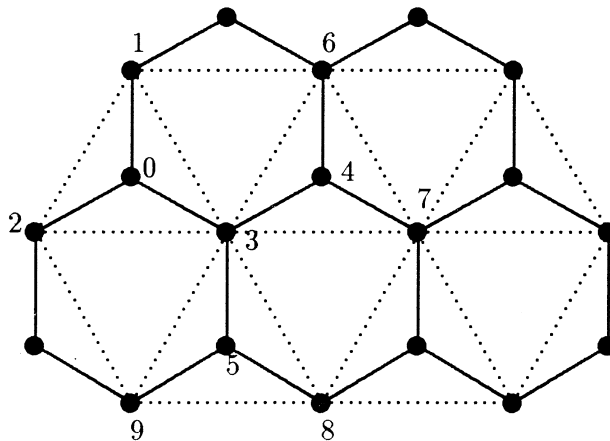


Honeycomb lattice correlation functions

The following definitions and systematic naming of correlation functions are accomplished by the vertex-number representation of graphs on the honeycomb lattice. For a given site-number representation, $\langle 3589 \rangle$, the vertex-number representation, $[(111), (122), (122), (122)]$, is found by a computer program. The numerical ordering of correlation functions, such as x_7, x_8, x_9 , is established by the increasing order of the vertex numbers:

$$\begin{aligned}x_7 &= \langle 3589 \rangle = [(111), (122), (122), (122)] \\x_8 &= \langle 3457 \rangle = [(112), (112), (123), (123)] \\x_9 &= \langle 3456 \rangle = [(112), (112), (124), (124)].\end{aligned}$$

1 Definitions of even-number correlation functions



Honeycomb lattice.

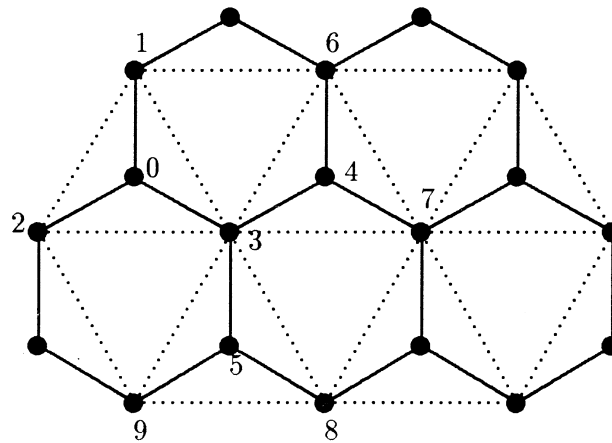
- $x_1 = \langle 01 \rangle, \langle 02 \rangle, \langle 03 \rangle, \langle 34 \rangle, \langle 35 \rangle, \langle 46 \rangle, \langle 47 \rangle, \langle 58 \rangle, \langle 59 \rangle$
- $x_2 = u_1 = \langle 04 \rangle, \langle 05 \rangle, \langle 12 \rangle, \langle 13 \rangle, \langle 16 \rangle, \langle 23 \rangle, \langle 29 \rangle, \langle 36 \rangle, \langle 37 \rangle, \langle 38 \rangle, \langle 39 \rangle, \langle 45 \rangle, \langle 67 \rangle, \langle 78 \rangle, \langle 89 \rangle$
- $x_3 = \langle 06 \rangle, \langle 09 \rangle, \langle 14 \rangle, \langle 25 \rangle, \langle 48 \rangle, \langle 57 \rangle$
- $x_4 = \langle 07 \rangle, \langle 08 \rangle, \langle 15 \rangle, \langle 24 \rangle, \langle 49 \rangle, \langle 56 \rangle$
- $x_5 = u_2 = \langle 17 \rangle, \langle 19 \rangle, \langle 26 \rangle, \langle 28 \rangle, \langle 68 \rangle, \langle 79 \rangle$
- $x_6 = u_3 = \langle 18 \rangle, \langle 27 \rangle, \langle 69 \rangle$
- $x_7 = \langle 0123 \rangle, \langle 0345 \rangle, \langle 3467 \rangle, \langle 3589 \rangle$
- $x_8 = \langle 0134 \rangle, \langle 0235 \rangle, \langle 0346 \rangle, \langle 0359 \rangle, \langle 3457 \rangle, \langle 3458 \rangle$
- $x_9 = \langle 0135 \rangle, \langle 0234 \rangle, \langle 0347 \rangle, \langle 0358 \rangle, \langle 3456 \rangle, \langle 3459 \rangle$
- $x_{10} = \langle 0124 \rangle, \langle 0125 \rangle, \langle 0348 \rangle, \langle 0349 \rangle, \langle 0356 \rangle, \langle 0357 \rangle, \langle 0467 \rangle, \langle 0589 \rangle, \langle 1345 \rangle, \langle 2345 \rangle, \langle 4567 \rangle, \langle 4589 \rangle$
- $x_{11} = \langle 0136 \rangle, \langle 0239 \rangle, \langle 1346 \rangle, \langle 2359 \rangle, \langle 3478 \rangle, \langle 3578 \rangle$
- $x_{12} = \langle 0126 \rangle, \langle 0129 \rangle, \langle 0139 \rangle, \langle 0236 \rangle, \langle 1347 \rangle, \langle 1467 \rangle, \langle 2358 \rangle, \langle 2589 \rangle, \langle 3468 \rangle, \langle 3579 \rangle, \langle 4678 \rangle, \langle 5789 \rangle$
- $x_{13} = \langle 0137 \rangle, \langle 0238 \rangle, \langle 1359 \rangle, \langle 2346 \rangle, \langle 3479 \rangle, \langle 3478 \rangle$
- $x_{14} = \langle 0138 \rangle, \langle 0237 \rangle, \langle 1348 \rangle, \langle 2347 \rangle, \langle 3469 \rangle, \langle 3569 \rangle$
- $x_{15} = \langle 0127 \rangle, \langle 0128 \rangle, \langle 1589 \rangle, \langle 2467 \rangle, \langle 4679 \rangle, \langle 5689 \rangle$
- $x_{16} = \langle 0369 \rangle, \langle 1348 \rangle, \langle 2357 \rangle$
- $x_{17} = \langle 0145 \rangle, \langle 0245 \rangle, \langle 0367 \rangle, \langle 0389 \rangle, \langle 0456 \rangle, \langle 0457 \rangle, \langle 0458 \rangle, \langle 0459 \rangle, \langle 1234 \rangle, \langle 1235 \rangle, \langle 3489 \rangle, \langle 3567 \rangle$
- $x_{18} = \langle 0368 \rangle, \langle 0379 \rangle, \langle 1349 \rangle, \langle 1357 \rangle, \langle 2348 \rangle, \langle 2356 \rangle$
- $x_{19} = \langle 0378 \rangle, \langle 1356 \rangle, \langle 2349 \rangle$
- $x_{20} = \langle 0146 \rangle, \langle 0259 \rangle, \langle 4578 \rangle$
- $x_{21} = \langle 0147 \rangle, \langle 0159 \rangle, \langle 0246 \rangle, \langle 0258 \rangle, \langle 4568 \rangle, \langle 4579 \rangle$
- $x_{22} = \langle 0156 \rangle, \langle 0249 \rangle, \langle 0478 \rangle, \langle 0578 \rangle, \langle 1456 \rangle, \langle 2459 \rangle$
- $x_{23} = \langle 0149 \rangle, \langle 0256 \rangle, \langle 0468 \rangle, \langle 0579 \rangle, \langle 1457 \rangle, \langle 2458 \rangle$
- $x_{24} = \langle 0158 \rangle, \langle 0247 \rangle, \langle 4569 \rangle$
- $x_{25} = \langle 0148 \rangle, \langle 0257 \rangle, \langle 0469 \rangle, \langle 0569 \rangle, \langle 1458 \rangle, \langle 2457 \rangle$
- $x_{26} = \langle 0157 \rangle, \langle 0248 \rangle, \langle 0479 \rangle, \langle 0568 \rangle, \langle 1459 \rangle, \langle 2456 \rangle$
- $x_{27} = \langle 0169 \rangle, \langle 0269 \rangle, \langle 1468 \rangle, \langle 1478 \rangle, \langle 2578 \rangle, \langle 2579 \rangle$
- $x_{28} = \langle 0167 \rangle, \langle 0289 \rangle, \langle 1246 \rangle, \langle 1259 \rangle, \langle 4789 \rangle, \langle 5678 \rangle$
- $x_{29} = \langle 0168 \rangle, \langle 0279 \rangle, \langle 1469 \rangle, \langle 1578 \rangle, \langle 2478 \rangle, \langle 2569 \rangle$
- $x_{30} = \langle 0179 \rangle, \langle 0268 \rangle, \langle 1479 \rangle, \langle 1579 \rangle, \langle 2468 \rangle, \langle 2568 \rangle$
- $x_{31} = \langle 0189 \rangle, \langle 0267 \rangle, \langle 1247 \rangle, \langle 1258 \rangle, \langle 4689 \rangle, \langle 5679 \rangle$
- $x_{32} = \langle 0178 \rangle, \langle 0278 \rangle, \langle 1569 \rangle, \langle 1569 \rangle, \langle 2469 \rangle, \langle 2479 \rangle$
- $x_{33} = u_6 = \langle 1236 \rangle, \langle 1239 \rangle, \langle 1367 \rangle, \langle 2389 \rangle, \langle 3678 \rangle, \langle 3789 \rangle$

