $T_s$  is the surface temperature, and  $T_{\infty}$  is the fluid temperature far from the wall in the bulk of the fluid. In this expression, a temperature difference is the driving potential for the heat transfer. A **local heat-transfer coefficient**,  $h_{\text{conv}, x}$ , also can be defined in terms of the local heat flux:

$$\dot{Q}_{\text{conv}}'' = h_{\text{conv}, x} (T_s - T_{\infty}), \quad (4.19)$$

where the value of each quantity appearing in Eq. 4.19 can vary with location on the surface. Some typical values for heat-transfer coefficients are presented in Table 4.3 for a wide range of situations.

The heat-transfer coefficient is not a thermo-physical property of the fluid but, rather, a proportionality coefficient relating the heat flow,  $\dot{Q}_{\text{conv}}$ , to a driving potential difference,  $T_s - T_{\infty}$ ; thus, Eqs. 4.18 and 4.19 can be considered to be definitions of  $\bar{h}_{\text{conv}}$  and  $h_{\text{conv}, x}$  rather than statements of some physical law. Heat-transfer coefficients depend on the specific geometry, the flow velocity, and several thermo-physical properties of the fluid (e.g., thermal conductivity, viscosity, and density). The wide range of values for  $h_{\text{conv}}$  shown in Table 4.3 results from the variation of these factors. Note that the temperature units associated with the heat-transfer coefficient are interchangeably K or  $^{\circ}\text{C}$ , since the units refer to a temperature difference, rather than a temperature, as can be seen from Eqs. 4.18 and 4.19.

Table 4.3 also categorizes convection heat transfer by general process: free convection, forced convection, and convection with phase change. **Free** (or **natural**) **convection** refers to flows that are driven naturally by buoyancy, whereas **forced convection** employs some external agent to drive the flow. The convection heat transfer from an incandescent light bulb is an example of free convection; the cooling of circuit boards in a computer with a fan is an example of forced convection. Figure 4.16 shows other examples of free and forced convection.

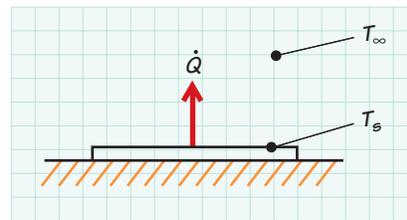
**Example 4.9**

Consider the cooling of an 80-mm by 120-mm computer circuit board that dissipates 6 W as heat from one side. Determine the mean surface temperature of the circuit board when the cooling is by free convection with  $\bar{h}_{\text{conv}} = 7.25 \text{ W/m}^2 \cdot ^\circ\text{C}$ . Compare this result with that obtained for forced convection ( $\bar{h}_{\text{conv}} = 57.7 \text{ W/m}^2 \cdot ^\circ\text{C}$ ) when a fan is employed. Assume the cooling air temperature in both cases is  $25^\circ\text{C}$ .

**Solution**

*Known*  $\dot{Q}, L, W, T_\infty, \bar{h}_{\text{conv}}$  (free convection),  $\bar{h}_{\text{conv}}$  (forced convection)

*Find*  $T_s$  for both free and forced convection

*Sketch**Assumptions*

- i. Uniform-temperature board
- ii. Heat transfer from one side only

*Analysis* We need only apply the definition of an average heat-transfer coefficient (Eq. 4.18) to find the circuit board temperature, that is,

$$\dot{Q} = \bar{h}_{\text{conv}} A (T_s - T_\infty).$$

Solving for  $T_s$  yields

$$T_s = \frac{\dot{Q}}{\bar{h}_{\text{conv}} A} + T_\infty.$$

Recognizing that the area  $A = LW$ , we numerically evaluate this expression for the two conditions. For free convection, we get

$$T_s = \left( \frac{6}{7.25(0.120)0.080} + 25 \right) ^\circ\text{C} = 111.2^\circ\text{C}$$

For forced convection, we get

$$T_s = \left( \frac{6}{57.7(0.120)0.080} + 25 \right) ^\circ\text{C} = 35.8^\circ\text{C}$$

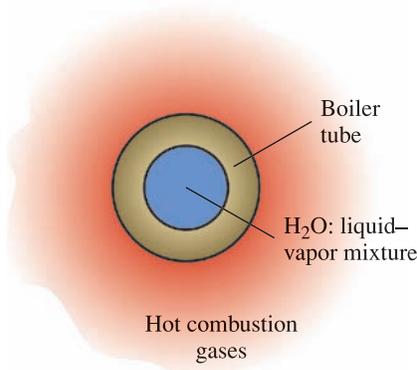
$$[=] \frac{\text{W}}{(\text{W/m}^2 \cdot ^\circ\text{C})\text{m}^2} = (\Delta) \text{K or } (\Delta) ^\circ\text{C}.$$

*Comments* We see that the circuit-board temperature is much lower with forced convection,  $35.8^\circ\text{C}$ , compared to  $111.2^\circ\text{C}$  for free convection. In fact, the  $111.2^\circ\text{C}$  value most likely exceeds a reliable operating temperature for many circuit components.

**Self Test**  
 4.8


Considering the discussion of free versus forced convection, explain why a person would seek relief on a hot day by standing in front of a fan.

(Answer: Since a forced-convection heat-transfer coefficient can be many times larger than that associated with free convection, standing in front of the fan results in a much greater heat-transfer rate from the body to the air.)

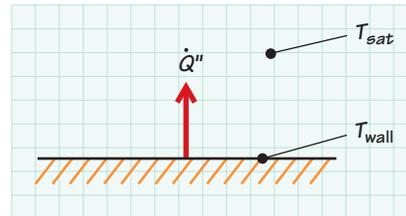
**Example 4.10 Steam Power Plant Application**


Consider a boiler generating steam at a pressure of 1 MPa. The heat flux at the inside surface of a boiler tube (see Fig. 1.2) is approximately  $25 \text{ kW/m}^2$ , and the corresponding local heat-transfer coefficient has a value of approximately  $6000 \text{ W/m}^2 \cdot \text{K}$ . Estimate the temperature of the inside surface of the boiler tube.

**Solution**

*Known*  $P_{\text{sat}}, \dot{Q}'' , h_{\text{conv},x}$

*Find*  $T_{\text{wall}}$

**Sketch**

**Assumptions**

$T_{\text{sat}}$  corresponds to  $T_{\infty}$  in Eq. 4.19.

*Analysis* We solve this problem by the straightforward application of the definition of the local heat-transfer coefficient (Eq. 4.19):

$$\dot{Q}'' = h_{\text{conv},x}(T_{\text{wall}} - T_{\infty}).$$

To apply Eq. 4.19, we assume that the ambient temperature  $T_{\infty} = T_{\text{sat}}$  ( $P_{\text{sat}} = 1 \text{ MPa}$ ). From the NIST database or Appendix D, we find  $T_{\text{sat}} (1 \text{ MPa}) = 179.88^{\circ}\text{C}$ . Solving Eq. 4.19 for the boiler tube inside surface temperature yields

$$T_{\text{wall}} = \frac{\dot{Q}''}{h_{\text{conv},x}} + T_{\text{sat}},$$

which we numerically evaluate as follows:

$$T_{\text{wall}} = \left( \frac{25,000}{6000} + 179.88 \right)^{\circ}\text{C} = (4.17 + 179.88)^{\circ}\text{C} \approx 184^{\circ}\text{C}.$$



Boiler tube sheet

Note that the units associated with  $h_{\text{conv},x}$ ,  $\text{W/m}^2 \cdot \text{K}$ , are equivalent, without modification, to units of  $\text{W/m}^2 \cdot ^{\circ}\text{C}$ , as a temperature *difference* is implied in the units of  $h_{\text{conv},x}$  (or  $\bar{h}_{\text{conv}}$ ), and  $\Delta T (\text{K}) \equiv \Delta T (^{\circ}\text{C})$ .

*Comments* The results of our calculation show that the tube-wall temperature is quite close to the temperature of the steam. This is a characteristic of most boilers, resulting from the very high values of  $h_{\text{conv}}$  on the steam side, and serves to keep metal tube temperatures low compared to the high temperatures of the combustion product gases surrounding the tubes. The low tube temperatures preserve the strength of the metal and permit a long service life.

### Example 4.11



Golden-crowned kinglets, small North American birds, fluff their feathers to stay warm on cold winter days. Estimate the convective heat-transfer rate from a kinglet in a 20 mile/hr (8.94 m/s) wind, treating the bird as a 60-mm-diameter sphere. The surface temperature of the kinglet's feathers is  $-7^{\circ}\text{C}$  and the temperature of the air is  $-10^{\circ}\text{C}$ . The barometric pressure is 100 kPa. The following empirical expression relates the average heat-transfer coefficient to the wind speed  $V$ :

$$\frac{\bar{h}_{\text{conv}}D}{k} = 0.53 \left( \frac{\rho VD}{\mu} \right)^{0.5},$$

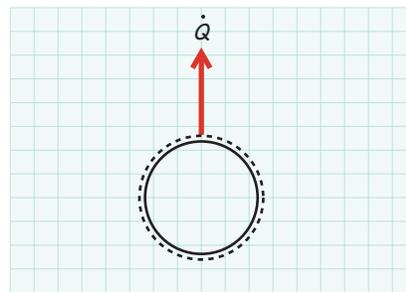
where  $D$  is the bird diameter and  $k$ ,  $\rho$ , and  $\mu$  are the thermal conductivity, density, and viscosity of the air, respectively.

