Worked solutions

AS Level practice paper

1 $r = 0.2$ m

$\omega = 6$ revolutions per minute $= 0.1$ rev s^{-1} $= 0.1 \times 2\pi$ rad s^{-1}

$v = r\omega = 0.2 \times 0.1 \times 2\pi = \dfrac{\pi}{25}$ m s^{-1} (Answer B)

[1 mark]

2 [mass] = **M**

[length] = **L**

[time] = **T**

[tension] = [ma] = $\mathbf{MLT}^{-2}$

[velocity] = $\mathbf{LT}^{-1}$

Since $v = k\, m^a\, l^b\, T^c$

$\mathbf{LT}^{-1} = \mathbf{M}^a\, \mathbf{L}^b\, \mathbf{M}^c\, \mathbf{L}^c\, \mathbf{T}^{-2c}$

Therefore: for **L**, $1 = b + c$; for **T**, $-1 = -2c$; and for **M**, $0 = a + c$

$a = \dfrac{-1}{2}, b = \dfrac{1}{2}, c = \dfrac{1}{2}$

So b, c and the absolute value of a are all equal. (Answer D)

[1 mark]

3 $m = 0.4, r = 0.8, v = 3$

Using Newton's second law:

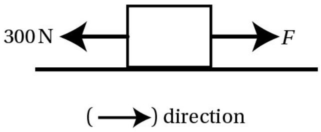
$T = ma$

$T = m\dfrac{v^2}{r}$

$T = 0.4 \times \dfrac{3^2}{0.8} = 4.5\,\text{N}$

[2 marks]

4



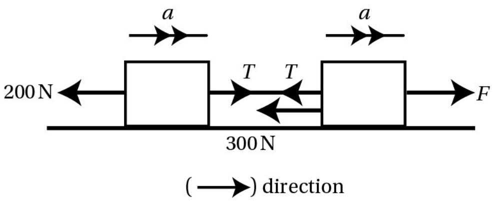
a At the maximum speed the resistance force is equal to the driving force.

Using $P = Fv$:

$12\,000 = 300v$ so $v = 40\ \text{m s}^{-1}$

[2 marks]

b i



Using $P = Fv$ for the car:

$12\,000 = 8F$ so that $F = 1500$ N

Using Newton's second law for the car in the direction of motion:

$$F = ma$$
$$F_D - 300 - T = 1000a$$
$$1500 - 300 - T = 1000a$$
$$\frac{1200 - T}{1000} = a \qquad (1)$$

Using Newton's second law for the trailer in the direction of motion:

$$F = ma$$
$$T - 200 = 250a$$
$$\frac{T - 200}{250} = a \qquad (2)$$

Substituting from (1):

$$\frac{1200-T}{1000}=\frac{T-200}{250}$$
$$1200-T=4T-800$$
$$2000=5T$$
$$T=400\text{ N}$$

[4 marks]

ii Substituting T = 400 into (2):

$$\frac{400-200}{250}=a$$

$$a=0.8\text{ m s}^{-2}$$

[2 marks]

c It is light and inextensible.

[1 mark]

5 a Using the equations of motion under constant acceleration, and choosing the direction of motion as vertically upwards:

$$u=\sqrt{6gl},\ v=v,\ s=l,\ a=-g$$

Giving:

$$v^2=u^2+2as$$
$$v^2=6gl-2gl=4gl$$
$$v=2\sqrt{gl}$$

[3 marks]

b If the two spheres are not next to each other you cannot assume that *A* moves a distance of *l* before the string becomes taut. The answer to part **a** would be larger since the height that *A* would travel would be smaller.

[1 mark]

c Using conservation of momentum:

Before:	**After:**
$m_A u_A + m_B u_B$	$m_A v_{AB} + m_B v_{AB}$

$$m_A u_A + m_B u_B = m_A v_{AB} + m_B v_{AB}$$
$$2m \times 2\sqrt{gl} + 0 = (2m + 3m)v$$
$$0.8\sqrt{gl} = v$$

[2 marks]

d As a whole system, all the kinetic energy is converted to gravitational potential energy:

$$\frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}v^2 = gh$$
$$\frac{1}{2} \times \frac{16}{25}gl = gh$$
$$\frac{8}{25}l = h$$
$$\frac{8}{25}l + l = \frac{33}{25}l$$

[3 marks]

e

Kinetic energy before:	**Kinetic energy after:**
$\frac{1}{2}2m4gl$	$\frac{1}{2}5m\frac{16}{25}gl$

$$\frac{1}{2}2m4gl - \frac{1}{2}5m\frac{16}{25}gl = 4mgl - \frac{8}{5}mgl = \frac{12}{5}mgl$$

[3 marks]