- $x_{34}=u_5=\langle 1237\rangle, \langle 1238\rangle, \langle 1368\rangle, \langle 1369\rangle, \langle 1378\rangle, \langle 1389\rangle, \langle 2367\rangle, \langle 2369\rangle, \langle 2378\rangle, \langle 2379\rangle, \langle 3679\rangle, \langle 3689\rangle$
- $x_{35}=u_4=\langle 1379\rangle, \langle 2368\rangle$
- $x_{36}=\langle 0678\rangle, \langle 0789\rangle, \langle 1249\rangle, \langle 1256\rangle, \langle 1567\rangle, \langle 2489\rangle$
- $x_{37}=u_8=\langle 1267\rangle, \langle 1269\rangle, \langle 1289\rangle, \langle 1678\rangle, \langle 2789\rangle, \langle 6789\rangle$
- $x_{38}=u_9=\langle 1268\rangle, \langle 1279\rangle, \langle 1679\rangle, \langle 1789\rangle, \langle 2678\rangle, \langle 2689\rangle$
- $x_{39}=\langle 0489\rangle, \langle 0567\rangle, \langle 1245\rangle$
- $x_{40}=\langle 0679\rangle, \langle 0689\rangle, \langle 1248\rangle, \langle 1257\rangle, \langle 1489\rangle, \langle 2567\rangle$
- $x_{41}=u_7=\langle 1278\rangle, \langle 1689\rangle, \langle 2679\rangle$
- $x_{42}=\langle 012345\rangle, \langle 034567\rangle, \langle 034589\rangle$
- $x_{43}=\langle 013456\rangle, \langle 023459\rangle, \langle 034578\rangle$
- $x_{44}=\langle 013457\rangle, \langle 013459\rangle, \langle 023456\rangle, \langle 023458\rangle, \langle 034568\rangle, \langle 034579\rangle$
- $x_{45}=\langle 013458\rangle, \langle 023457\rangle, \langle 034569\rangle$
- $x_{46}=\langle 012346\rangle, \langle 012359\rangle, \langle 013467\rangle, \langle 023589\rangle, \langle 345678\rangle, \langle 345789\rangle$
- $x_{47}=\langle 012349\rangle, \langle 012356\rangle, \langle 034678\rangle, \langle 035789\rangle, \langle 134567\rangle, \langle 234589\rangle$
- $x_{48}=\langle 012347\rangle, \langle 012358\rangle, \langle 013589\rangle, \langle 023467\rangle, \langle 345679\rangle, \langle 345689\rangle$
- $x_{49}=\langle 012348\rangle, \langle 012357\rangle, \langle 034679\rangle, \langle 035689\rangle, \langle 134589\rangle, \langle 234567\rangle$
- $x_{50}=\langle 012369\rangle, \langle 134678\rangle, \langle 235789\rangle$
- $x_{51}=\langle 012367\rangle, \langle 012389\rangle, \langle 123467\rangle, \langle 123589\rangle, \langle 346789\rangle, \langle 356789\rangle$
- $x_{52}=\langle 012368\rangle, \langle 012389\rangle, \langle 134679\rangle, \langle 125789\rangle, \langle 234678\rangle, \langle 235689\rangle$
- $x_{53}=\langle 012378\rangle, \langle 135689\rangle, \langle 234679\rangle$
- $x_{54}=\langle 013468\rangle, \langle 013469\rangle, \langle 023569\rangle, \langle 023579\rangle, \langle 134578\rangle, \langle 234578\rangle$
- $x_{55}=\langle 013478\rangle, \langle 013569\rangle, \langle 023469\rangle, \langle 023578\rangle, \langle 134568\rangle, \langle 234579\rangle$
- $x_{56}=\langle 013479\rangle, \langle 013579\rangle, \langle 023468\rangle, \langle 023568\rangle, \langle 134579\rangle, \langle 234568\rangle$
- $x_{57}=\langle 013568\rangle, \langle 013578\rangle, \langle 023478\rangle, \langle 023479\rangle, \langle 134569\rangle, \langle 234569\rangle$