#### Solution

*Known*  $T_s, T_{\infty}, P, V$ , expression for  $h_{\text{conv}}$

*Find*  $\dot{Q}$

#### Sketch



#### Assumptions

- i. Ideal-gas behavior
- ii. Spherical geometry approximates complex shape (given)

*Analysis* To estimate the heat-transfer rate, we apply the basic convective rate law (Eq. 4.18):

$$\dot{Q} = \bar{h}_{\text{conv}} A_{\text{surf}} (T_s - T_{\infty}).$$

With the assumption of a spherical shape for the bird, the surface area is  $\pi D^2$ . We calculate the average heat-transfer coefficient using the given correlation. The needed properties are obtained from the NIST software

using  $T = -10^\circ\text{C}$  and  $P = 0.1\text{ MPa}$ :

$$\begin{aligned}\rho &= 1.3245\text{ kg/m}^3, \\ \mu &= 16.753 \times 10^{-6}\text{ N}\cdot\text{s/m}^2, \\ k &= 0.02361\text{ W/m}\cdot\text{K}.\end{aligned}$$

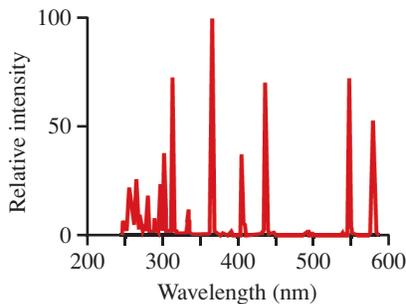
We now evaluate  $\bar{h}_{\text{conv}}$ :

$$\begin{aligned}\bar{h}_{\text{conv}} &= \left(\frac{k}{D}\right)0.53\left(\frac{\rho VD}{\mu}\right)^{0.5} \\ &= \frac{0.02361(0.53)}{0.06}\left[\frac{1.3245(8.94)0.06}{16.753 \times 10^{-6}}\right]^{0.5} \\ &= 0.2086(42,408)^{0.5} = 42.9 \\ [=] &\frac{\text{W}}{\text{m}\cdot\text{K}}\frac{1}{\text{m}}\left(\frac{\text{kg}}{\text{m}^3}\frac{\text{m}}{\text{s}}\frac{\text{m}}{\text{N}\cdot\text{s}}\left[\frac{\text{m}^2}{\text{kg}\cdot\text{m/s}^2}\right]\right)^{0.5} \\ &= \frac{\text{W}}{\text{m}^2\cdot\text{K}}(1).\end{aligned}$$

The heat loss rate from the kinglet is thus

$$\begin{aligned}\dot{Q} &= 42.9\pi(0.06)^2[-7 - (-10)] \\ &= 1.46 \\ [=] &\frac{\text{W}}{\text{m}^2\cdot\text{K}}\text{m}^2\text{C}\left[\frac{1\text{ K}}{1^\circ\text{C}}\right] = \text{W}.\end{aligned}$$

*Comments* An interesting aspect of this example is approximating the small bird as a sphere. Although imprecise, approximations such as these are frequently used to obtain “ballpark” estimates for hard-to-calculate quantities. Notice also the treatment of units where we recognize that a temperature difference expressed in kelvins and  $^\circ\text{C}$  are equivalent (i.e.,  $\Delta T = 3\text{ K} = 3^\circ\text{C}$ ).

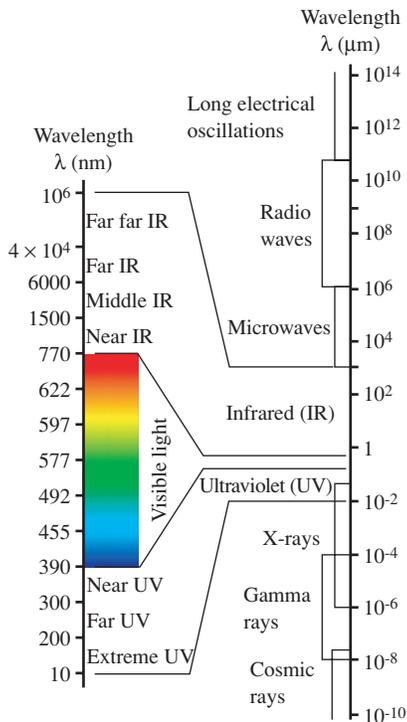


Radiant energy from a mercury vapor lamp is concentrated at several narrow regions of the electromagnetic spectrum.

### 4.5c Radiation<sup>9</sup>

Of the three modes of heat transfer, radiation is the most difficult to describe in general terms. This difficulty arises, in part, because of the wavelength dependence of both the radiant energy and the radiant properties of substances. Also contributing is the action-at-a-distance nature of radiation. Unlike conduction, which depends only on the local material conductivity and local temperature gradient (see Eq. 4.13), radiant energy can be exchanged without any intervening medium and at distances that may be long relative to the dimensions of the device under study. A fortunate example of this is the radiant energy that travels 93 million miles from the sun to the earth through the essential vacuum of outer space. In this chapter, we limit our scope in calculating radiation to a special situation in which many simplifying assumptions have been made to produce a relatively simple result. A more expanded view can be found in Ref. [16]. Several comprehensive textbooks are available for those interested in a detailed treatment [17, 18].

<sup>9</sup> This section can be skipped without loss of continuity.



Visible light occupies only a small region of the complete electromagnetic spectrum.

Before presenting a radiation “rate law,” we need to define what is meant by a blackbody and a gray surface. You may have encountered the concept of a blackbody in a physics course, although the concept of a gray surface is likely to be new. A **blackbody** has the following properties:

1. It absorbs all incident radiation (hence, the appellation black).
2. It emits the maximum possible radiation at every wavelength for its temperature.
3. It emits radiation in accord with the Stefan–Boltzmann law,

$$E_b = \sigma T^4, \tag{4.20}$$

where  $E_b$  is the radiant energy emitted per unit area by the blackbody, or **blackbody emissive power** ( $[=] \text{ W/m}^2$ ),  $T$  is the absolute temperature, and  $\sigma$  is the Stefan–Boltzmann constant, given by

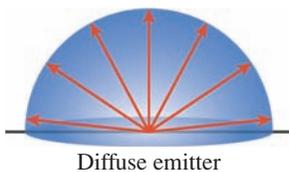
$$\sigma = 5.67051 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4.$$

Furthermore, the intensity of the emitted radiation is independent of direction; thus, a blackbody is said to be a **diffuse emitter**.

To understand the concept of a **gray surface**, we need more information about the characteristics of a blackbody. Planck’s distribution law gives the spectral distribution of the radiation emitted by a blackbody as follows:<sup>10</sup>

$$E_{\lambda b} = \frac{2\pi \hbar c_0^2}{\lambda^5 (e^{\hbar c_0 / \lambda k_B T} - 1)}, \tag{4.21}$$

where  $E_{\lambda b}$  is the spectral blackbody radiant emission per unit area per unit wavelength, or **blackbody spectral emissive power** ( $[=] \text{ W/m}^2 \cdot \mu\text{m}$ );  $\lambda$  is the wavelength;  $\hbar$  is Planck’s constant;  $c_0$  is the vacuum speed of light; and  $k_B$  is the Boltzmann constant. Figure 4.17 is a plot of Eq. 4.21 for blackbody radiation from surfaces at 1000 and 2000 K. The integral of  $E_{\lambda b}$  over all wavelengths yields the Stefan–Boltzmann law,



$$\int_0^\infty E_{\lambda b} d\lambda = \sigma T^4. \tag{4.22}$$

Graphically, the area under each curve in Fig. 4.17 equals  $\sigma T^4$  for its respective temperature.

Because of the fourth-power dependence on temperature, a relatively small change in temperature results in a large change in emission. For example, the emission from a black surface at furnace temperatures, say, 1800 K, is 1330 times more than the emission from the same surface at room temperature (298 K). The significant effect of lowering the temperature from 2000 K to 1000 K is clearly seen in Fig. 4.17.

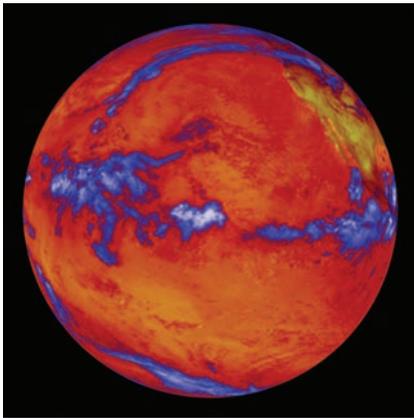
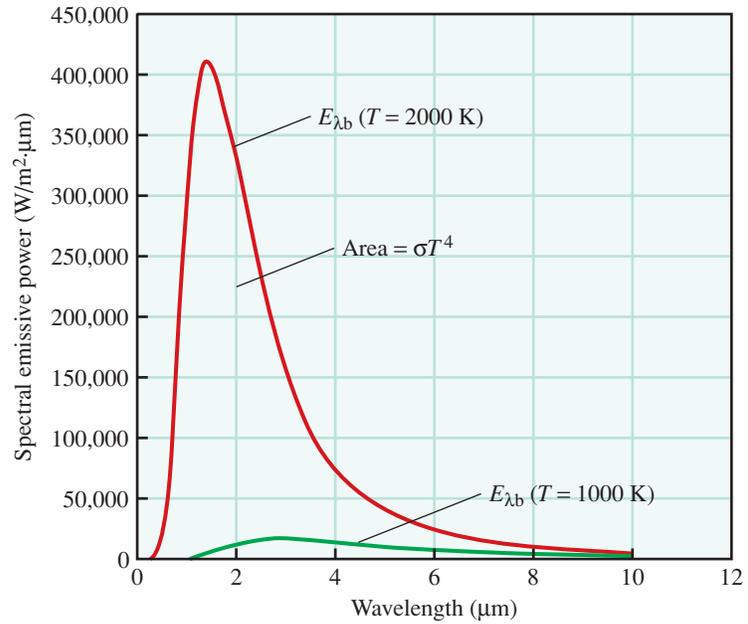
Many surfaces of engineering interest do not necessarily radiate as blackbodies. In fact, the emission from real surfaces frequently displays a complex wavelength dependence that greatly complicates a detailed analysis [17, 18]. A common simplifying assumption is that the emission from a surface is a constant fraction of that from a blackbody. Such a surface is termed a **gray**



In this iron bar, the highest temperatures occur where the visible radiation is the brightest.

