Finance: A Quantitative Introduction
Chapter 11
Hedging

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Hedging

Is to protect oneself from losing or failing by a counterbalancing action

In finance:

- offset price risk of one asset
- by taking opposite position in another asset

Was original meaning of the term 'hedge fund' (no longer)

We have extensively looked at hedging before:

- hedge portfolio in option pricing gives a perfect hedge
- (hedging: opposite position, pricing: same position)

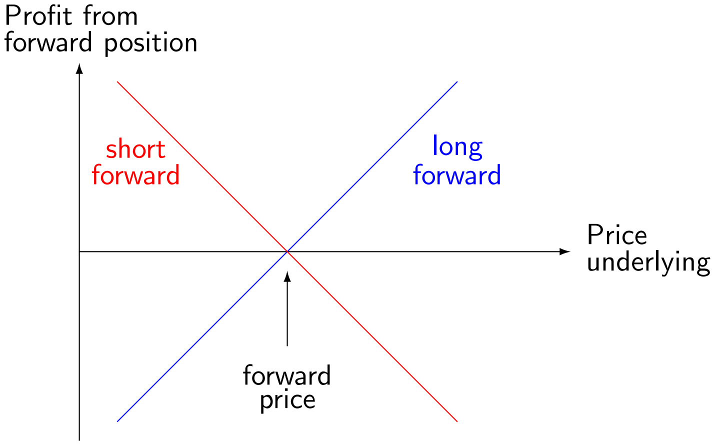
Some more characteristics of hedging:

- redistributes risk at market prices
- many forms of hedging:
  - buying insurance
  - matching (e.g. currency of costs & revenues)
  - delta hedging in options trading
- in perfect markets:
  - zero NPV deal
- in practice:
  - costs money in good times
  - makes money in bad times

Instruments (contracts) for hedging:

- **Spot contract** for immediate delivery and payment
  price is called **spot price**
- A long (short) **forward contract**
  - gives the obligation to buy (sell) something (a security or a commodity)
  - at a price determined now (the **forward price**)
  - on some future date.
  - At maturity (or settlement date)
    - long investor pays forward price to short investor
    - short investor delivers asset underlying the forward contract

- A **future contract** is a standardized forward contract
- price is called **future price**
- is traded in a special, organized way:
    - created by intermediary (exchange)
    - fixed quantities, maturity dates
    - intermediary guarantees all trades (no default risk)
    - intermediary creates a secondary market with traders and speculators to provide liquidity
    - must deposit a margin (e.g. 15%-20%) after opening a contract (levered position, very volatile)
    - contracts are marked to market every day: price changes paid - deposited
    - usually no delivery at maturity, contracts settled financially (by taking opposite position)



Profit diagram for a forward contract

- Futures are easier to use (standardization)
- Forwards are easier to price (no marking-to-market)

But: future and forward prices are the same if:

- interest rates are certain (non-stochastic)

In practice: differences ignored for short lived contracts

- but currency futures can have maturities of years

What is the difference between forwards and options?

- A forward contains no flexibility:
- you buy (or sell) 'for better or worse'.

Classic example of Farmer and Baker

It is March, spot price wheat is €70 per tonne, October futures price is €73.50

Farmer has sown wheat, expects to harvest 500 tonnes in October.

- At current price range of €70-75 tonne he can:
  - pay all his bills
  - make a living
- If October price is €80 he has a very good year
- If October price is €60 he cannot pay all his bills $\rightarrow$ ends in financial distress

Baker has made her production plans for Christmas cakes, needs 500 tonnes of wheat

- At current price range of €70-75 tonne she can:
  - sell whole production
  - pay all her bills
  - make a living
- If October price is €80 she cannot cover all costs$\rightarrow$ ends in financial distress
- If October price is €60 she has a very good year

Both farmer and baker can eliminate price risk through the futures market:

- Farmer sells 500 October future contracts of 1 tonne at €73.50 per tonne
  - he has a long position in wheat: expects to own 500 tonnes in October
  - offsets price risk by selling (shorting) same quantity in futures market
- Baker buys 500 October future contracts of 1 tonne at €73.50 per tonne
  - she has a short position in wheat: needs 500 tonnes in October
  - offsets price risk by buying same quantity (long position) in futures market

In October, Farmer:

- sells his wheat on the spot market
- closes his futures position with an offsetting trade:
  - sold futures in March
  - closes by buying spot in Oct.