6 Y is moving in the opposite direction to its original motion before the collision.

Using conservation of momentum in the direction of motion of X:

$$m_X u_X + m_Y u_Y = m_X v_X + m_Y v_Y$$

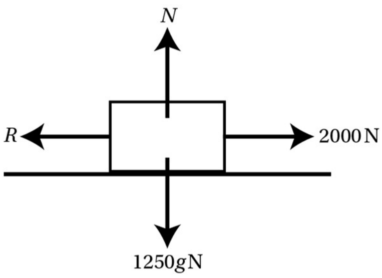
$$0.6 \times 5 - 0.4 \times 5 = -2.4 \times 0.6 + 0.4 \times v_Y$$

$$v_Y = \frac{2.44}{0.4} = 6.1\,\mathrm{m\,s^{-1}}$$

$$e = \frac{v_Y - v_X}{u_X - u_Y} = \frac{6.1 - (-2.4)}{5 - (-5)} = 0.85$$

[4 marks]

7



Resolving forces in the direction of the motion gives:

$F = 2000 - R$, where R is the resistance force.

$F(x) = 2000 - kx^2$, since R is directly proportional to the square of the distance from A and k is a constant of proportionality.

At $x = 20$, $F = 1920$, therefore:

$F(20) = 2000 - k \times 20^2 = 1920$

$20^2 k = 80$

$k = 0.2$

Resultant force $= 2000 - \dfrac{x^2}{5}$

Using $\int F(x)\,\mathrm{d}x$ equals work done for a variable force:

$$\int F(x)\,\mathrm{d}x = \int_0^{50}\left(2000 - \frac{x^2}{5}\right)\mathrm{d}x = \left[2000x - \frac{x^3}{15}\right]_0^{50}$$

$$(2000 \times 50 - \frac{50^3}{15}) - 0 = 91.7 \text{ kJ (3 s.f.)}$$

[5 marks]

8 Since the string is elastic it means that you can model it as obeying Hooke's law and so the total work done in stretching the spring an extension of x from its natural length l is given by $\frac{kx^2}{2}$, where k is the stiffness of the string. You can assume that the spring is light.

Assuming there are no external forces acting, you can use the conservation of energy.

Set the gravitational potential energy level as zero when the particle is pulled down and let x be the extension for the spring.

Energy at the start position:

KE: 0 J (started from rest)

GPE: 0 J

EPE: $\frac{40 \times 0.3^2}{2}$ J

Energy when the ball is at its highest point:

KE: 0 J (at the highest point it is when the ball is a rest again)

GPE: $(0.3 - x) \times 0.4 \times 10$ J

EPE: $\frac{40 \times x^2}{2}$ J

$$(0.3 - x) \times 0.4 \times 10 + \frac{40 \times x^2}{2} = \frac{40 \times 0.3^2}{2}$$

Simplifying this quadratic:

$$\frac{6}{5} - 4x + 20x^2 = \frac{9}{5}$$

Rearranging:

$100x^2 - 20x - 3 = 0$.

Factorising:

$(10x + 1)(10x - 3) = 0$.

$x = 0.3$ or -0.1

Since x is the extension, -0.1 means that the spring is compressed by 0.1 m.

Therefore the length of the spring is $0.6 - 0.1 = 0.5$ m.

[6 marks]

Worked solutions

A Level practice paper

1 You want the magnitude of the impulse on the remote control car. If you take the direction for the question to be in the same direction as the motion of the car:

$$I = mv - mu$$
$$I = 0 - 35 \times 5$$
$$I = -175\,\text{N s (Answer D)}$$

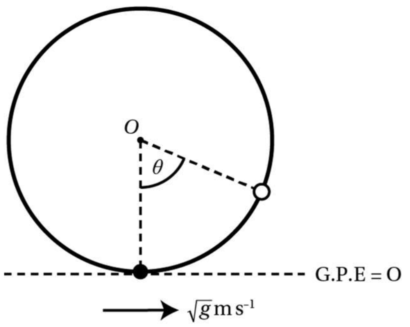
[1 mark]

2 Given that:

$$T = \frac{\lambda x}{l}$$
$$6 = \frac{\lambda \times 0.03}{0.45}$$
$$\lambda = 90\,\text{N (Answer A)}$$

[1 mark]

3 **a**



Energy at start:	**Energy at end:**
$\frac{1}{2}m\left(\sqrt{g}\right)^2$	
0	$mg(1-\cos\theta)$

By conservation of energy:

$$\frac{1}{2}mg = mg(1-\cos\theta)$$
$$\frac{1}{2} = (1-\cos\theta)$$
$$\cos\theta = \frac{1}{2}$$

So the height above the point of projection is 0.5 m.