<sup>10</sup> Max Planck won the Nobel Prize in 1918 for the discovery of energy quanta. Such quanta were necessary to explain the spectral radiation characteristics of a blackbody.

**FIGURE 4.17**  
 Planck's spectral distribution of radiation from a blackbody at temperatures of 1000 and 2000 K.



Satellite image of long-wavelength (infrared) radiation from Earth. Radiation from cold clouds is indicated in blue, whereas that from the relatively warm oceans is shown in red. Radiation from the southwestern United States exceeds that from the oceans. Image courtesy of NASA Goddard

surface. Figure 4.18 illustrates the spectral distribution of radiant emission from black and gray surfaces at 2000 K. The primary property characterizing a gray surface is the **emissivity**. This dimensionless quantity expresses the ratio of the gray surface spectral emissive power ( $E_\lambda$ ) to that of a blackbody ( $E_{\lambda b}$ ):

$$\varepsilon \equiv \frac{E_\lambda}{E_{\lambda b}} = \text{constant.} \quad (4.23)$$

Since the emissivity is independent of wavelength, it follows that

$$\int_0^\infty E_\lambda d\lambda = \varepsilon \sigma T^4, \quad (4.24)$$

which simply states that the total emissive power of a gray surface is the product of the emissivity and the blackbody emissive power at the same temperature.

We now define a simplified rate law for radiation heat transfer for a solid surface that exchanges radiant energy with its surroundings. The net radiation heat transfer from the surface to the surroundings can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon_s A_s \sigma (T_s^4 - T_{\text{surr}}^4), \quad (4.25)$$

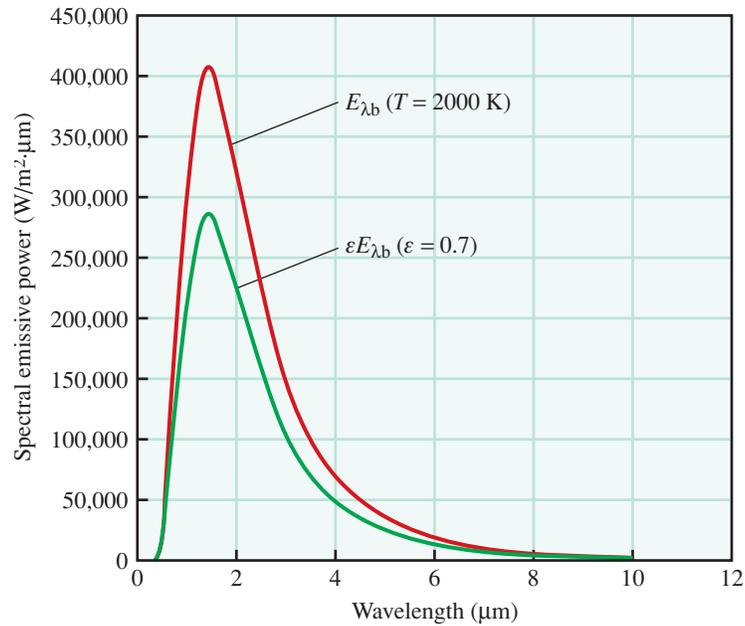
where  $\varepsilon_s$  is the emissivity of the surface,  $A_s$  is the area of the surface, and  $T_s$  and  $T_{\text{surr}}$  are the absolute temperatures of the surface and surroundings, respectively.

Use of Eq. 4.25 to estimate radiation heat-transfer rates relies on the following conditions being met:

1. The surface and the surroundings are, respectively, isothermal; thus, one single temperature characterizes the surface and another single temperature characterizes the surroundings.
2. All of the radiation that leaves  $A_s$  is incident upon the surroundings; that is, there are no intervening surfaces that intercept a portion of the radiation from  $A_s$  and there is no radiation leaving any one region of  $A_s$  that is

**FIGURE 4.18**

The radiant emission from a gray surface is a fixed fraction of that from a blackbody at every wavelength. The emissivity  $\epsilon$  is the ratio of gray surface emission to the blackbody emission.



incident upon any other region of  $A_s$ . Furthermore, any medium between the two surfaces (e.g., air) neither absorbs nor emits any radiation.

3. The surroundings behave as a blackbody *or* the area of the surroundings is much greater than  $A_s$ .
4. Surface  $A_s$  is a diffuse-gray emitter and reflector.

All of the examples involving radiation in this book are such that these four conditions are met or approximated. To deal with more complex situations, we refer the interested reader to Ref. [16].

### Example 4.12



Miniature thermocouple fits through the eye of a needle. Reproduced with the permission of Omega Engineering, Inc., Stamford, CT 06907 [www.omega.com](http://www.omega.com). This image is a registered trademark of Omega Engineering, Inc.

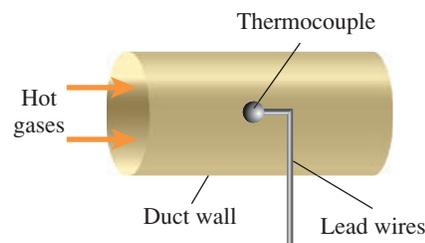
A spherical (0.75-mm-diameter) thermocouple with an emissivity of 0.41 is used to measure the temperature of hot combustion products flowing through a duct as shown in the sketch. The walls of the duct are cooled and maintained at 52°C. The thermocouple temperature is 1162°C. Ignoring the presence of the thermocouple lead wires, determine the rate of radiation heat transfer from the thermocouple to the duct walls.

#### Solution

*Known*  $D_{TC}$ ,  $\epsilon_{TC}$ ,  $T_{\text{wall}}$ ,  $T_{TC}$

*Find*  $\dot{Q}_{\text{rad, TC-wall}}$

#### Sketch



*Assumptions*

- i. The effects of lead wires are negligible.
- ii. All of the restrictions associated with Eq. 4.25 are met.
- iii. The combustion product gases neither absorb nor emit any radiation.

*Analysis* We apply Eq. 4.25, recognizing that  $T_s$  is the thermocouple temperature,  $T_{TC}$ , and that  $T_{surr}$ , the surroundings temperature, is identified with the duct wall temperature, that is,  $T_{surr} \equiv T_{wall}$ . Since absolute temperatures are required for Eq. 4.25, we convert the given temperatures:

$$\begin{aligned} T_{wall} &= 52 + 273 = 325 \text{ K}, \\ T_{TC} &= 1162 + 273 = 1435 \text{ K}. \end{aligned}$$

The radiant heat-transfer rate is calculated from

$$\dot{Q} = \varepsilon_{TC} A_{TC} \sigma (T_{TC}^4 - T_{wall}^4),$$

where  $A_{TC}$  is the surface area of the spherical thermocouple ( $= \pi D^2$ ). Substituting numerical values, we obtain

$$\begin{aligned} \dot{Q} &= 0.41 \pi (0.00075)^2 5.67 \times 10^{-8} [(1435)^4 - (325)^4] \\ &= 0.174 \\ &[=] \text{m}^2 (\text{W}/\text{m}^2 \cdot \text{K}^4) \text{K}^4 = \text{W}. \end{aligned}$$

*Comments* In our analysis, we assumed that the hot gases themselves do not participate in the exchange of radiation and that the only exchange occurs between the two solid surfaces, the thermocouple and the duct wall. This is an excellent assumption for diatomic molecules such as  $\text{N}_2$  and  $\text{O}_2$ , the primary components of air. Asymmetrical molecules, such as  $\text{CO}$ ,  $\text{CO}_2$ , and  $\text{H}_2\text{O}$ , which are found in combustion products, can also participate in the radiation process; thus our analysis is a first approximation to the actual radiation exchange associated with the thermocouple. Treatment of participating media (i.e., gas molecules and particulate matter), can found in advanced textbooks [17, 18].

**Self Test**  
4.9

Consider a cast-iron stove with emissivity  $\varepsilon = 0.44$  and surface area  $A_s = 16 \text{ ft}^2$  located in a large room. If the stove surface temperature is 500 F and the room wall temperature is 60 F, determine the radiation heat-transfer rate to the room walls.

(Answer:  $\dot{Q}_{rad} = 9380 \text{ Btu/h}$ )

**Example 4.13**

Consider the same situation described in Example 4.12. For steady state to be achieved, the heat lost by the thermocouple must be balanced by an energy input. This input comes from convection from the hot gases to the thermocouple, so,

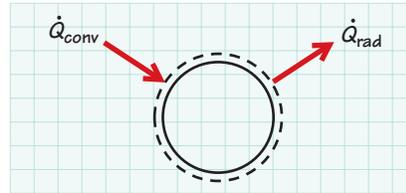
$$\dot{Q}_{conv} = \dot{Q}_{rad}.$$

Assuming an average heat-transfer coefficient of  $595 \text{ W}/\text{m}^2 \cdot \text{K}$ , estimate the temperature of the hot flowing gases.

**Solution**

*Known*  $\dot{Q}_{\text{rad}}$  (Ex. 4.12),  $\bar{h}_{\text{conv}}$ ,  $D_{\text{TC}}$

*Find*  $T_{\text{gas}}$

*Sketch**Assumptions*

- i. The situation is steady state.
- ii. The effects of the lead wires are negligible.
- iii. The combustion products neither absorb nor emit any radiation.