- $x_{58} = \langle 013567 \rangle, \langle 023489 \rangle, \langle 034789 \rangle, \langle 035678 \rangle, \langle 123456 \rangle, \langle 123459 \rangle$
- $x_{59} = \langle 013489 \rangle, \langle 023567 \rangle, \langle 034689 \rangle, \langle 035679 \rangle, \langle 123457 \rangle, \langle 123458 \rangle$
- $x_6 = \langle 012456 \rangle, \langle 012459 \rangle, \langle 014567 \rangle, \langle 024589 \rangle, \langle 045678 \rangle, \langle 045789 \rangle$
- $x_{61} = \langle 012457 \rangle, \langle 012458 \rangle, \langle 014589 \rangle, \langle 024567 \rangle, \langle 045679 \rangle, \langle 045689 \rangle$
- $x_{62} = \langle 012469 \rangle, \langle 012569 \rangle, \langle 014678 \rangle, \langle 025789 \rangle, \langle 145678 \rangle, \langle 245789 \rangle$
- $x_{63} = \langle 012467 \rangle, \langle 012589 \rangle, \langle 456789 \rangle$
- $x_{64} = \langle 012468 \rangle, \langle 012579 \rangle, \langle 014679 \rangle, \langle 025689 \rangle, \langle 145789 \rangle, \langle 245678 \rangle$
- $x_{65} = \langle 012489 \rangle, \langle 012567 \rangle, \langle 046789 \rangle, \langle 056789 \rangle, \langle 124567 \rangle, \langle 124589 \rangle$
- $x_{66} = \langle 012479 \rangle, \langle 012568 \rangle, \langle 015789 \rangle, \langle 024678 \rangle, \langle 145679 \rangle, \langle 245689 \rangle$
- $x_{67} = \langle 012478 \rangle, \langle 012578 \rangle, \langle 015689 \rangle, \langle 024679 \rangle, \langle 145689 \rangle, \langle 245679 \rangle$
- $x_{68} = \langle 013679 \rangle, \langle 023689 \rangle, \langle 123468 \rangle, \langle 123579 \rangle, \langle 134789 \rangle, \langle 235678 \rangle$
- $x_{69} = \langle 013689 \rangle, \langle 023679 \rangle, \langle 123478 \rangle, \langle 123578 \rangle, \langle 134689 \rangle, \langle 235679 \rangle$
- $x_7 = \langle 012679 \rangle, \langle 012689 \rangle, \langle 124678 \rangle, \langle 125789 \rangle, \langle 146789 \rangle, \langle 256789 \rangle$
- $x_{71} = \langle 013678 \rangle, \langle 023789 \rangle, \langle 123469 \rangle, \langle 123569 \rangle, \langle 135678 \rangle, \langle 234789 \rangle$
- $x_{72} = \langle 013789 \rangle, \langle 023678 \rangle, \langle 123479 \rangle, \langle 123568 \rangle, \langle 135679 \rangle, \langle 234689 \rangle$
- $x_{73} = \langle 012678 \rangle, \langle 012789 \rangle, \langle 124679 \rangle, \langle 125689 \rangle, \langle 156789 \rangle, \langle 246789 \rangle$
- $x_{74} = \langle 036789 \rangle, \langle 123489 \rangle, \langle 123567 \rangle$
- $x_{75} = \langle 014568 \rangle, \langle 014569 \rangle, \langle 014578 \rangle, \langle 024569 \rangle, \langle 024578 \rangle, \langle 024579 \rangle$
- $x_{76} = \langle 014579 \rangle, \langle 024568 \rangle$
- $x_{77} = \langle 014689 \rangle, \langle 025679 \rangle, \langle 124578 \rangle$
- $x_{78} = \langle 014789 \rangle, \langle 015679 \rangle, \langle 024689 \rangle, \langle 025678 \rangle, \langle 124568 \rangle, \langle 124579 \rangle$
- $x_{79} = \langle 015678 \rangle, \langle 024789 \rangle, \langle 124569 \rangle$

- $x_{80} = \langle 016789 \rangle, \langle 026789 \rangle, \langle 124689 \rangle, \langle 124789 \rangle, \langle 125678 \rangle, \langle 125679 \rangle$
- $x_{81} = u_{11} = \langle 123678 \rangle, \langle 123679 \rangle, \langle 123689 \rangle, \langle 123789 \rangle, \langle 136789 \rangle, \langle 236789 \rangle$
- $x_{82} = u_{10} = \langle 126789 \rangle$
- $x_{83} = \langle 01234569 \rangle, \langle 01345678 \rangle, \langle 02345789 \rangle$
- $x_{84} = \langle 01234567 \rangle, \langle 01234589 \rangle, \langle 03456789 \rangle$
- $x_{85} = \langle 01234568 \rangle, \langle 01234579 \rangle, \langle 01345679 \rangle, \langle 01345789 \rangle, \langle 02345678 \rangle, \langle 02345689 \rangle$
- $x_{86} = \langle 01234578 \rangle, \langle 01345689 \rangle, \langle 02345679 \rangle$
- $x_{87} = \langle 01234678 \rangle, \langle 01234679 \rangle, \langle 01235689 \rangle, \langle 01235789 \rangle, \langle 13456789 \rangle, \langle 23456789 \rangle$
- $x_{88} = \langle 01234689 \rangle, \langle 01234679 \rangle, \langle 01346789 \rangle, \langle 02356789 \rangle, \langle 12345678 \rangle, \langle 12345789 \rangle$
- $x_{89} = \langle 01234789 \rangle, \langle 01235678 \rangle, \langle 01356789 \rangle, \langle 02346789 \rangle, \langle 12345679 \rangle, \langle 12345689 \rangle$
- $x_9 = \langle 01236789 \rangle, \langle 12346789 \rangle, \langle 12356789 \rangle$
- $x_{91} = \langle 01245678 \rangle, \langle 01245679 \rangle, \langle 01245689 \rangle, \langle 01245789 \rangle, \langle 01456789 \rangle, \langle 02456789 \rangle$
- $x_{92} = \langle 01246789 \rangle, \langle 01256789 \rangle, \langle 12456789 \rangle$

2 Definitions of odd-number correlation functions



Honeycomb lattice.

- $y_2 = \langle 012 \rangle, \langle 013 \rangle, \langle 023 \rangle, \langle 034 \rangle, \langle 035 \rangle, \langle 345 \rangle, \langle 346 \rangle, \langle 347 \rangle, \langle 367 \rangle, \langle 358 \rangle, \langle 359 \rangle, \langle 467 \rangle, \langle 589 \rangle$