[3 marks]

b Include friction in the model.

[1 mark]

4 a Mass = m, $\lambda = mg$ and natural length = l. Let $x = r - l$ be the extension for the string.

The tension in the string is given by $T = \frac{\lambda x}{l}$:

$$T = \frac{(r-l)\lambda}{l}$$
$$= \frac{(r-l)mg}{l}$$

[2 marks]

b $\frac{k}{\pi}$ rev s^{-1} is $2k$ rad s^{-1} = ω.

Using Newton's second law towards the centre of the circular motion:

$$F = ma$$

$$\frac{(r-l)mg}{l} = mr\omega^2$$

$$(r-l)g = lr4k^2$$

$$rg - lg = lr4k^2$$

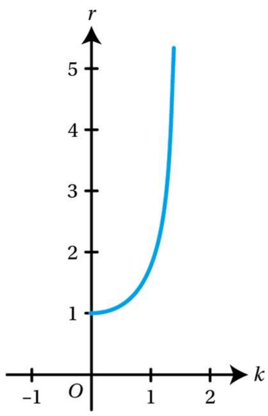
$$rg - lr4k^2 = lg$$

$$r(g - l4k^2) = lg$$

$$r = \frac{lg}{g - l4k^2}$$

[4 marks]

c You need to let $l = 1$ and sketch the graph of $r = \dfrac{g}{g - 4k^2}$ with $k > 0$, as you can consider $2k$ rad s^{-1} as the angular speed.



[2 marks]

d There is an asymptote at $k = \frac{1}{2}\sqrt{g}$.

Given that $T = \frac{(r-1)mg}{1} = (r-1)mg$, if the string breaks when the tension is mg then:

$(r - 1)mg < mg$ and so $(r - 1) < 1$

then $r < 2$.

Given that the string has natural length 1, you have a bound for r : $1 < r < 2$.

This restricts the graph from part **c**. Looking at the values for k that give this bound: $0 < k < \frac{1}{2}\sqrt{\frac{g}{2}}$.

[2 marks]

5 $$\bar{x} = \frac{\int_a^b \pi y^2 x\,dx}{\int_a^b \pi y^2\,dx} = \frac{\int_a^b x^{-2} x\,dx}{\int_a^b x^{-2}\,dx} = \frac{\int_a^b x^{-1}\,dx}{\int_a^b x^{-2}\,dx}$$

$$= \frac{[\ln x]_a^b}{\left[-x^{-1}\right]_a^b} = \frac{\ln b - \ln a}{\frac{1}{a} - \frac{1}{b}}$$

$$= \frac{ab\ln\left(\frac{b}{a}\right)}{b-a}$$

a This is the coordinate of the centre of mass on the x-axis. To find the distance from a you must subtract a:

$$\frac{ab\ln\left(\frac{b}{a}\right)}{b-a} - a = a\left(\frac{b\ln\left(\frac{b}{a}\right)}{b-a} - 1\right) = a\left(\frac{b\ln\left(\frac{b}{a}\right)}{b-a} - \frac{b-a}{b-a}\right) = a\left(\frac{b\ln\left(\frac{b}{a}\right) - b + a}{b-a}\right)$$

$$= \frac{a\left(a - b - b\ln\left(\frac{a}{b}\right)\right)}{b-a}$$

[5 marks]

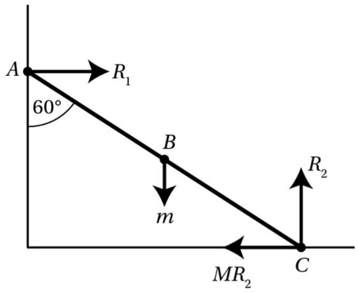
b For $a = 1$ and $b = 5$, the centre of mass lies on the axis of symmetry of the object and is $\dfrac{5\ln 5 - 4}{4}$ units from its base. The solid shape will be on the point of toppling when the perpendicular from the centre of mass passes through a corner on the base of the solid shape.

At this point the angle is given by:

$$\theta = \tan^{-1}\frac{4}{5\ln 5 - 4} = 44.7°$$

[2 marks]

6



Let the length of the ladder be $2x$ and the mass be m.

Taking clockwise moments about A:

$$x \times m\sin 60 + 2x \times \mu R_2 \sin 30 - 2x \times R_2 \sin 60 = 0$$

Resolving forces vertically:

$m = R_2$

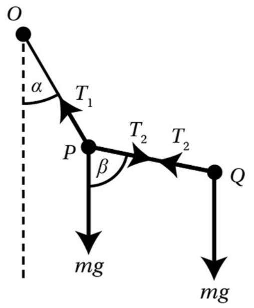
Therefore,

$$xm\frac{\sqrt{3}}{2} + 2x\mu m\frac{1}{2} - 2xm\frac{\sqrt{3}}{2} = 0 \text{ and } \frac{\sqrt{3}}{2} + \mu - \sqrt{3} = 0,$$

so $\mu = \dfrac{\sqrt{3}}{2} = 0.87$ (2 s.f.)