*Analysis* Given that

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{rad}},$$

we substitute the convection rate law (Eq. 4.18) and write

$$\bar{h}_{\text{conv}} A_{\text{TC}} (T_{\text{gas}} - T_{\text{TC}}) = \dot{Q}_{\text{rad}}.$$

Here we recognize that the gas must be hotter than the thermocouple for the heat transfer to be in the direction shown; thus, the appropriate temperature difference is  $T_{\text{gas}} - T_{\text{TC}}$ . Solving for  $T_{\text{gas}}$  yields

$$T_{\text{gas}} = \frac{\dot{Q}_{\text{rad}}}{\bar{h}_{\text{conv}} A_{\text{TC}}} + T_{\text{TC}}.$$

Recognizing that the surface area of the thermocouple is  $\pi D_{\text{TC}}^2$ , we numerically evaluate this as

$$\begin{aligned} T_{\text{gas}} &= \frac{0.174}{595 \pi (0.00075)^2} + 1435 \\ &= 1600 \\ &[=] \frac{\text{W}}{(\text{W}/\text{m}^2 \cdot \text{K}) \text{m}^2} = \text{K}. \end{aligned}$$

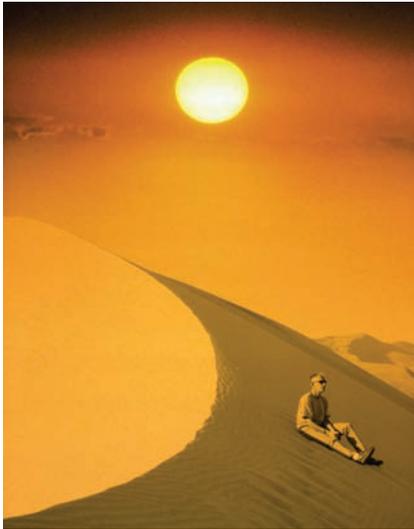
*Comments* This example foreshadows the power of the principle of conservation of energy, from which the statement that  $\dot{Q}_{\text{conv}}$  equals  $\dot{Q}_{\text{rad}}$  was derived. We also see the inherent difficulty in making temperature measurements in high-temperature environments, as the presence of radiation from the thermocouple to the relatively cold wall results in a substantial error in the hot-gas temperature measurement. In this specific example, the error is 165 K.

**Self Test**  
4.10



The air temperature of a house is kept at 22°C year-round. Determine the rate of radiation heat transfer from a person ( $A_s = 1.2 \text{ m}^2$ ) to the walls of a room (a) during summer when the walls are at 28°C and (b) during winter when the walls are at 18°C. Discuss.

(Answer: (a) 66.4 W, (b) 133.6 W. With more heat loss by radiation during the winter than in the summer, a person will feel colder, even though the room air temperature is the same.)

**Example 4.14**

Consider the sun as a blackbody radiator at 5800 K. The diameter of the sun is approximately  $1.39 \times 10^9$  m (864,000 miles), the diameter of the earth is approximately  $1.29 \times 10^7$  m (8020 miles), and the distance between the sun and earth is approximately  $1.5 \times 10^{11}$  m ( $9.3 \times 10^7$  miles).

- Calculate the total energy emitted by the sun in watts.
- Calculate the amount of the sun's energy intercepted by the earth.
- Calculate the solar flux in  $\text{W/m}^2$  at the upper reaches of the earth's atmosphere for the sun's rays striking a surface perpendicularly. Compare this quantity with the maximum solar flux of  $720 \text{ W/m}^2$  recorded during a clear day in Fort Collins, Colorado.

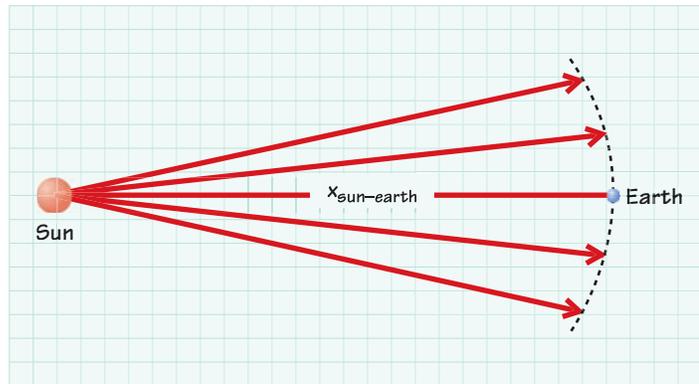
**Solution**

*Known*  $T_{\text{sun}}, D_{\text{sun}}, D_{\text{earth}}, X_{\text{sun-earth}}$

*Find*  $\dot{Q}_{\text{sun}}, \dot{Q}_{\text{sun-earth}}, \dot{Q}_{\text{sun @ earth}}''$

**Assumptions**

- The sun emits as a blackbody.
- The sun's energy is radiated uniformly in all directions.
- The sun is sufficiently far from the earth that the sun's rays are essentially parallel when they strike the earth.

**Sketch****Analysis**

- Given that the sun is a blackbody, the total energy emitted is the product of the sun's emissive power (Eq. 4.20) and surface area,  $\pi D_{\text{sun}}^2$ :

$$\begin{aligned}\dot{Q}_{\text{sun}} &= E_{\text{b, sun}} A_{\text{surf, sun}} \\ &= \sigma T_{\text{sun}}^4 \pi D_{\text{sun}}^2.\end{aligned}$$

Substituting numerical values yields

$$\begin{aligned}\dot{Q}_{\text{sun}} &= 5.67 \times 10^{-8} (5800)^4 \pi (1.39 \times 10^9)^2 = 3.89 \times 10^{26} \\ &[\text{=}] (\text{W/m}^2 \cdot \text{K}^4) \text{K}^4 \text{m}^2 = \text{W}.\end{aligned}$$

- Referring to our sketch, we see that all of the energy from the sun is distributed over the interior surface of a sphere having a radius equal to the sun-earth distance. The fraction of this energy intercepted by

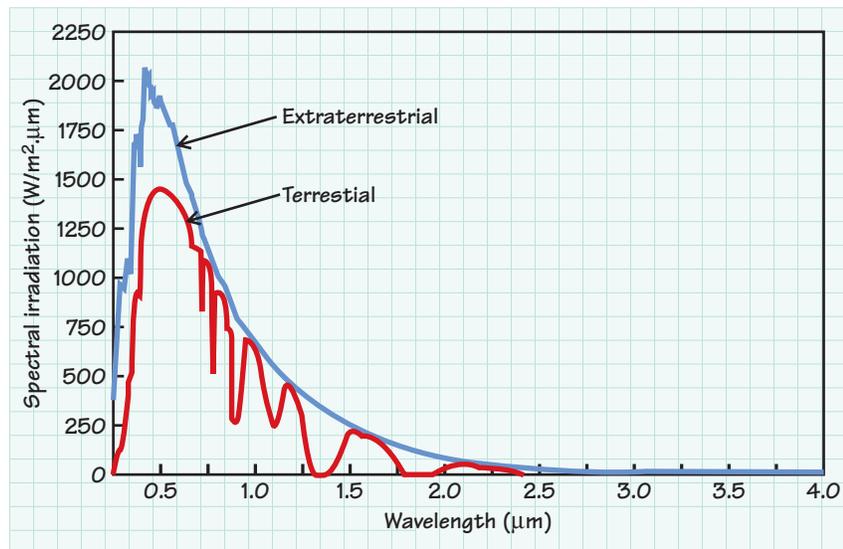
the earth is the ratio of the projected area of the earth,  $\pi D_{\text{earth}}^2/4$ , to the total area,  $4\pi x_{\text{sun-earth}}^2$ . Thus, the energy intercepted by the earth is

$$\begin{aligned}\dot{Q}_{\text{sun-earth}} &= \dot{Q}_{\text{sun}} \frac{\pi D_{\text{earth}}^2/4}{4\pi x_{\text{sun-earth}}^2} \\ &= \frac{\dot{Q}_{\text{sun}}}{16} \left( \frac{D_{\text{earth}}}{x_{\text{sun-earth}}} \right)^2 \\ &= \frac{3.89 \times 10^{26}}{16} \left( \frac{1.29 \times 10^7}{1.5 \times 10^{11}} \right)^2 \text{ W} \\ &= 1.80 \times 10^{17} \text{ W}.\end{aligned}$$

C. We now apply the definition of a heat flux (Eq. 4.15) and evaluate this expression using the surface area associated with the sphere having the sun–earth distance as its radius:

$$\begin{aligned}\dot{Q}''_{\text{sun@earth}} &= \frac{\dot{Q}_{\text{sun}}}{4\pi x_{\text{sun-earth}}^2} \\ &= \frac{3.89 \times 10^{26} \text{ W}}{4\pi(1.5 \times 10^{11})^2 \text{ m}^2} \\ &= 1376 \text{ W/m}^2.\end{aligned}$$

*Comments* First we note the incredibly large rate at which energy is emitted by the sun. The energy emitted by the sun in just 38 ns is equivalent to the U.S. electricity production for an entire year (cf. Table 1.1)!



Comparing our calculated solar flux of  $1376 \text{ W/m}^2$  with the measured peak value for a North American location, we see that the measured value ( $720 \text{ W/m}^2$ ) is substantially less. There are several factors that contribute to this difference. First, the sun's rays at a northern latitude strike the earth's surface obliquely, thus spreading the energy over a greater area. In addition, the sun's radiation is attenuated from absorption and scattering by molecules and particles in the atmosphere. (See sketch.) In the design of solar heating equipment, historical records of measured solar energy at the location of interest are used because of the near impossibility of making theoretical predictions. Local weather conditions also play a major role in determining the average solar flux at a location.

## SUMMARY

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This chapter began by considering the various ways in which energy is stored in systems and control volumes. We saw the importance of distinguishing between the bulk system energy, which is associated with the system as a whole, and the internal energy, which is associated with the constituent molecules. We also carefully defined heat and work and their time rates. We saw that both of these quantities are energy transfers that occur only at boundaries of systems and control volumes. Neither is possessed by or a property of a system or control volume. Various modes of working were discussed along with the three principal modes of heat transfer: conduction, convection, and radiation. To consolidate your knowledge further, reviewing the learning objectives presented at the beginning of this chapter is recommended.

# Chapter 4

## Key Concepts & Definitions Checklist<sup>11</sup>

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### 4.2 System and Control-Volume Energy

- Bulk versus internal energies ▶ **Q4.4**
- Linear and rotational kinetic energies ▶ **4.1**
- Potential energy ▶ **4.1**
- Molecular description of internal energy ▶ **Q4.5**

### 4.3 Energy Transfer Across Boundaries

#### 4.3a Heat

- Definition of heat (or heat transfer) ▶ **Q4.6**
- Driving force for heat transfer ▶ **Q4.7, Q4.8**
- Distinctions among heat, heat-transfer rate, and heat flux ▶ **Q4.9, Q4.22**
- Identification of heat interactions in real devices and systems ▶ **4.47**

#### 4.3b Work

- Definition of work ▶ **Q4.13**
- Definition of power ▶ **Q4.14**
- Five common types of work (list) ▶ **Q4.15**
- $P-dV$  (compression and expansion) work ▶ **4.8, 4.9**
- Viscous work ▶ **4.26**
- Shaft work ▶ **4.15**
- Electrical work ▶ **4.27**
- Flow work ▶ **4.28**
- Identification of work interactions in real devices and systems ▶ **4.31**

### 4.4 Sign Conventions and Units

- Units for energy and energy rates ▶ **Q4.16**

### 4.5 Rate Laws for Heat Transfer

#### 4.5a Conduction

- Fourier's law: 1-D Cartesian form ▶ **Q4.19, 4.33**
- Fourier's law: 1-D cylindrical and spherical forms ▶ **4.34**
- Fourier's law: general vector form ▶ **Q4.18**
- Heat flux vector ▶ **Q4.18**
- Temperature gradient vector ▶ **Q4.18**

#### 4.5b Convection ▶ 4.38, 4.44

- Convection definition (Eq.4.17) ▶ **4.37**
- Average heat-transfer coefficient ▶ **Q4.21**
- Local heat-transfer coefficient ▶ **Q4.21**
- Forced convection ▶ **Q4.25**
- Free convection ▶ **Q4.25**

#### 4.5c Radiation

- Blackbody ▶ **Q4.26, 4.46**
- Emissive power ▶ **Q4.28**
- Gray surface ▶ **Q4.29, Q4.30**
- Spectral emissive power ▶ **Q4.29, Q4.30**
- Emissivity ▶ **Q4.29, Q4.30**
- Net radiation exchange rate (Eq. 4.25) ▶ **Q4.31, 4.47**

<sup>11</sup> Numbers following arrows refer to Questions (prefaced with a Q) and Problems at the end of the chapter.

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$P$	pressure
rad	radiation
s	surface
shaft	shaft
surr	surroundings
sys	system
visc	viscous
wall	at the wall

$x$	local value
1	station 1
2	station 2
$\infty$	ambient or freestream value

**OTHER**

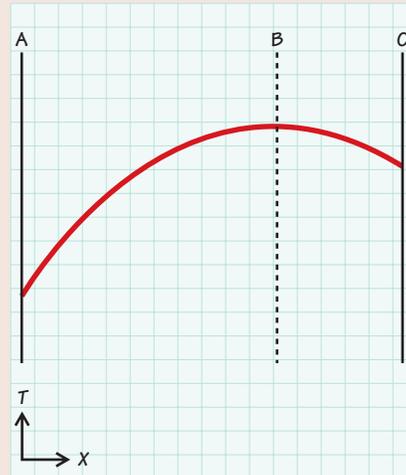
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$\nabla()$	Gradient operator
$(\bar{\quad})$	Average value

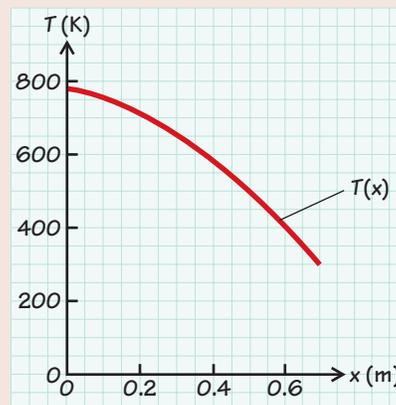
## QUESTIONS

- 4.1 Review the most important equations presented in this chapter (i.e., those with a red background). What physical principles do they express? What restrictions apply?
- 4.2 Name three pioneers in the thermal-fluid sciences and indicate their major contributions. Provide an approximate date for these contributions.
- 4.3 List and define all of the boldfaced words in this chapter.
- 4.4 Distinguish between the bulk and internal energies associated with a thermodynamic system. Give a concrete example of a system that possesses both.
- 4.5 From a molecular point of view, how is energy stored in a volume of gas composed of monatomic species? How is energy stored in diatomic gas molecules? In triatomic gas molecules?
- 4.6 From memory, write out a formal definition of heat or heat transfer. Compare your definition with that in the text.
- 4.7 What thermodynamic property “drives” a heat-transfer process?
- 4.8 Explain the difference between a temperature difference,  $\Delta T$ , and a temperature gradient,  $\nabla T$ .
- 4.9 How does heat (or heat transfer)  $Q$  differ from the heat-transfer rate  $\dot{Q}$ ?
- 4.10 List the units for  $Q$ ,  $\dot{Q}$ , and  $\dot{Q}''$ .
- 4.11 What distinguishes work from internal energy? What distinguishes heat from internal energy? Discuss.
- 4.12 What distinguishes work from heat? Discuss.
- 4.13 From memory, write out a formal definition of work. Compare your definition with that in the text.
- 4.14 How does work  $W$  differ from power  $\dot{W}$  (or  $\mathcal{P}$ )? What units are associated with each?
- 4.15 List, define, and discuss five types of work. For example, one of these should be shaft work.
- 4.16 Provide the units associated with the following quantities:  $E$ ,  $u$ ,  $W$ ,  $\dot{W}$ ,  $Q$ , and  $\dot{Q}''$ .
- 4.17 List the three modes of heat transfer. Compare and contrast these modes.
- 4.18 Define all of the terms appearing in Fourier’s law (Eqs. 4.13 and 4.14). What is the origin of the minus sign?
- 4.19 Consider the steady-state temperature distribution  $T(x)$  in a solid wall as shown in the sketch. Assume the thermal conductivity is uniform throughout the wall.

- A. At what location is the heat-transfer rate largest? Smallest? Explain.
- B. What is the direction of the heat transfer at location A? At location C? Explain.
- C. What special significance, if any, is associated with the plane B? Discuss.



- 4.20 The temperature distribution  $T(x)$  through a solid wall is shown in the sketch. Using the grid given, estimate numerical values for the temperature gradient  $dT/dx$  at  $x = 0$  and  $x = 0.5$  m. Give units. Would the numerical values for  $dT/dx$  change if  $T(x)$  were given in degrees Celsius rather than kelvins? Discuss.



- 4.21 Using symbols, define the local heat-transfer coefficient  $h_{\text{conv},x}$  and the average heat-transfer coefficient  $\bar{h}_{\text{conv}}$ . Under what conditions would  $h_{\text{conv},x}$  and  $\bar{h}_{\text{conv}}$  have the same numerical values? Discuss.
- 4.22 Explain the difference between heat-transfer rate  $\dot{Q}$  and heat flux  $\dot{Q}''$ .
- 4.23 Consider the one-dimensional, steady-state heat conduction through a long hollow cylinder of

insulating material (see Fig. 4.14b). The temperature at the inner radius,  $T_1 = T(r_1)$ , is higher than the temperature at the outer radius,  $T_2 = T(r_2)$ . If the heat-transfer rate is the same at all radial locations, sketch  $\dot{Q}$ ,  $\dot{Q}''$ , and  $T$  as functions of  $r$ . Discuss.

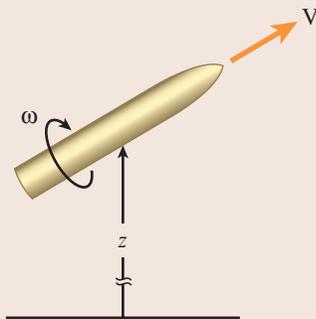
- 4.24 Repeat Question 4.23 for a planar (Cartesian) geometry. Discuss.
- 4.25 Compare typical values of heat-transfer coefficients for free convection and forced convection. What factor(s) account for the differences?
- 4.26 List the properties of a blackbody radiator.
- 4.27 Radiation from the sun strikes a black surface. Is it possible to determine the fraction of the solar energy that is reflected from the surface? If so, what is the fraction?
- 4.28 Define the blackbody emissive power. What units are associated with this quantity?
- 4.29 Discuss the attributes of the radiant energy emitted by a gray surface.
- 4.30 Explain the concept of emissivity.
- 4.31 What are the restrictions implied in the use of the expression  $\dot{Q}_{\text{rad}} = \varepsilon_s A_s \sigma (T_s^4 - T_{\text{surr}}^4)$ ?