Closing position Farmer

wheat price:		€ 60		€ 80
Sell spot:		30 000		40 000
Close future position:				
future (short)	73.50		73.50	
buy spot	-60.00		-80.00	
500×	13.50	6 750	-6.50	-3 250
Total position:		36 750		36 750

In October, Baker:

- buys her wheat on the spot market
- closes her futures position with an offsetting trade:
  - bought futures in March
  - closes by selling spot in Oct.

Closing position Baker

wheat price:		€ 60		€ 80
Buy spot:		-30 000		-40 000
Close future position:				
future (long)	-73.50		-73.50	
sell spot	60.00		80.00	
500×	-13.50	-6 750	6.50	3 250
Total position:		-36 750		-36 750

In practice, hedges are not perfect

- there is uncertainty about volume
- **basis risk** remains: risk that spot price and futures price do not move in line because:
  - location: North sea oil, Paris wheat
  - product: hedging gasoline with crude oil
  - time: maturity future $\neq$ underlying

Example: basis risk in crude oil

Most crude oil contracts have as underlying:

- West Texas Intermediate oil (WTI), traded on New York Mercantile Exchange
- Brent, traded on London's International Petroleum Exchange

Example (continued)

Both are light, sweet oil, containing little sulphur
Middle east accounts for one third of oil production, mostly with heavy, sour crude

- has different price development than WTI or Brent
- latter arguably ill-suited to hedge Middle East oil contracts

Plans are to establish new oil exchange in Dubai

- trading hours between London and Asia-Pacific
- Middle East sour future contracts
- also Middle East based jet fuel future contracts

(Petroleum Economist, Sept. 2006)

Reasons for hedging

Rationale for hedging looks obvious, is not

- Well diversified investors hold shares in Farmer and Baker, don't care who wins or loses, don't want to pay hedging costs for both
- Investor X thinks wheat price will go up $\rightarrow$ buys shares in Farmer; wheat prices goes up, no effect on shares because of hedging
- only obvious for not well diversified shareholders (owner-manager, founding family)

Economic reasons for hedging:

- Reduces expected costs of financial distress
  - allows more leverage
  - larger tax advantage
- Allows separation of performance and market prices
  - oil companies do well if oil price goes up
  - no reason to reward managers
- Allows firms to concentrate on risks they can influence
  - oil companies manage oil price risk
  - leave Forex risk to bank

- Reduces agency costs of underinvestment, risk seeking and asset substitution
  - removes peak and troughs in returns
  - makes risky investments less attractive
- Cost and information advantages of firms over investors
  - firms can hedge cheaper

These reasons are also found in empirical research

- but hedging covers only small part of risks

Pricing forward contracts

The general pricing procedure we have used so far is:

$$Forward = \frac{Exp[Price_T]}{(1+r)^T}$$

The forward price can be calculated

- using real probabilities and a risk adjusted discount rate
- or using risk neutral probabilities and the risk free rate.

Forward prices traditionally derived in a different way

- based on different way to get the underlying a maturity

Example:

The price of a share ZX Co is now €500.

At what price would you be willing to buy a forward contract on the share

- with a maturity of 3 months and
- a forward price of €510?

Assume that

- the stock pays no dividends
- the 3 months' risk free interest rate is 2,5%

Compare the following 2 strategies:

1. Buy the forward
2. Buy 1 share ZX Co and borrow PV(510)=510/1.025 = 497.6

Let the price of ZX Co in 3 months be S_T. Then:

Strategy	Costs today	Value at maturity
1	?	S_T - 510
2	share-PV(510) = 500-497.6	S_T - 510

- Any other forward price than 2.40 gives arbitrage opportunities
- Therefore, 2.40 has to be the forward price

It is usual to set forward prices such that contract requires no payment today
If we call that forward price F we get:

Strategy	Costs today	Value at maturity
1	0	S_T - F
2	share-PV(F) = 500-F/1.025	S_T - F

- From this we get a forward price of 512.5.
- Any other price gives arbitrage opportunities.

In the example we created a synthetic forward using the principle of a cash-and-carry market

In cash-and-carry markets you can :

- buy the asset by
  - borrowing risk free, collateralizing the asset
  - = cash
- store and insure the asset
  - = carry it forward
- until expiration of any derivative contract.