[5 marks]

7



a The string is light and inextensible.

[1 mark]

b Resolving forces vertically at Q:

$T_2 \cos\beta = mg$ (1)

Using $F = ma$ towards the centre of the circular motion for Q:

$$F = mr\omega^2$$

$$T_2 \sin\beta = m(l\sin\alpha + l\sin\beta)\omega^2$$

Dividing this equation by equation (1):

$$\frac{T_2 \sin\beta}{T_2 \cos\beta} = \frac{m(l\sin\alpha + l\sin\beta)\omega^2}{mg}$$

$\tan\beta = \dfrac{l\omega^2}{g}(\sin\alpha + \sin\beta)$ (2)

[4 marks]

c Resolving forces vertically at P:

$mg + T_2 \cos\beta = T_1 \cos\alpha$ (3)

Using $F = ma$ towards the centre of the circular motion for P:

$T_1 \sin\alpha - T_2 \sin\beta = ml\omega^2 \sin\alpha$ (4)

Substituting from (1) into (3):

$$mg + mg = T_1 \cos\alpha$$

$$\frac{2mg}{\cos\alpha} = T_1 \qquad (5)$$

Substituting (5) and $\frac{mg}{\cos\beta} = T_2$ into (4):

$$\frac{2mg}{\cos\alpha}\sin\alpha - \frac{mg}{\cos\beta}\sin\beta = ml\omega^2\sin\alpha$$

$$2g\tan\alpha - g\tan\beta = l\omega^2\sin\alpha$$

From part **b**:

$$2g\tan\alpha - g\frac{l\omega_2}{g}(\sin\alpha + \sin\beta) = l\omega^2\sin\alpha$$

$$2g\tan\alpha - l\omega^2\sin\alpha - l\omega^2\sin\beta = l\omega^2\sin\alpha$$

$$2g\tan\alpha = 2l\omega^2\sin\alpha + l\omega^2\sin\beta$$

$$\tan\alpha = \frac{l\omega^2}{2g}(2\sin\alpha + \sin\beta)$$

[4 marks]

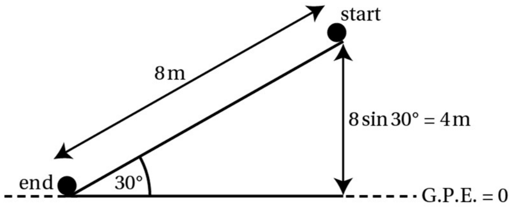
8 a Using conservation of energy:

Kinetic energy before:	**Kinetic energy after:**
$0.5 \times 25 \times 3^2$	$0.5 \times 25 \times 5^2$

Gain in kinetic energy = $0.5 \times 25 \times 5^2 - 0.5 \times 25 \times 3^2 = 200$ J

[2 marks]

b



Using conservation of energy:

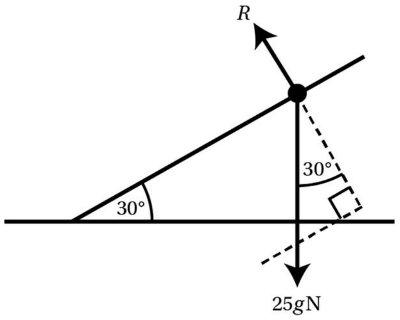
Energy at start:	**Energy at end:**
$0.5 \times 25 \times 3^2$	$0.5 \times 25 \times 5^2$
$4 \times 25 \times g$	0

$0.5 \times 25 \times 3^2 + 4 \times 25 \times g = 0.5 \times 25 \times 5^2 +$ work done

Work done = 780 J

[5 marks]

c



Frictional force at a maximum is given by $F = \mu R$, where R is the normal reaction force.

Using work done = force × distance:

$$F = \frac{780}{8} = 97.5\text{ N}$$

$R = 25g\cos 30°$

$$F = \mu R = 97.5\text{ N}$$
$$\mu \times 25g\cos 30° = 97.5$$
$$\mu = 0.46 \text{ (2 s.f.)}$$

[3 marks]

d For a constant speed, the work done against friction must be equal to the loss in gravitational potential energy. If the particle travels x metres down the slope, then:

$$mgx\sin 30° = \mu Rx$$
$$mgx\sin 30° = \mu mg\cos 30° \times x$$
$$\tan 30° = \mu$$
$$\mu = 0.58 \text{ (2 s.f.)}$$

[3 marks]