### Chapter 4 Problem Subject Areas

- 4.1–4.7 Macroscopic and microscopic energies  
 4.8–4.31 Heat and work  
 4.32–4.48 Conduction, convection, and radiation

## PROBLEMS

- 4.1 A 3.084-kg steel projectile is fired from a gun. At a particular point in its trajectory it has a velocity of 300 m/s, an altitude of 700 m, and a temperature of 350 K. The projectile is spinning about its longitudinal axis at an angular speed of 100 rad/s. Treating the projectile as a thermodynamic system, calculate the total system energy relative to a reference state of zero velocity, zero altitude, and a temperature of 298 K. The rotational moment of inertia for the projectile is  $9.64 \times 10^{-4} \text{ kg} \cdot \text{m}^2$  and the specific heat is  $460.5 \text{ J/kg} \cdot \text{K}$ .



- 4.2 Determine the kinetic energy (kJ) of a 90-kg object moving at a velocity of 10 m/s on a planet where  $g = 7 \text{ m/s}^2$ .
- 4.3 Initially, 1 lb<sub>m</sub> of water is at  $P_1 = 14.7 \text{ psia}$ ,  $T_1 = 70 \text{ F}$ ,  $v_1 = 0 \text{ ft/s}$ , and  $z_1 = 0 \text{ ft}$ . The water then undergoes a

process that ends at  $P_2 = 30 \text{ psia}$ ,  $T_2 = 700 \text{ F}$ ,  $v_2 = 100 \text{ ft/s}$ , and  $z_2 = 100 \text{ ft}$ . Determine the increases in internal energy, potential energy, and kinetic energy of the water in Btu/lb<sub>m</sub>. Compare the increases in internal energy, potential energy, and kinetic energy.

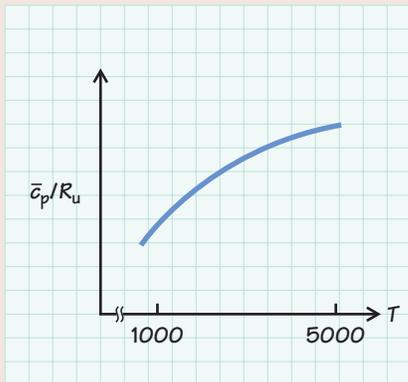
- 4.4 A 100-kg man wants to lose weight by increasing his energy consumption through exercise. He proposes to do this by climbing a 6-m staircase several times a day. His current food consumption provides him an energy input of 4000 Cal/day. (Note: 1 Cal = unit for measuring the energy released by food when oxidized in the body = 1 kcal.) Determine the vertical distance (m) the man must climb to convert his food intake into potential energy. What is your advice regarding exercise versus a reduced food intake to effect a weight loss?
- 4.5 Consider a 25-mm-diameter steel ball that has a density of  $7870 \text{ kg/m}^3$  and a constant-volume specific heat of the form

$$c_v \text{ (J/kg} \cdot \text{K)} = 447 + 0.4233 [T(\text{K}) - 300].$$



What velocity would the ball have to achieve such that its bulk kinetic energy equals its internal energy at 500 K? Use an internal energy reference state of 300 K.

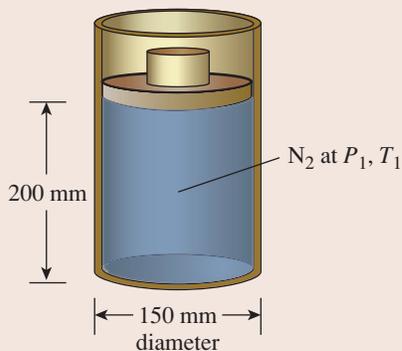
- 4.6 For the temperature range 1000–5000 K, the ratio of the molar constant-pressure specific heat to the universal gas constant is given as follows for  $\text{CO}_2$ :



$$\begin{aligned} \bar{c}_p/R_u = & 4.45362 + (3.140168 \times 10^{-3})T \\ & - (1.278410 \times 10^{-6})T^2 + (2.393996 \\ & \times 10^{-10})T^3 - (1.6690333 \times 10^{-14})T^4, \end{aligned}$$

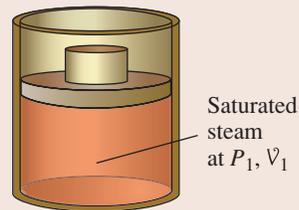
where  $T[=]$  K. For 1 kg of  $\text{CO}_2$ , plot  $U(T) - U(1000 \text{ K})$  versus  $T$  up to 5000 K. Discuss.

- 4.7 Consider nitrogen gas contained in a piston-cylinder arrangement as shown in the sketch. The cylinder diameter is 150 mm. At the initial state the temperature of the  $\text{N}_2$  is 500 K, the pressure is 100 kPa, and the height of the gas column within the cylinder is 200 mm. Heat is removed from the  $\text{N}_2$  by a quasi-static, constant-pressure process until the temperature of the  $\text{N}_2$  is 300 K. Calculate the work done by the  $\text{N}_2$  gas.



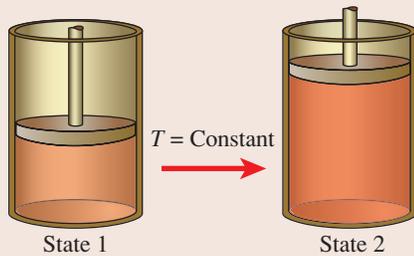
- 4.8 Saturated steam at 2 MPa is contained in a piston-cylinder arrangement as shown in the sketch. The initial volume is  $0.25 \text{ m}^3$ . Heat is removed from the steam during a quasi-static, constant-pressure process such that 197.6 kJ of work is done by the surroundings

on the steam. Determine the quality of the steam at the final state.



- 4.9 A polytropic process is one that obeys the relationship  $PV^n = \text{constant}$ , where  $n$  may take on various values.
- A. Define the type of process associated with each of the following values of  $n$  assuming the working fluid is an ideal gas (i.e., determine which state variable is constant):
- $n = 0$
  - $n = 1$
  - $n = \gamma$  ( $\equiv c_p/c_v$ )
- B. Starting from the same initial state, which process results in the most amount of work performed by the gas when the gas is expanded quasi-statically to a final volume twice as large as the initial volume? Which process results in the least amount of work?
- C. Sketch the three processes on  $P$ - $v$  coordinates.
- 4.10 A 500-lb<sub>m</sub> box is towed at a constant velocity of 5 ft/s along a frictional, horizontal surface by a 200-lb<sub>f</sub> horizontal force. Consider the box as a system.
- A. Draw a force and motion sketch showing directions and magnitudes of all forces (lb<sub>f</sub>) acting on the system and the velocity (ft/s) vector.
- B. Draw an energy sketch showing magnitudes and directions of all rates of work (hp).
- 4.11 Initially, 0.05 kg of air is contained in a piston-cylinder device at 200°C and 1.6 MPa. The air then expands at constant temperature to a pressure of 0.4 MPa. Assume the process occurs slowly enough that the acceleration of the piston can be neglected. The ambient pressure is 101.35 kPa.
- A. Determine the work (kJ) performed by the air in the cylinder on the piston.
- B. Determine the work (kJ) performed by the piston on the ambient environment. Neglect the cross-sectional area of the connecting rod.
- C. Determine the work transfer (kJ) from the piston to the connecting rod. Neglect friction between the piston and cylinder. Assume that no heat transfer to or from the piston occurs and that the energy of the piston does not change.

- D. Discuss why the assumption of negligible piston acceleration was made.
- E. Which of the work interactions might be useful for driving a car? Why?



- 4.12 Initially, 0.5 kmol of an ideal gas at 20°C and 2 atm is contained in a piston–cylinder device. The piston is held in place by a pin. The pin is then removed and the gas expands rapidly. After a new equilibrium is attained, the gas is at 20°C and 1 atm. Taking the gas and piston as the system, determine the net work transfer (kJ) to the atmosphere.

- 4.13 A spherical balloon is inflated from a diameter of 10 in to a diameter of 20 in by forcing air into the balloon with a tire pump. The initial pressure inside the balloon is 20 psia. During the process the pressure is proportional to the balloon diameter.

- A. Determine the amount of work transfer (Btu) from the air inside the balloon to the balloon.
- B. Determine the amount of work transfer (Btu) from the balloon to the surrounding atmosphere.

- 4.14 Initially, 3 lb<sub>m</sub> of steam is contained in a piston–cylinder device at 500 F with a quality of 0.70. Heat is added at constant pressure until all of the liquid is vaporized. The steam then expands adiabatically at constant temperature to a pressure of 400 psia, behaving essentially as an ideal gas. The process occurs slowly enough that the acceleration of the piston may be neglected.

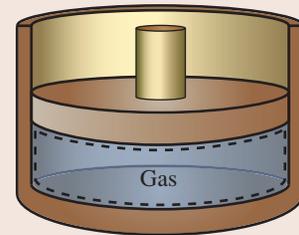
- A. Determine the work transfer (Btu) from the steam in the cylinder to the piston.
- B. Determine the work transfer (Btu) between the piston and the ambient environment. Neglect the cross-sectional area of the connecting rod.
- C. Determine the work transfer (Btu) from the piston to the connecting rod. Neglect friction between the piston and cylinder. Assume that no heat transfer to or from the piston occurs and that the energy of the piston does not change.
- D. Discuss why the assumption of negligible piston acceleration was made.
- E. Which of the work transfers might be useful for driving a car? Why?

- F. How good is the approximation that the steam can be treated as an ideal gas?

- 4.15 An automobile drive shaft rotates at 3000 rev/min and delivers 75 kW of power from the engine to the wheels. Determine the torque (N·m and ft·lb<sub>f</sub>) in the drive shaft.

- 4.16 A compression process occurs in two steps. From state 1 at 15.0 psia and 3.00 ft<sup>3</sup>, compression is to state 2 at 45.0 psia following the path  $PV^2 = \text{constant}$ . From state 2, the compression occurs at constant pressure to a final volume at state 3 of 1.00 ft<sup>3</sup>. Determine the work (Btu) for each step of this two-step process.

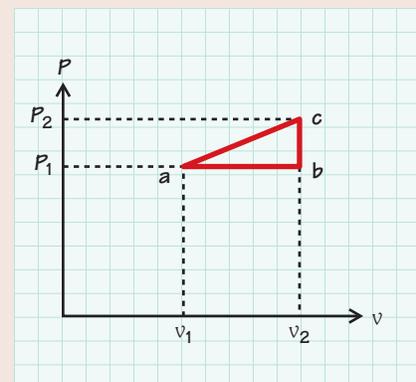
- 4.17 A gas is trapped in a cylinder–piston arrangement as shown in the sketch. The initial pressure and volume are 13,789.5 Pa and 0.02832 m<sup>3</sup>, respectively. Determine the work (kJ) assuming that the volume is increased to 0.08496 m<sup>3</sup> in a constant-pressure process.



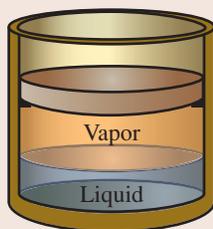
- 4.18 For the same conditions as in Problem 4.17, determine the work done if the process is polytropic with  $n = 1$  (i.e.,  $PV = \text{constant}$ ).

- 4.19 For the same conditions as in Problem 4.17, determine the work done if the process is polytropic with  $n = 1.4$  (i.e.,  $PV^{1.4} = \text{constant}$ ).

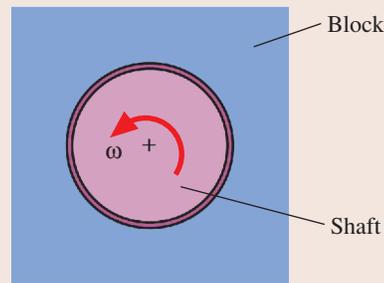
- 4.20 Consider the two processes shown in the sketch:  $ac$  and  $abc$ . Determine the work done by an ideal gas executing these reversible processes if  $P_2 = 2P_1$  and  $v_2 = 2v_1$ ? Assume you are dealing with a closed system and express your answer in terms of the gas constant  $R$  and the temperature  $T_1$ .



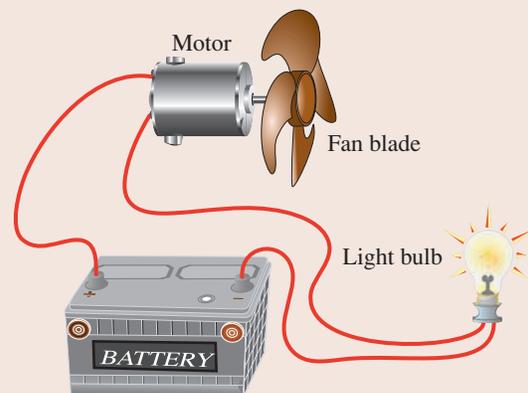
- 4.21 Determine the work done by an ideal gas in a reversible adiabatic expansion from  $T_1$  to  $T_2$ .
- 4.22 Determine the work done in  $\text{ft} \cdot \text{lb}_f$  by a  $2\text{-lb}_m$  steam system as it expands slowly in a cylinder–piston arrangement from the initial conditions of 324 psia and  $12.44 \text{ ft}^3$  to the final conditions of 25.256  $\text{ft}^3$  in accordance with the following relations: (a)  $P = 20V + 75.12$ , where  $V$  and  $P$  are expressed in units of  $\text{ft}^3$  and psia, respectively; and (b)  $PV = \text{constant}$ .
- 4.23 A vertical cylinder–piston arrangement contains  $0.3 \text{ lb}_m$  of  $\text{H}_2\text{O}$  (liquid and vapor in equilibrium) at 120 F. Initially, the volume beneath the  $250\text{-lb}_m$  piston is  $1.054 \text{ ft}^3$ . The piston area is  $120 \text{ in}^2$ . With an atmospheric pressure of  $14.7 \text{ lb}_f/\text{in}^2$ , the piston is resting on the stops. The gravitational acceleration is  $30.0 \text{ ft/s}^2$ . Heat is transferred to the arrangement until only saturated vapor exists inside. (a) Show this process on a  $P$ – $V$  diagram and (b) determine the work done (Btu).



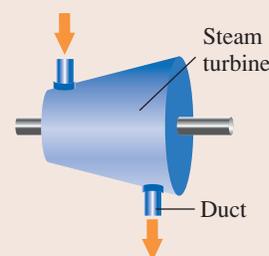
- 4.24 One cubic meter of an ideal gas expands in an isothermal process from 760 to 350 kPa. Determine the work done by this gas in kJ and Btu.
- 4.25 Air is compressed reversibly in a cylinder by a piston. The  $0.12 \text{ lb}_m$  of air in the cylinder is initially at 15 psia and 80 F, and the compression process takes place isothermally to 120 psia. Assuming ideal-gas behavior, determine the work required to compress the air (Btu).
- 4.26 Consider a 30-mm-diameter shaft rotating concentrically in a hole in a block as shown in the sketch. The clearance between the shaft and the block is 0.2 mm and is filled with motor oil at 310 K. The length of the annular space between the shaft and the block is 60 mm. Since the clearance is quite small compared to the shaft radius, the flow produced by the rotating shaft is a Couette flow (see Example 3.6); that is, a linear velocity distribution exists in the oil between the stationary surface ( $v_x = 0$  at the block) to the surface of the rotating shaft ( $v_x = R\omega$ ). The shear stress in the fluid film obeys the relation  $\tau = \mu dv_x/dy$ , where  $y$  is the distance from the stationary wall. Estimate the viscous shear work performed on the fluid in the annular space between the shaft and the block for a shaft speed of 1750 rev/min.



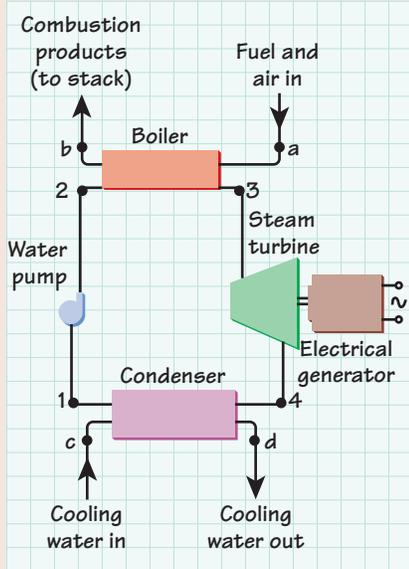
- 4.27 Consider the electrical circuit shown in the sketch in which a light bulb and a motor are wired in series with a battery. The motor drives a fan. Analyze the following thermodynamic systems for all heat and work interactions. Sketch the system boundaries and use labeled arrows for the various  $\dot{Q}$ s and  $\dot{W}$ s. Use subscripts to denote the type of work and make sure that your arrows point in the correct directions.
- Light bulb alone
  - Light bulb and motor, with the system boundary cutting through the motor shaft
  - Light bulb and motor, but the system boundary is now contiguous with the fan blade rather than cutting through the shaft
  - Battery alone
  - Battery, light bulb, motor, and fan



- 4.28 Steam exits a turbine and enters a 1.435-m-diameter duct. The steam is at 4.5 kPa and has a quality of 0.90. The mean velocity of the steam through the duct is 40 m/s. Determine the flow work required to push the steam into the duct.

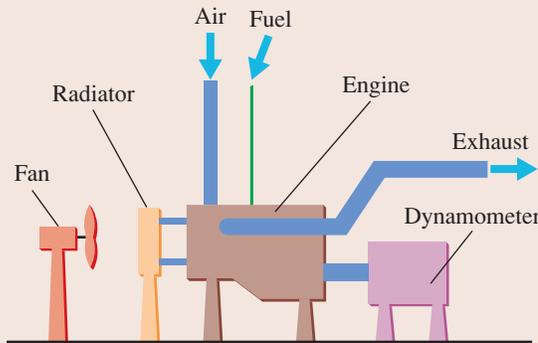


4.29 Consider the Rankine cycle power plant as shown in the sketch. Using a control volume that includes *only* the working fluid (steam/water), draw and label a sketch showing all of the heat and work interactions associated with the cycle control volume.



4.30 Consider the solar water heating scheme illustrated in Fig. 4.12. To analyze this scheme, choose a control surface that includes all of the components shown and cuts through the cold water supply and hot water out lines. Assume hot water is being drawn for a shower. Sketch the control volume and show using labeled arrows all of the heat and work interactions (i.e., all  $\dot{Q}$ s and  $\dot{W}$ s). Use appropriate subscripts to denote the type of work or heat exchanges.

4.31 To conduct a laboratory test, a multicylinder spark-ignition engine is connected to a dynamometer (a device that measures the shaft power output) as shown in the sketch.



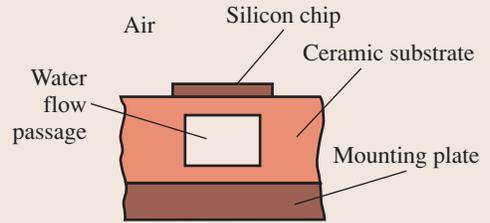
For the following control volumes, draw a sketch indicating all heat and work interactions (i.e.,  $\dot{Q}$ s and  $\dot{W}$ s):

A. The control volume contains only the engine, and the control surface cuts through the hoses to

and from the radiator, through the air and fuel supply lines, through the exhaust pipe, and through the shaft connecting the engine and dynameter.

B. Same as in Part A but now the radiator is included within the control volume.

4.32 In a high-performance computer, silicon chips are actively cooled by water flowing through the ceramic substrate as shown in the sketch. The chip assembly is inside a case through which air freely circulates.

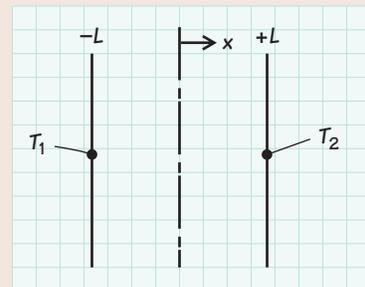


Consider the following thermodynamic systems (or control volumes), sketching each and showing all of the heat interactions with arrows. Label each heat interaction to indicate the mode of heat transfer (i.e.,  $\dot{Q}_{\text{cond}}$ ,  $\dot{Q}_{\text{conv}}$ , or  $\dot{Q}_{\text{rad}}$ ).

- A. Silicon chip only
- B. Silicon chip and substrate (excluding the water)
- C. Ceramic substrate only
- D. Ceramic substrate and the flowing water

4.33 The temperature distribution in a planar slab of nuclear generating material is given by

$$T(x) = 50 \left( 1 - \frac{x^2}{L^2} \right) + \frac{T_2 - T_1}{2} \frac{x}{L} + \frac{T_2 + T_1}{2},$$



where  $L$  is the half-thickness of the slab and  $T_1$  and  $T_2$  are the surface temperatures as shown in the sketch. Note that  $x$  is measured from the center plane and  $T(x)$  has units of kelvins.

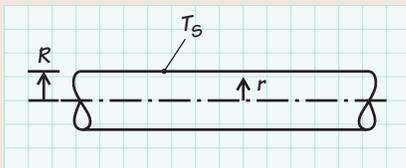
- A. Draw a graph of  $T$  versus  $x$  when  $T_1 = 350$  K,  $T_2 = 450$  K, and  $L = 22$  mm.
- B. What is the value of the temperature gradient at  $x = -L$ ? Give units.

- C. Calculate the conduction heat flux at  $x = -L$  for the conditions in Part A. Give units. The thermal conductivity of the slab is  $25 \text{ W/m}\cdot\text{K}$ . What is the direction of the heat transfer at this location?
- D. Determine the location of the adiabatic plane within the slab for the same conditions as Part A [i.e., the location at which  $\dot{Q}''(x) = 0$ ]. Also determine the temperature at this location.
- E. At  $x = +L$ , is the heat transfer from the slab to the surroundings, or from the surroundings to the slab? Discuss.

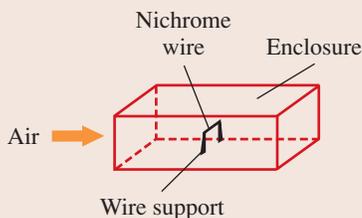
- 4.34 A long cylindrical nuclear fuel rod has the following steady-state temperature distribution:

$$T(r) = 500\left(1 - \frac{r^2}{R^2}\right) + T_s,$$

where  $R$  is the fuel-rod radius and  $T_s$  is the surface temperature.  $T(r)$  is expressed in kelvins. For a fuel rod with  $R = 25 \text{ mm}$ ,  $T_s = 800 \text{ K}$ , and thermal conductivity of  $40 \text{ W/m}\cdot\text{K}$ , determine both the heat flux at  $r = R$  and the heat-transfer rate per unit length of rod, (i.e.,  $\dot{Q}/L [=] \text{ W/m}$ ). Plot the temperature distribution.



- 4.35 A 35-mm length of 1-mm-diameter nichrome (80% Ni, 20% Cr) wire is heated to 915 K by the passage of an electrical current through the wire (Joule heating). The wire is in an enclosure, the walls of which are maintained at 300 K. Warm air at 325 K flows slowly through the enclosure. The emissivity of the wire is 0.5 and the convective heat-transfer coefficient has a value of  $54.9 \text{ W/m}^2\cdot\text{K}$ . Determine both the heat flux and the heat-transfer rate from the nichrome wire.



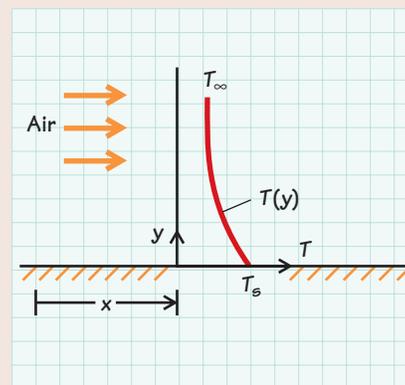
- 4.36 Consider the same physical situation as in Problem 4.35, except that the electrical current is reduced such that the nichrome wire temperature is now 400 K. This temperature reduction also

results in a reduction of the heat-transfer coefficient associated with forced convection, which now has a value of  $36.9 \text{ W/m}^2\cdot\text{K}$ . Compare the fraction of the total heat transfer due to radiation for this situation to the fraction from Problem 4.35. Discuss.

- 4.37 Air at 300 K ( $\equiv T_\infty$ ) flows over a flat plate maintained at 400 K ( $\equiv T_s$ ). At a particular location on the plate, the temperature distribution in the air measured from the plate surface is given by

$$T(y) = (T_s - T_\infty)\exp[-22,000 y(\text{m})] + T_\infty,$$

where  $y$  is the perpendicular distance from the plate, as shown in the sketch.



- A. Determine the conduction heat flux through the air at the plate surface where the no-slip condition applies (i.e., at  $y = 0^+$ ).
- B. The physical mechanism of convection heat transfer starts with conduction in the fluid at the wall, as described in Part A. The details of the flow, however, determine the temperature distribution through the fluid. Use your result from Part A to determine a value for the local heat-transfer coefficient  $h_{\text{conv},x}$  at the  $x$  location where  $T(y)$  is given.
- 4.38 Determine the instantaneous rate of heat transfer from a 1.5-cm-diameter ball bearing with a surface temperature of  $150^\circ\text{C}$  submerged in an oil bath at  $75^\circ\text{C}$  if the surface convective heat transfer coefficient is  $850 \text{ W/m}^2\cdot^\circ\text{C}$ .
- 4.39 Air at 480 F flows over a 20-in by 12-in flat surface. The surface is maintained at 75 F and the convective heat-transfer coefficient at the surface is  $45 \text{ Btu/hr}\cdot\text{ft}^2\cdot\text{F}$ . Find the rate of heat transfer to the surface.
- 4.40 A convective heat flux of  $20 \text{ W/m}^2$  (free convection) is observed between the walls of a room and the ambient air. What is the heat-transfer coefficient when the air temperature is  $32^\circ\text{C}$  and the wall temperature is  $35^\circ\text{C}$ ?

- 4.41 A 30-cm-diameter, 5-m-long steam pipe passes through a room where the air temperature is  $20^{\circ}\text{C}$ . If the exposed surface of the pipe is a uniform  $40^{\circ}\text{C}$ , find the rate of heat loss from the pipe to the air if the surface heat-transfer coefficient is  $8.5\text{ W/m}^2\cdot^{\circ}\text{C}$ .
- 4.42 A 10-ft-diameter spherical tank is used to store petroleum products. The products in the tank maintain the exposed surface of the tank at 75 F. Air at 60 F blows over the surface, resulting in a heat-transfer coefficient of  $10\text{ Btu/hr}\cdot\text{ft}^2\cdot\text{F}$ . Determine the rate of heat transfer from the tank.
- 4.43 The heat-transfer coefficient for water flowing normal to a cylinder is measured by passing water over a 3-cm-diameter, 0.5-m-long electric resistance heater. When water at a temperature of  $25^{\circ}\text{C}$  flows across the cylinder with a velocity of 1 m/s, 30 W of electrical power are required to maintain the heater surface temperature at  $60^{\circ}\text{C}$ . Estimate the heat-transfer coefficient at the heater surface. What is the significance of the water velocity in determining the answer?
- 4.44 A cylindrical electric resistance heater has a diameter of 1 cm and a length of 0.25 m. When water at  $30^{\circ}\text{C}$  flows across the heater a heat-transfer coefficient of  $15\text{ W/m}^2\cdot^{\circ}\text{C}$  exists at the surface. If the electrical input to the heater is 5 W, what is the surface temperature of the heater?
- 4.45 A typical value of the heat-transfer coefficient for water boiling on a flat surface is  $900\text{ Btu/hr}\cdot\text{ft}^2\cdot\text{F}$ . Estimate the heat flux on such a surface when the surface is maintained at 222 F and the water is at 212 F.
- 4.46 Find the rate of radiant energy emitted by an ideal blackbody having a surface area of  $10\text{ m}^2$  when the surface temperature is maintained at (a)  $50^{\circ}\text{C}$ , (b)  $100^{\circ}\text{C}$ , and (c)  $500^{\circ}\text{C}$ .
- 4.47 A surface is maintained at  $100^{\circ}\text{C}$  and is enclosed by very large surrounding surfaces at  $80^{\circ}\text{C}$ . What is the net radiant flux ( $\text{W/m}^2$ ) from this surface if its emissivity is (a) 1.0 or (b) 0.8?
- 4.48 Repeat Problem 4.47 for a surface temperature of  $150^{\circ}\text{C}$ , with all other data remaining unchanged.