- $y_3 = \langle 014 \rangle, \langle 016 \rangle, \langle 025 \rangle, \langle 029 \rangle, \langle 036 \rangle, \langle 039 \rangle, \langle 046 \rangle, \langle 059 \rangle, \langle 134 \rangle, \langle 146 \rangle, \langle 235 \rangle, \langle 259 \rangle, \langle 357 \rangle, \langle 457 \rangle, \langle 458 \rangle, \langle 478 \rangle, \langle 578 \rangle$
- $y_4 = \langle 015 \rangle, \langle 024 \rangle, \langle 037 \rangle, \langle 038 \rangle, \langle 047 \rangle, \langle 058 \rangle, \langle 135 \rangle, \langle 234 \rangle, \langle 349 \rangle, \langle 356 \rangle, \langle 456 \rangle, \langle 459 \rangle$
- $y_5 = \langle 019 \rangle, \langle 026 \rangle, \langle 147 \rangle, \langle 258 \rangle, \langle 468 \rangle, \langle 579 \rangle$
- $y_6 = \langle 017 \rangle, \langle 028 \rangle, \langle 159 \rangle, \langle 246 \rangle, \langle 479 \rangle, \langle 568 \rangle$
- $y_7 = \langle 018 \rangle, \langle 027 \rangle, \langle 138 \rangle, \langle 247 \rangle, \langle 469 \rangle, \langle 569 \rangle$
- $y_8 = \langle 045 \rangle, \langle 123 \rangle, \langle 136 \rangle, \langle 239 \rangle, \langle 367 \rangle, \langle 378 \rangle, \langle 389 \rangle$
- $y_9 = \langle 126 \rangle, \langle 129 \rangle, \langle 137 \rangle, \langle 139 \rangle, \langle 167 \rangle, \langle 236 \rangle, \langle 238 \rangle, \langle 289 \rangle, \langle 368 \rangle, \langle 379 \rangle, \langle 678 \rangle, \langle 789 \rangle$
- $y_{10} = \langle 138 \rangle, \langle 237 \rangle, \langle 369 \rangle$
- $y_{11} = \langle 048 \rangle, \langle 049 \rangle, \langle 056 \rangle, \langle 057 \rangle, \langle 067 \rangle, \langle 089 \rangle, \langle 124 \rangle, \langle 125 \rangle, \langle 145 \rangle, \langle 245 \rangle, \langle 489 \rangle, \langle 567 \rangle$
- $y_{12} = \langle 078 \rangle, \langle 156 \rangle, \langle 249 \rangle$
- $y_{13} = \langle 127 \rangle, \langle 128 \rangle, \langle 168 \rangle, \langle 169 \rangle, \langle 178 \rangle, \langle 189 \rangle, \langle 267 \rangle, \langle 269 \rangle, \langle 278 \rangle, \langle 279 \rangle, \langle 679 \rangle, \langle 689 \rangle$
- $y_{14} = \langle 069 \rangle, \langle 148 \rangle, \langle 257 \rangle$
- $y_{15} = \langle 068 \rangle, \langle 079 \rangle, \langle 149 \rangle, \langle 157 \rangle, \langle 248 \rangle, \langle 256 \rangle$
- $y_{16} = \langle 179 \rangle, \langle 268 \rangle$
- $y_{17} = \langle 01234 \rangle, \langle 01235 \rangle, \langle 01345 \rangle, \langle 02345 \rangle, \langle 03456 \rangle, \langle 03457 \rangle, \langle 03458 \rangle, \langle 03459 \rangle, \langle 03467 \rangle, \langle 03589 \rangle, \langle 34567 \rangle, \langle 34589 \rangle$
- $y_{18} = \langle 01236 \rangle, \langle 01239 \rangle, \langle 13467 \rangle, \langle 23589 \rangle, \langle 34678 \rangle, \langle 35789 \rangle$
- $y_{19} = \langle 01237 \rangle, \langle 01238 \rangle, \langle 13589 \rangle, \langle 23467 \rangle, \langle 34679 \rangle, \langle 35689 \rangle$
- $y_{20} = \langle 01346 \rangle, \langle 02359 \rangle, \langle 34578 \rangle$
- $y_{21} = \langle 01347 \rangle, \langle 01359 \rangle, \langle 02346 \rangle, \langle 02358 \rangle, \langle 34568 \rangle, \langle 34579 \rangle$
- $y_{22} = \langle 01356 \rangle, \langle 02349 \rangle, \langle 03478 \rangle, \langle 03578 \rangle, \langle 13456 \rangle, \langle 23459 \rangle$
- $y_{23} = \langle 01349 \rangle, \langle 02356 \rangle, \langle 03468 \rangle, \langle 03579 \rangle, \langle 13457 \rangle, \langle 23458 \rangle$
- $y_{24} = \langle 01358 \rangle, \langle 02347 \rangle, \langle 34569 \rangle$
- $y_{25} = \langle 01349 \rangle, \langle 02357 \rangle, \langle 03469 \rangle, \langle 03569 \rangle, \langle 13458 \rangle, \langle 23457 \rangle$
- $y_{26} = \langle 01357 \rangle, \langle 02348 \rangle, \langle 03479 \rangle, \langle 03568 \rangle, \langle 13459 \rangle, \langle 23456 \rangle$

- $y_{27} = \langle 01245 \rangle, \langle 03489 \rangle, \langle 03567 \rangle, \langle 04567 \rangle, \langle 04589 \rangle, \langle 12345 \rangle$
- $y_{28} = \langle 01246 \rangle, \langle 01259 \rangle, \langle 01467 \rangle, \langle 02589 \rangle, \langle 45678 \rangle, \langle 45789 \rangle$
- $y_{29} = \langle 01249 \rangle, \langle 01256 \rangle, \langle 04678 \rangle, \langle 05789 \rangle, \langle 14567 \rangle, \langle 24589 \rangle$
- $y_{30} = \langle 01247 \rangle, \langle 01258 \rangle, \langle 01589 \rangle, \langle 02467 \rangle, \langle 45679 \rangle, \langle 45689 \rangle$
- $y_{31} = \langle 01248 \rangle, \langle 01257 \rangle, \langle 04679 \rangle, \langle 05689 \rangle, \langle 14589 \rangle, \langle 24567 \rangle$
- $y_{32} = \langle 01367 \rangle, \langle 02679 \rangle, \langle 13468 \rangle, \langle 13478 \rangle, \langle 23578 \rangle, \langle 23579 \rangle$
- $y_{33} = \langle 01269 \rangle, \langle 14678 \rangle, \langle 25789 \rangle$
- $y_{34} = \langle 01367 \rangle, \langle 02389 \rangle, \langle 12346 \rangle, \langle 12359 \rangle, \langle 34789 \rangle, \langle 35678 \rangle$
- $y_{35} = \langle 01368 \rangle, \langle 02379 \rangle, \langle 13469 \rangle, \langle 13578 \rangle, \langle 23478 \rangle, \langle 23569 \rangle$
- $y_{36} = \langle 01379 \rangle, \langle 01468 \rangle, \langle 13479 \rangle, \langle 13579 \rangle, \langle 23468 \rangle, \langle 23568 \rangle$
- $y_{37} = \langle 01389 \rangle, \langle 02367 \rangle, \langle 12347 \rangle, \langle 12358 \rangle, \langle 34689 \rangle, \langle 35679 \rangle$
- $y_{38} = \langle 01267 \rangle, \langle 01289 \rangle, \langle 12467 \rangle, \langle 12589 \rangle, \langle 46789 \rangle, \langle 56789 \rangle$
- $y_{39} = \langle 01268 \rangle, \langle 01278 \rangle, \langle 14679 \rangle, \langle 15789 \rangle, \langle 24678 \rangle, \langle 25689 \rangle$
- $y_{40} = \langle 01378 \rangle, \langle 02378 \rangle, \langle 13568 \rangle, \langle 13569 \rangle, \langle 23469 \rangle, \langle 23479 \rangle$
- $y_{41} = \langle 01278 \rangle, \langle 15689 \rangle, \langle 24679 \rangle$
- $y_{42} = \langle 03679 \rangle, \langle 03689 \rangle, \langle 12348 \rangle, \langle 12357 \rangle, \langle 13489 \rangle, \langle 23567 \rangle$
- $y_{43} = \langle 03678 \rangle, \langle 03789 \rangle, \langle 12349 \rangle, \langle 12356 \rangle, \langle 13567 \rangle, \langle 23489 \rangle$
- $y_{44} = \langle 01456 \rangle, \langle 02459 \rangle, \langle 04578 \rangle$
- $y_{45} = \langle 01457 \rangle, \langle 01459 \rangle, \langle 02456 \rangle, \langle 02458 \rangle, \langle 04568 \rangle, \langle 04579 \rangle$
- $y_{46} = \langle 01458 \rangle, \langle 02457 \rangle, \langle 04569 \rangle$
- $y_{47} = \langle 01468 \rangle, \langle 01469 \rangle, \langle 02569 \rangle, \langle 02579 \rangle, \langle 14578 \rangle, \langle 24678 \rangle$
- $y_{48} = \langle 01478 \rangle, \langle 01569 \rangle, \langle 02469 \rangle, \langle 02578 \rangle, \langle 14568 \rangle, \langle 24579 \rangle$