We can derive the value of a forward contract more formally using the cash-and-carry argumentation as follows:

Construct the portfolio

At maturity, this portfolio has as payoff:

Portfolio	Costs today	Value at maturity
1 long share	S_0	S_T
borrowing	$-S_0$	$-S_0(1+r_f)^T$
1 short forward	0	$F - S_T$
Total	0	$F - S_0(1+r_f)^T$

What can we say about the value of this portfolio at time T?

- Contains only values now
  - because S_T drops out
- The portfolio involves no risk.
- no net investment + no risk ⇒ value must be zero

More formally:

- To avoid arbitrage possibilities

  F **must** be such that portfolio value=0.

- Any other value for F gives arbitrage opportunities

This means we have:

$$F - S_0(1+r)^T = 0$$

so

$$F = S_0(1+r)^T$$

Known as the **cash-and-carry relationship** between spot and forward prices.

Why is this so much simpler than option pricing?

- With a forward, we are *certain* to end up with the share
  - can price by simpler ways of ending up with share
- With options we are *not* certain to end up with the share
  - need probability calculations
  - need to price risk

Look at Black and Scholes' option pricing formula:

$$O_{c,0} = S_0 N(d_1) - X e^{-rT} N(d_2)$$

If the option **must** be exercised, then

- option can have negative value
- probability terms $N(d_1)$ and $N(d_2)$ both become 1

$$O_{c,0} = S_0 - X e^{-rT}$$

If exercise price is chosen such that initial value is zero:

$$S_0 - X e^{-rT} = 0 \quad \Rightarrow \quad X = S_0 e^{rT}$$

the continuous time equivalent of cash-and-carry relation

Forward price is no prediction of price underlying at maturity easy to demonstrate with one period forward:

- has forward price of $F = S_0(1 + r_f)$
- value of underlying grows at higher rate, includes risk premium rp:

$$E(S_T) = S_0(1 + r_f + rp)$$

- only equal if risk premium is zero, if underlying has no systematic risk
- means cash-and-carry forward price contains no information on S_T beyond S_0
- if cash-and-carry strategy is not available, then forward price can convey information not reflected in S_0

Expected return on a forward also calculated from one period example
At maturity, holder of forward contract:

- pays the forward price F
- receives the underlying $E(S_T)$

Expected payoff is

$$E(S_T) - F = S_0(1 + r_f + rp) - S_0(1 + r_f) = S_0(rp)$$

- There is no initial investment
- holder of long forward does not earn time value of money r_f
- can expect to earn risk premium on underlying
- is the market price for bearing price risk of the underlying

Example:

- Price share ZX Co is 500
- 90 days forward costs 507.50
- Risk free interest rate 5% per year.

Is forward correctly priced? If not, how do we profit from it?

Start with c&c relation:

$$F = S_0(1+r)^T = 500 \times (1+.05)^{\frac{90}{365}} = 506.1$$

Forward is priced too high!

How do we profit from the mispricing?

Sell what is overvalued, buy what is undervalued.

$\Rightarrow$ Simply sell forward?

Wrong!

Exposes us to stock price movements, we lose money if stock price falls.

Have to make a portfolio with the stock to neutralize stock price risk:

- We have a short position in the forward
- Hedge short forward with an opposite = long position in the stock

Portfolio	Costs today	Value at maturity
1 share	500	S_T
borrowing	-500	-506.1
1 short forward	0	-(S_T-507.5)
Total	0	1.4

Dividends

Holders of forward contracts miss out on dividends stock pays before maturity
Same as with call option

$$S_0 = PV(S_T) + PV(dividends)$$

Only future value stock is delivered, not dividends.
Solution also same as call options: subtract PV(div) from stock price:

$$F = (S_0 - PV(dividends))(1 + r)^T$$

Example:

- Price ZX Co is 500
- ZX Co pays dividends of 10 after 45 days
- Risk free interest rate 5% per year.

What is price of 90 days forward ZX Co?

Start by calculating PV(div):

$$10/(1+.05)^{45/365} = 9.94$$

Continue with c&c relation:

$$F = (S_0 - PV(dividends))(1+r)^T$$

$$F = (500 - 9.94) \times (1+.05)^{\frac{90}{365}} = 496$$

Commodities

Holders of forward contracts on commodities miss out on something else too:

- cannot bake Christmas cakes of wheat forwards
- need real wheat for that

Hence, production companies store commodities

- face large costs if they run out of stock

Benefit of having the real thing is called: **convenience yield**

Convenience yield treated same way as dividends:

- subtract PV(cy) from spot price

Downside of storing commodities is **storage costs**
Holders of forward contracts on commodities miss out on these costs too

- increases the value of the forward.

Include in same way:

$$F = [S_0 + PV(sc) - PV(cy)](1 + r)^T$$

Called **cost-of-carry relationship**

Convenience yield changes nature of relationship

- cash-and-carry relationship is for financial investments
- are traded when profitable, when $F \neq S_0(1+r)^T$

Cost-of-carry relation is for commodities

- are held for production purposes
- not always traded when profitable
- Baker won't disrupt production of Christmas cakes to profit from (small) price difference spot-forward

Convenience yield expresses degree in which arbitrage relation does not work

We often use net convenience yield:

$$ncy = PV(cy) - PV(sc)$$

NCY can be positive or negative

- Typically positive for metals (low storage costs, e.g. for copper $<$\$1 tonne per month)
- varies with harvest cycle for agricultural commodities
- negative when e.g. fuel storage tanks are full for a cold winter that doesn't come

NCY is often 'backed out' (inferred) from spot price and discounted forward price

Example:

On LME (London Metal Exchange, www.lme.com) we found the following copper prices in US$/tonne:

- Cash buyer: 4109
- 3 month buyer 3943

The annualized 3 month interest rate is 2,5%

$$F = [S_0 - ncy](1 + r)^T$$

$$3943/(1 + .025)^{3/12} = 3918.7$$

$$3918.7 - 4109 = -190.3 = -ncy$$

So the net convenience yield is \$190.3 per tonne $\approx 4.5\%$

Forwards and futures are extensively used in:

- currencies (foreign currency forwards contracts)
- interest rates (forwards rate agreements or FRA)
- commodities (see list)

Markets are measured in $ trillions

For one important product the cost-of-carry relationship cannot be used. Can you guess which?
Electricity cannot be stored!

Commodities contract list EuroNext LIFFE April 2011:

- Cocoa Futures
- Cocoa Options
- Robusta Coffee Futures (No. 409)
- Robusta Coffee Options
- Corn Futures
- Corn Options
- Malting Barley Futures
- Malting Barley Options
- Rapeseed Futures
- Rapeseed Options
- Raw Sugar Futures
- Raw Sugar Options
- White Sugar Futures
- White Sugar Options
- Feed Wheat Futures
- Feed Wheat Options
- Milling Wheat Futures
- Milling Wheat Options
- SMP Futures (skimmed milk powder)

Setting up a hedge with basis risk

Recall: basis risk is risk spot price and forward price differ at maturity

- because of location, product, time, etc.
- hedging with different product is called **cross hedging**
    - e.g. jet fuel with sweet crude futures

Problem is:

- finding relation spot - forward
- means: finding optimal hedge ratio

How do we find the optimal hedge ratio?
Return of the hedged portfolio at maturity is:

$$R_{pT} = R_{ST} - \Delta R_{FT}$$

R_{pT} = return hedged portfolio
R_{ST} = return portfolio at spot prices
R_{FT} = return portfolio at future prices
Δ = hedge ratio

Two criteria are used to choose Δ :

- minimize variance of R_{pT} (used here)
- maximize utility of R_{pT}

Minimum variance Δ calculated as:

$$var(R_{pT}) = var(R_{ST} - \Delta R_{FT})$$

$var(R_{ST} - R_{FT})$ is variance of a 2-asset portfolio:

$$var(R_{pT}) = var\,(R_{ST}) + \Delta^2 var(R_{FT}) - 2\Delta covar(R_{ST}, R_{FT})$$

Δ that minimizes $var(R_{pT})$ found with first derivative:

$$\frac{\partial var(R_{pT})}{\partial \Delta} = 2\Delta var(R_{FT}) - 2covar(R_{ST}, R_{FT})$$

minimal when first derivative $= 0$

Solving

$$2\Delta var(R_{FT}) - 2covar(R_{ST}, R_{FT}) = 0$$

for Δ gives:

$$\Delta = \frac{covar(R_{ST}, R_{FT})}{var(R_{FT})}$$

We have seen this ratio of covariance / variance before:

- formula of β in portfolio theory and CAPM
- also formula slope-coefficient in OLS

That is how we find optimal hedge ratio:

- regress spot-price changes on futures-prices changes
- slope coefficient β is minimum variance hedge ratio

Farmer example again:

- sells wheat on local spot market
- hedges with forwards in NYSE Liffe Paris

Find relation local price - Paris, run regression:

$$\Delta P_{local} = \alpha + \beta(\Delta P_{Paris}) + \varepsilon$$

β is then hedge ratio. Let $\beta = 1.2$

- Farmer has to short $1.2 \times 500 = 600$ tonnes in Paris
- if March spot price was €70 both local/Paris
- with $\Delta P_{Paris} = +, -10 \Rightarrow \Delta P_{local} = +, -12$
- end positions become:

wheat price Euronext:		€ 60		€ 80
wheat price local:		€ 58		€ 82
Sell spot:	500×58	29 000	500×82	41 000
Close future position:				
future (short)	73.50		73.50	
buy spot	-60.00		-80.00	
600×	13.50	8 100	-6.50	-3 900
Total position:		37 100		37 100

Still a perfect hedge, why?
Local prices are exactly on regression line ($\varepsilon = 0$).

Suppose local market 'overshoots' Paris

- both upwards and downwards with 3
- means $\varepsilon = +, -3$
- local price development differs from futures market

If $\Delta P_{Paris} = -10$, from €70 to €60

- $\Delta P_{local} = 1.2 \times -10 - 3 = -15 \Rightarrow P_{local} = 55$

If $\Delta P_{Paris} = +10$, from €70 to €80

- $\Delta P_{local} = 1.2 \times 10 + 3 = 15 \Rightarrow P_{local} = 85$

End positions then become:

wheat price Euronext:		€ 60		€ 80
wheat price local:		€ 55		€ 85
Sell spot:	500×55	27 500	500×85	42 500
Close future position:				
future (short)	73.50		73.50	
buy spot	-60.00		-80.00	
600×	13.50	8 100	-6.50	-3 900
Total position:		35 600		38 600

- End position was 37 100 with perfect hedge
- becomes 35 600 or 38 600 with imperfect hedge

Impossible to avoid all losses with imperfect hedge

- closing position changes with unexpected price change
- so with €3 per tonne
- or €1500

Difficulty of cross hedging is estimating hedge ratio

Foreign exchange is gigantic market

- global *daily* turnover is larger than
- *annual* spending US government

Like options, foreign exchange has own terminology and traditions

- some currencies in 1s, others in 100s
- both €/\$ and \$/€ occur
- nicknames are used:
  - kiwi for New Zealand dollar
  - fiber for €/\$
  - chunnel for €/£

Very easy to get confused

Different ways to quote currencies

Country/ Currency	BBC Website	Financial Times Currency cross rates				
	£1 buys		GPB £	EUR €	USD $	JPY ¥
CA, Dollar	1.7058	CAD	0.5862	0.6680	0.9473	85.480
EU, Euro	1.1392	EUR	0.8775	N/A	1.4177	127.94
JP, Yen	145.82	JPY	0.0069	0.0078	0.0111	N/A
CH, Franc	1.6771	CHF	0.5959	0.6789	0.9627	86.890
GB, Pound	-	GBP	N/A	1.1392	1.6157	145.78
US, Dollar	1.6149	USD	0.6189	0.7051	N/A	90.210

CA = Canada, EU = European Union, JP = Japan, CH = Switzerland, GB = United Kingdom, US = United States

We assume euro is domestic currency
and use exchange rates like prices:

- rate is price of 1 foreign currency unit in domestic currency
  - like stock price is price of 1 stock
- so euro/dollar rate is €0.714/$
  - takes 0.714 euro to buy 1 dollar
- Newspapers do it other way around
  - EUR/USD=1.4
  - means value of €1=$1.4

Different conventions rich source of errors

Forward rate

Forward currency contracts traded and priced like other forwards

- but: also studied in international finance
- uses different approach and terminology

Show both approaches:

- first derive forward rate as in international finance
- then demonstrate equivalence with cash-and-carry relation

Start with numerical example

European company has to pay \$100 000 in 1 year

- spot exchange rate is €0.714/\$ or $S_{€\$}=0.714$
- euro interest rate is 8%
- dollar interest rate is 4%

Company can hedge exchange rate risk in 2 ways:

1. Buy PV(\$100 000) spot, place money in US bank to earn dollar interest rate
2. