Process identification—First order with dead time function

Identification of the model function and its parameters is a topic of its own. In an introductory course, our problems tend to be trivial by comparison. We usually only need to fit our data to a first order function with dead time, or a second order underdamped function. In Chapter 6, we use a first order with dead time function in empirical tuning relations. Analysis of an underdamped response is useful to test our controller settings, or in order words, our system response.[1]

This section is a review of simple calculus. Since experiments must be performed in the time-domain, we need the time-domain solutions. We also must be extra careful—we measure the actual variable, but equations here are based on the deviation variables.

For a first order model

$$\tau_p \frac{dy}{dt} + y = K_p f(t) \quad , \qquad \text{with} \qquad y(0) = 0,$$

and with a step input, $f(t) = Mu(t)$,

$$Y(s) = \frac{MK_p}{s} \frac{1}{(\tau_p s + 1)} \, ,$$

the time-domain solution is

$$\frac{y(t)}{MK_p} = 1 - e^{-t/\tau_p} .$$

To find the process gain, we use the fact that as $t \to \infty$, $y(t) = MK_p$. Hence, using the new steady state measurement, we calculate:

$$K_p = \frac{y}{M} = \frac{\hat{y}^\infty - \hat{y}^o}{\hat{M}^\infty - \hat{M}^o} = \frac{\Delta \hat{y}}{\Delta \hat{M}} \left[\frac{\text{units of } \hat{y}}{\text{units of } \hat{M}} \right]$$

In this equation, y and M are deviation variables, but the others with a ^ are actual measured values. The superscripts o and ∞ are used to denote the initial and final (steady-state) values.

Now we need to find the time constant, and if present, the dead time. There are several ways to estimate the time constant and more often than not, they yield different results. The main reason is that we frequently "force fit" the first order model to data that are obtained from higher order or nonlinear processes.

The following are methods that can be used to estimate the first order time constant:

[1] Use the 63% response.
When $t = \tau_p$, $y(t)/MK_p = 1 - e^{-1} = 0.632$. The time constant can be calculated from the 63.2% response.

[2] Use the initial slope.
The derivative of the response is

$$\frac{d}{dt}\left[\frac{y(t)}{MK_p}\right] = \frac{1}{\tau_p} e^{-t/\tau_p}$$

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[1] What we will skip is the data analysis of a second order overdamped function. Its utility is not overwhelming. If you need to, you can find such methods in the books by Harriott [*Process Control*, McGraw-Hill, 1964] and Smith method [*Digital Computer Process Control*, 1972]. These methods are also summarized in Seborg et al. [Process Dynamics and Control, 1989].

which suggests that the use of the *initial slope* at $t = 0$ can be used to estimate the time constant τ_p.

$$\frac{d}{dt}\left[\frac{y(t)}{MK_p}\right]_{t=0} = \frac{1}{\tau_p}$$

[3] Use a least-squares fit.

Here, we use what we learned in chemical kinetics. Write $y^{\infty} = MK_p$ and rearrangement of the time-domain solution gives

$$\ln\left(\frac{y^{\infty} - y}{y^{\infty}}\right) = -\frac{1}{\tau_p}\,t$$

In control jargon, this is referred to as a fraction (in)complete response method.

When there is a time delay, we eyeball the inflection point and extrapolate back to the time axis. The intercept is the dead time. Like the time constant, we often find it difficult to obtain a good precise value for the dead time. The final step response simulation is quite sensitive to the ratio of t_d/τ_p that we pick.

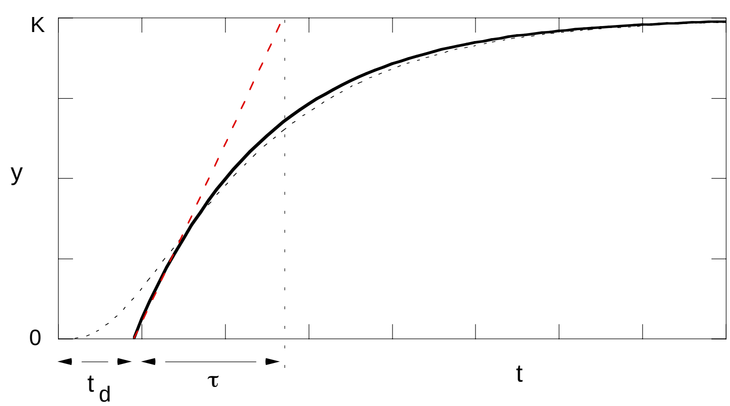

Illustration of fitting a first order with dead time function (solid curve) to data representative of self-regulating and multi-capacity processes (dotted curve, with Y∞ = MK). The time constant estimation shown here is based on the initial slope and a visual estimation of dead time. The Ziegler-Nichols tuning relations also use the slope through the inflection point of the data (not shown).

[4] Use two-point percent response. Option I

We know it is difficult to draw an accurate tangent or to make an estimate based on one time point. Hence, there are methods that try to utilize the response at two different times. One example is that of Sundaresan and Krishnaswamy.[1] Their method uses the times that reach 35.3% and 85.3% response (y/MK_p versus t curve). The recipe is

$$t_d = 1.3\,t_{35.3} - 0.29\,t_{85.3}$$

$$\tau_p = 0.67\,(t_{85.3} - t_{35.3})$$

[1] *Can. J. Chem. Eng.*, **56**, 257 (1977).

[5] Use two-point percent response. Option II

A second example uses the times that reach 28.3% and 63.2% response.[1] The recipe is

$$t_d = t_{63.2} - t_{28.3}$$

$$\tau_p = 1.5\,(t_{63.2} - t_{28.3})$$

So which estimation is the best? It takes trial and error and this is where MATLAB comes in handy.

Final remark:

Because we are fitting data (very likely on a nonlinear process) with no physical reasons, the approximations on the gain, time constant(s), damping ratio, and dead time can be different depending on the specific conditions, input step size, and the direction of change.

The results also depend on the estimation method, since by definition, these methods are neither accurate nor foolproof. Nevertheless, they do give us a quick and dirty estimate. How you use them is subject to our judgment.

Generally speaking, with noisy and scattered data, the precision of fitting a second-order plus dead-time model is low. This is one reason why many of the empirical controller design methods are based on the simpler first order plus time delay function.

Finally, recall from Eq. (2-40) that for a nonlinear process, the coefficients of the linearized model depends on the steady state. If we change the operating conditions, the coefficients likely will change too, and whatever that we have measured at the original steady state condition would no longer be valid.

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[1] Smith and Corripio (1997), Section 6-3.