- $y_{49} = \langle 01479 \rangle, \langle 01579 \rangle, \langle 02468 \rangle, \langle 02568 \rangle, \langle 14579 \rangle, \langle 24568 \rangle$
- $y_{50} = \langle 01568 \rangle, \langle 01578 \rangle, \langle 02478 \rangle, \langle 02479 \rangle, \langle 14569 \rangle, \langle 24569 \rangle$
- $y_{51} = \langle 01567 \rangle, \langle 02489 \rangle, \langle 04789 \rangle, \langle 05678 \rangle, \langle 12456 \rangle, \langle 12459 \rangle$
- $y_{52} = \langle 01489 \rangle, \langle 02567 \rangle, \langle 04689 \rangle, \langle 05679 \rangle, \langle 12457 \rangle, \langle 12458 \rangle$
- $y_{53} = \langle 01679 \rangle, \langle 02689 \rangle, \langle 12468 \rangle, \langle 12579 \rangle, \langle 14789 \rangle, \langle 25678 \rangle$
- $y_{54} = \langle 01689 \rangle, \langle 02679 \rangle, \langle 12478 \rangle, \langle 12578 \rangle, \langle 14689 \rangle, \langle 25679 \rangle$
- $y_{55} = \langle 01678 \rangle, \langle 02789 \rangle, \langle 12469 \rangle, \langle 12569 \rangle, \langle 15678 \rangle, \langle 24789 \rangle$
- $y_{56} = \langle 01789 \rangle, \langle 02678 \rangle, \langle 12479 \rangle, \langle 12568 \rangle, \langle 15679 \rangle, \langle 24689 \rangle$
- $y_{57} = \langle 12367 \rangle, \langle 12369 \rangle, \langle 12389 \rangle, \langle 13678 \rangle, \langle 23789 \rangle, \langle 36789 \rangle$
- $y_{58} = \langle 12368 \rangle, \langle 12379 \rangle, \langle 13679 \rangle, \langle 13789 \rangle, \langle 23678 \rangle, \langle 23689 \rangle$
- $y_{59} = \langle 12378 \rangle, \langle 13689 \rangle, \langle 23679 \rangle$
- $y_{60} = \langle 06789 \rangle, \langle 12489 \rangle, \langle 12567 \rangle$
- $y_{61} = \langle 12678 \rangle, \langle 12679 \rangle, \langle 12689 \rangle, \langle 12789 \rangle, \langle 16789 \rangle, \langle 26789 \rangle$
- $y_{62} = \langle 0123456 \rangle, \langle 0123459 \rangle, \langle 0134567 \rangle, \langle 0134589 \rangle, \langle 0345678 \rangle, \langle 0345789 \rangle$
- $y_{63} = \langle 0123457 \rangle, \langle 0123458 \rangle, \langle 0134589 \rangle, \langle 0234567 \rangle, \langle 0345679 \rangle, \langle 0345689 \rangle$
- $y_{64} = \langle 0134568 \rangle, \langle 0134569 \rangle, \langle 0134578 \rangle, \langle 0234569 \rangle, \langle 0234578 \rangle, \langle 0234579 \rangle$
- $y_{65} = \langle 0134579 \rangle, \langle 0234568 \rangle$
- $y_{66} = \langle 0123469 \rangle, \langle 0123569 \rangle, \langle 0134678 \rangle, \langle 0235789 \rangle, \langle 1345678 \rangle, \langle 2345789 \rangle$
- $y_{67} = \langle 0123467 \rangle, \langle 0123589 \rangle, \langle 3456789 \rangle$
- $y_{68} = \langle 0123468 \rangle, \langle 0123579 \rangle, \langle 0134679 \rangle, \langle 0235689 \rangle, \langle 1345789 \rangle, \langle 2345678 \rangle$
- $y_{69} = \langle 0123479 \rangle, \langle 0123568 \rangle, \langle 0135789 \rangle, \langle 0234678 \rangle, \langle 1345679 \rangle, \langle 2345689 \rangle$
- $y_{70} = \langle 0123489 \rangle, \langle 0123567 \rangle, \langle 0346789 \rangle, \langle 0356789 \rangle, \langle 1234567 \rangle, \langle 1234589 \rangle$

- $y_{71} = \langle 0123478 \rangle, \langle 0123578 \rangle, \langle 0135689 \rangle, \langle 0234679 \rangle, \langle 1345689 \rangle, \langle 2345679 \rangle$
- $y_{72} = \langle 0123679 \rangle, \langle 0123689 \rangle, \langle 1234678 \rangle, \langle 1235789 \rangle, \langle 1346789 \rangle, \langle 2356789 \rangle$
- $y_{73} = \langle 0123678 \rangle, \langle 0123789 \rangle, \langle 1234679 \rangle, \langle 1235689 \rangle, \langle 1356789 \rangle, \langle 2346789 \rangle$
- $y_{74} = \langle 0134689 \rangle, \langle 0235679 \rangle, \langle 2346789 \rangle$
- $y_{75} = \langle 0134789 \rangle, \langle 0135679 \rangle, \langle 0234689 \rangle, \langle 0235678 \rangle, \langle 1234568 \rangle, \langle 1234579 \rangle$
- $y_{76} = \langle 0135678 \rangle, \langle 0234789 \rangle, \langle 1234569 \rangle$
- $y_{77} = \langle 0124569 \rangle, \langle 0145678 \rangle, \langle 0245789 \rangle$
- $y_{78} = \langle 0124567 \rangle, \langle 0124589 \rangle, \langle 0456789 \rangle$
- $y_{79} = \langle 0124568 \rangle, \langle 0124579 \rangle, \langle 0145679 \rangle, \langle 0145789 \rangle, \langle 0245678 \rangle, \langle 0245689 \rangle$
- $y_8 = \langle 0124578 \rangle, \langle 0145689 \rangle, \langle 0245679 \rangle$
- $y_{81} = \langle 0124678 \rangle, \langle 0124679 \rangle, \langle 0125689 \rangle, \langle 0125789 \rangle, \langle 1456789 \rangle, \langle 2456789 \rangle$
- $y_{82} = \langle 0124689 \rangle, \langle 0125679 \rangle, \langle 0146789 \rangle, \langle 0256789 \rangle, \langle 1245678 \rangle, \langle 1245789 \rangle$
- $y_{83} = \langle 0124789 \rangle, \langle 0125678 \rangle, \langle 0156789 \rangle, \langle 0246789 \rangle, \langle 1245679 \rangle, \langle 1245689 \rangle$
- $y_{84} = \langle 0136789 \rangle, \langle 0236789 \rangle, \langle 1234689 \rangle, \langle 1234789 \rangle, \langle 1235678 \rangle, \langle 1235679 \rangle$
- $y_{85} = \langle 0126789 \rangle, \langle 1246789 \rangle, \langle 1256789 \rangle$
- $y_{86} = \langle 1236789 \rangle$
- $y_{87} = \langle 012345678 \rangle, \langle 012345679 \rangle, \langle 012345689 \rangle, \langle 012345789 \rangle, \langle 013456789 \rangle, \langle 023456789 \rangle$
- $y_{88} = \langle 012346789 \rangle, \langle 01235679 \rangle, \langle 123456789 \rangle$
- $y_{89} = \langle 012456789 \rangle$

3 Computer generated linear identities [0]

[0] is chosen as the origin site.

$$x_3 = A(x_2 + x_2 + x_5) + B^*x_{33} \quad (1)$$

$$x_4 = A(x_2 + x_5 + x_6) + B^*x_{34} \quad (2)$$

$$x_{36} = A(x_{33} + x_{37} + x_{38}) + B^*x_{81} \quad (3)$$

$$x_{40} = A(x_{34} + x_{38} + x_{41}) + B^*x_{81} \quad (4)$$

$$x_2 = A(x_1 + x_3 + x_4) + B^*x_{17} \quad (5)$$

$$x_{10} = A(x_7+x_{12}+x_{15})+B^*x_{51} \quad (6)$$

$$x_{23} = A(x_{12}+x_{27}+x_{30})+B^*x_{68} \quad (7)$$

$$x_{22} = A(x_{11}+x_{27}+x_{29})+B^*x_{69} \quad (8)$$

$$x_{25} = A(x_{14}+x_{29}+x_{32})+B^*x_{71} \quad (9)$$

$$x_{26} = A(x_{13}+x_{30}+x_{32})+B^*x_{72} \quad (10)$$

$$x_{39} = A(x_{17}+x_{36}+x_{40})+B^*x_{74} \quad (11)$$

$$x_{65} = A(x_{51}+x_{70}+x_{73})+B^*x_{90} \quad (12)$$

$$x_{17} = A(x_9+x_{22}+x_{26})+B^*x_{58} \quad (13)$$

$$x_{17} = A(x_8+x_{23}+x_{25})+B^*x_{59} \quad (14)$$

$$x_{60} = A(x_{46}+x_{62}+x_{64})+B^*x_{88} \quad (15)$$

$$x_{61} = A(x_{48}+x_{66}+x_{67})+B^*x_{89} \quad (16)$$

$$x_1 = A(x_2+x_2+_1)+B^*x_2 \quad (17)$$

$$x_{17} = A(x_2+x_{33}+x_{34})+B^*x_{37} \quad (18)$$

$$x_{18} = A(x_5+x_{34}+x_{35})+B^*x_{38} \quad (19)$$

$$x_{19} = A(x_2+x_{34}+x_{34})+B^*x_{41} \quad (20)$$

$$x_{16} = A(x_6+x_{34}+x_{34})+B^*x_{37} \quad (21)$$

$$x_{74} = A(x_{37}+x_{81}+x_{81})+B^*x_{82} \quad (22)$$

$$x_8 = A(x_1+x_{11}+x_{13})+B^*x_{28} \quad (23)$$

$$x_9 = A(x_1+x_{12}+x_{14})+B^*x_{31} \quad (24)$$

$$x_{10} = A(x_3+x_{16}+x_{18})+B^*x_{40} \quad (25)$$

$$x_{47} = A(x_{12}+x_{50}+x_{52})+B^*x_{70} \quad (26)$$

$$x_{10} = A(x_4+x_{18}+x_{19})+B^*x_{36} \quad (27)$$

$$x_{49} = A(x_{15}+x_{52}+x_{53})+B^*x_{73} \quad (28)$$

$$x_{59} = A(x_{31}+x_{69}+x_{72})+B^*x_{80} \quad (29)$$

$$x_{58} = A(x_{28}+x_{68}+x_{71})+B^*x_{80} \quad (30)$$

$$x_7 = A(x_2+x_{10}+x_{10})+B^*x_{39} \quad (31)$$

$$x_{42} = A(x_{10}+x_{47}+x_{49})+B^*x_{65} \quad (32)$$

$$x_{44} = A(x_{21}+x_{55}+x_{56})+B^*x_{78} \quad (33)$$

$$x_{43} = A(x_{20}+x_{54}+x_{54})+B^*x_{77} \quad (34)$$

$$x_{45} = A(x_{24}+x_{57}+x_{57})+B^*x_{79} \quad (35)$$

$$x_{84} = A(x_{63}+x_{87}+x_{87})+B^*x_{92} \quad (36)$$

$$x_{28} = A(x_2+x_{33}+x_{37})+B^*x_{37} \quad (37)$$

$$x_{29} = A(x_5 + x_{34} + x_{38}) + B^*x_{35} \quad (38)$$

$$x_{32} = A(x_2 + x_{34} + x_{41}) + B^*x_{34} \quad (39)$$

$$x_{27} = A(x_6 + x_{34} + x_{37}) + B^*x_{34} \quad (40)$$

$$x_{30} = A(x_5 + x_{35} + x_{38}) + B^*x_{34} \quad (41)$$

$$x_{31} = A(x_2 + x_{34} + x_{37}) + B^*x_{33} \quad (42)$$

$$x_{80} = A(x_{37} + x_{81} + x_{82}) + B^*x_{81} \quad (43)$$

$$x_{20} = A(x_1 + x_{11} + x_{28}) + B^*x_{13} \quad (44)$$

$$x_{21} = A(x_1 + x_{12} + x_{31}) + B^*x_{14} \quad (45)$$

$$x_{25} = A(x_3 + x_{16} + x_{40}) + B^*x_{18} \quad (46)$$

$$x_{62} = A(x_{12} + x_{50} + x_{70}) + B^*x_{52} \quad (47)$$

$$x_{23} = A(x_4 + x_{18} + x_{36}) + B^*x_{19} \quad (48)$$

$$x_{64} = A(x_{15} + x_{52} + x_{73}) + B^*x_{53} \quad (49)$$

$$x_{77} = A(x_{31} + x_{69} + x_{80}) + B^*x_{72} \quad (50)$$

$$x_{78} = A(x_{28} + x_{68} + x_{80}) + B^*x_{71} \quad (51)$$

$$x_{22} = A(x_4 + x_{19} + x_{36}) + B^*x_{18} \quad (52)$$

$$x_{26} = A(x_3 + x_{18} + x_{40}) + B^*x_{16} \quad (53)$$

$$x_{24} = A(x_1 + x_{14} + x_{31}) + B^*x_{12} \quad (54)$$

$$x_{79} = A(x_{28} + x_{71} + x_{80}) + B^*x_{68} \quad (55)$$

$$x_{21} = A(x_1 + x_{13} + x_{28}) + B^*x_{11} \quad (56)$$

$$x_{78} = A(x_{31} + x_{72} + x_{80}) + B^*x_{69} \quad (57)$$

$$x_{67} = A(x_{15} + x_{53} + x_{73}) + B^*x_{52} \quad (58)$$

$$x_{66} = A(x_{12} + x_{52} + x_{70}) + B^*x_{50} \quad (59)$$

$$x_{17} = A(x_2 + x_{10} + x_{39}) + B^*x_{10} \quad (60)$$

$$x_{60} = A(x_{10} + x_{47} + x_{65}) + B^*x_{49} \quad (61)$$

$$x_{75} = A(x_{21} + x_{55} + x_{78}) + B^*x_{56} \quad (62)$$

$$x_{75} = A(x_{20} + x_{54} + x_{77}) + B^*x_{54} \quad (63)$$

$$x_{75} = A(x_{24} + x_{57} + x_{79}) + B^*x_{57} \quad (64)$$

$$x_{76} = A(x_{21} + x_{56} + x_{78}) + B^*x_{55} \quad (65)$$

$$x_{61} = A(x_{10} + x_{49} + x_{65}) + B^*x_{47} \quad (66)$$

$$x_{91} = A(x_{63} + x_{87} + x_{92}) + B^*x_{87} \quad (67)$$

$$x_{11} = A(x_2 + x_2 + x_{33}) + B^*x_5 \quad (68)$$

$$x_{13} = A(x_2 + x_5 + x_{34}) + B^*x_6 \quad (69)$$

$$x_{14} = A(x_2 + x_6 + x_{34}) + B^*x_5 \quad (70)$$

$$x_{71} = A(x_{33} + x_{37} + x_{81}) + B^*x_{38} \quad (71)$$

$$x_{12} = A(x_2 + x_5 + x_{33}) + B^*x_2 \quad (72)$$

$$x_{68} = A(x_{34} + x_{38} + x_{81}) + B^*x_{41} \quad (73)$$

$$x_{69} = A(x_{34} + x_{41} + x_{81}) + B^*x_{38} \quad (74)$$

$$x_{72} = A(x_{33} + x_{38} + x_{81}) + B^*x_{37} \quad (75)$$

$$x_8 = A(x_1 + x_3 + x_{17}) + B^*x_4 \quad (76)$$

$$x_{46} = A(x_7 + x_{12} + x_{51}) + B^*x_{15} \quad (77)$$

$$x_{54} = A(x_{12} + x_{27} + x_{68}) + B^*x_{30} \quad (78)$$

$$x_{55} = A(x_{11} + x_{27} + x_{69}) + B^*x_{29} \quad (79)$$

$$x_{54} = A(x_{14} + x_{29} + x_{71}) + B^*x_{32} \quad (80)$$

$$x_{56} = A(x_{13} + x_{30} + x_{72}) + B^*x_{32} \quad (81)$$

$$x_{59} = A(x_{17} + x_{40} + x_{74}) + B^*x_{36} \quad (82)$$

$$x_{88} = A(x_{51} + x_{70} + x_{90}) + B^*x_{73} \quad (83)$$

$$x_9 = A(x_1 + x_4 + x_{17}) + B^*x_3 \quad (84)$$

$$x_{58} = A(x_{17} + x_{36} + x_{74}) + B^*x_{40} \quad (85)$$

$$x_{57} = A(x_{13} + x_{32} + x_{72}) + B^*x_{30} \quad (86)$$

$$x_{57} = A(x_{11} + x_{29} + x_{69}) + B^*x_{27} \quad (87)$$

$$x_{55} = A(x_{14} + x_{32} + x_{71}) + B^*x_{29} \quad (88)$$

$$x_{56} = A(x_{12} + x_{30} + x_{68}) + B^*x_{27} \quad (89)$$

$$x_{48} = A(x_7 + x_{15} + x_{51}) + B^*x_{12} \quad (90)$$

$$x_{89} = A(x_{51} + x_{73} + x_{90}) + B^*x_{70} \quad (91)$$

$$x_{43} = A(x_9 + x_{22} + x_{58}) + B^*x_{26} \quad (92)$$

$$x_{44} = A(x_8 + x_{23} + x_{59}) + B^*x_{25} \quad (93)$$

$$x_{45} = A(x_8 + x_{25} + x_{59}) + B^*x_{23} \quad (94)$$

$$x_{83} = A(x_{46} + x_{62} + x_{88}) + B^*x_{64} \quad (95)$$

$$x_{44} = A(x_9 + x_{26} + x_{58}) + B^*x_{22} \quad (96)$$

$$x_{85} = A(x_{48} + x_{66} + x_{89}) + B^*x_{67} \quad (97)$$

$$x_{86} = A(x_{48} + x_{67} + x_{89}) + B^*x_{66} \quad (98)$$

$$x_{85} = A(x_{46} + x_{64} + x_{88}) + B^*x_{62} \quad (99)$$

$$x_{15} = A(x_5 + x_6 + x_{34}) + B^*x_2 \quad (100)$$

$$x_{73} = A(x_{37} + x_{38} + x_{81}) + B^*x_{33} \quad (101)$$

$$x_{70} = A(x_{38} + x_{41} + x_{81}) + B^*x_{34} \quad (102)$$

$$x_{10} = A(x_3 + x_4 + x_{17}) + B^*x_1 \quad (103)$$

$$x_{63} = A(x_{12} + x_{15} + x_{51}) + B^*x_7 \quad (104)$$

$$x_{64} = A(x_{27} + x_{30} + x_{68}) + B^*x_{12} \quad (105)$$

$$x_{67} = A(x_{27} + x_{29} + x_{69}) + B^*x_{11} \quad (106)$$

$$x_{62} = A(x_{29} + x_{32} + x_{71}) + B^*x_{14} \quad (107)$$

$$x_{66} = A(x_{30} + x_{32} + x_{72}) + B^*x_{13} \quad (108)$$

$$x_{65} = A(x_{36} + x_{40} + x_{74}) + B^*x_{17} \quad (109)$$

$$x_{92} = A(x_{70} + x_{73} + x_{90}) + B^*x_{51} \quad (110)$$

$$x_{60} = A(x_{22} + x_{26} + x_{58}) + B^*x_9 \quad (111)$$

$$x_{61} = A(x_{23} + x_{25} + x_{59}) + B^*x_8 \quad (112)$$

$$x_{91} = A(x_{62} + x_{64} + x_{88}) + B^*x_{46} \quad (113)$$

$$x_{91} = A(x_{66} + x_{67} + x_{89}) + B^*x_{48} \quad (114)$$

$$x_7 = A(x_2 + x_2 + x_2) + B \quad (115)$$

$$x_{51} = A(x_{33} + x_{34} + x_{37}) + B^*x_2 \quad (116)$$

$$x_{52} = A(x_{34} + x_{35} + x_{38}) + B^*x_5 \quad (117)$$

$$x_{53} = A(x_{34} + x_{34} + x_{41}) + B^*x_2 \quad (118)$$

$$x_{50} = A(x_{34} + x_{34} + x_{37}) + B^*x_6 \quad (119)$$

$$x_{90} = A(x_{81} + x_{81} + x_{82}) + B^*x_{37} \quad (120)$$

$$x_{46} = A(x_{11} + x_{13} + x_{28}) + B^*x_1 \quad (121)$$

$$x_{48} = A(x_{12} + x_{14} + x_{31}) + B^*x_1 \quad (122)$$

$$x_{49} = A(x_{16} + x_{18} + x_{40}) + B^*x_3 \quad (123)$$

$$x_{87} = A(x_{50} + x_{52} + x_{70}) + B^*x_{12} \quad (124)$$

$$x_{47} = A(x_{18} + x_{19} + x_{36}) + B^*x_4 \quad (125)$$

$$x_{87} = A(x_{52} + x_{53} + x_{73}) + B^*x_{15} \quad (126)$$

$$x_{88} = A(x_{69} + x_{72} + x_{80}) + B^*x_{31} \quad (127)$$

$$x_{89} = A(x_{68} + x_{71} + x_{80}) + B^*x_{28} \quad (128)$$

$$x_{42} = A(x_{10} + x_{10} + x_{39}) + B^*x_2 \quad (129)$$

$$x_{84} = A(x_{47} + x_{49} + x_{65}) + B^*x_{10} \quad (130)$$

$$x_{85} = A(x_{55} + x_{56} + x_{78}) + B^*x_{21} \quad (131)$$

$$x_{86} = A(x_{54} + x_{54} + x_{77}) + B^*x_{20} \quad (132)$$

$$x_{83} = A(x_{57} + x_{57} + x_{79}) + B^*x_{24} \quad (133)$$

4 Computer generated linear identities [3]

[3] is chosen as the origin site.

$$x_2 = A(x_1 + x_3 + x_4) + B^*x_{17} \quad (134)$$

$$x_{33} = A(x_{12} + x_{28} + x_{36}) + B^*x_{60} \quad (135)$$

$$x_{34} = A(x_{27} + x_{29} + x_{32}) + B^*x_{75} \quad (136)$$

$$x_{35} = A(x_{30} + x_{30} + x_{30}) + B^*x_{76} \quad (137)$$

$$x_{34} = A(x_{15} + x_{31} + x_{40}) + B^*x_{61} \quad (138)$$

$$x_{81} = A(x_{70} + x_{73} + x_{80}) + B^*x_{91} \quad (139)$$

$$x_1 = A(x_2 + x_2 + x_1) + B^*x_2 \quad (140)$$

$$x_{19} = A(x_2 + x_{22} + x_{22}) + B^*x_{20} \quad (141)$$

$$x_{18} = A(x_5 + x_{23} + x_{26}) + B^*x_{21} \quad (142)$$

$$x_{17} = A(x_2 + x_{10} + x_{39}) + B^*x_{10} \quad (143)$$

$$x_{14} = A(x_6 + x_{24} + x_{25}) + B^*x_{25} \quad (144)$$

$$x_{13} = A(x_5 + x_{21} + x_{26}) + B^*x_{23} \quad (145)$$

$$x_{11} = A(x_2 + x_{20} + x_{22}) + B^*x_{22} \quad (146)$$

$$x_{71} = A(x_{37} + x_{62} + x_{79}) + B^*x_{62} \quad (147)$$

$$x_{12} = A(x_5 + x_{21} + x_{23}) + B^*x_{26} \quad (148)$$

$$x_{72} = A(x_{38} + x_{66} + x_{78}) + B^*x_{64} \quad (149)$$

$$x_7 = A(x_2 + x_{10} + x_{10}) + B^*x_{39} \quad (150)$$

$$x_{53} = A(x_{41} + x_{67} + x_{67}) + B^*x_{77} \quad (151)$$

$$x_{52} = A(x_{38} + x_{64} + x_{66}) + B^*x_{78} \quad (152)$$

$$x_{51} = A(x_{37} + x_{63} + x_{65}) + B^*x_{65} \quad (153)$$

$$x_{16} = A(x_6 + x_{25} + x_{25}) + B^*x_{24} \quad (154)$$

$$x_{74} = A(x_{37} + x_{65} + x_{65}) + B^*x_{63} \quad (155)$$

$$x_{69} = A(x_{41} + x_{67} + x_{77}) + B^*x_{67} \quad (156)$$

$$x_{68} = A(x_{38} + x_{64} + x_{78}) + B^*x_{66} \quad (157)$$

$$x_{50} = A(x_{37} + x_{62} + x_{62}) + B^*x_{79} \quad (158)$$

$$x_{90} = A(x_{82} + x_{92} + x_{92}) + B^*x_{92} \quad (159)$$

$$x_{10} = A(x_3 + x_4 + x_{17}) + B^*x_1 \quad (160)$$

$$x_9 = A(x_1 + x_4 + x_{17}) + B^*x_3 \quad (161)$$

$$x_8 = A(x_1 + x_3 + x_{17}) + B^*x_4 \quad (162)$$

$$x_{47} = A(x_{12} + x_{36} + x_{60}) + B^*x_{28} \quad (163)$$

$$x_{55} = A(x_{27} + x_{32} + x_{75}) + B^*x_{29} \quad (164)$$

$$x_{54} = A(x_{27} + x_{29} + x_{75}) + B^*x_{32} \quad (165)$$

$$x_{46} = A(x_{12} + x_{28} + x_{60}) + B^*x_{36} \quad (166)$$

$$x_{57} = A(x_{29} + x_{32} + x_{75}) + B^*x_{27} \quad (167)$$

$$x_{56} = A(x_{30} + x_{30} + x_{76}) + B^*x_{30} \quad (168)$$

$$x_{48} = A(x_{15} + x_{31} + x_{61}) + B^*x_{40} \quad (169)$$

$$x_{49} = A(x_{15} + x_{40} + x_{61}) + B^*x_{31} \quad (170)$$

$$x_{87} = A(x_{70} + x_{73} + x_{61}) + B^*x_{80} \quad (171)$$

$$x_{58} = A(x_{28} + x_{36} + x_{60}) + B^*x_{12} \quad (172)$$

$$x_{59} = A(x_{31} + x_{40} + x_{61}) + B^*x_{15} \quad (173)$$

$$x_{88} = A(x_{70} + x_{80} + x_{91}) + B^*x_{73} \quad (174)$$

$$x_{89} = A(x_{73} + x_{80} + x_{91}) + B^*x_{70} \quad (175)$$

$$x_7 = A(x_2 + x_2 + x_2) + B \quad (176)$$

$$x_{43} = A(x_{20} + x_{22} + x_{22}) + B^*x_2 \quad (177)$$

$$x_{44} = A(x_{21} + x_{23} + x_{26}) + B^*x_5 \quad (178)$$

$$x_{42} = A(x_{10} + x_{10} + x_{39}) + B^*x_2 \quad (179)$$

$$x_{45} = A(x_{24} + x_{25} + x_{25}) + B^*x_6 \quad (180)$$

$$x_{83} = A(x_{62} + x_{62} + x_{79}) + B^*x_{37} \quad (181)$$

$$x_{85} = A(x_{64} + x_{66} + x_{78}) + B^*x_{38} \quad (182)$$

$$x_{86} = A(x_{67} + x_{67} + x_{77}) + B^*x_{41} \quad (183)$$

$$x_{84} = A(x_{63} + x_{65} + x_{65}) + B^*x_{37} \quad (184)$$

5 A computer program for u_1 , u_6 and u_7

The linear identities for the even correlation functions generated in the previous section are not self-closed, i.e., the number of equations is always smaller than the number of correlation functions no matter where the hierarchy of equations is terminated. If, however, nearest- u_1 , second- u_6 , and third-neighbor u_7 pair correlation functions of the triangle lattice are found by the Pfaffian technique, in addition to the known nonlinear exact identities (J. Groeneveld, R. J. Boel, and P. W. Kasteleyn, *Physica A* **93**, 503 (1978)) are utilized, all the exact even correlation functions can be calculated, i.e., even the functional relations can be found in principle. In the following a scheme will be given by which all the remaining correlation functions are successively found.

The nonlinear identities utilized are:

$$\begin{aligned}x_8 &= x_1x_1 - x_2x_2 + x_1x_3, \\x_{11} &= x_1x_2 + x_1x_2 - x_2x_3, \\x_{20} &= x_1x_1 + x_2x_2 - x_3x_3.\end{aligned}$$

When the triangular reciprocal temperature AL is given, the following program will calculate the nearest-neighbor u_1 , second-neighbor u_6 and third-neighbor u_7 , correlation functions.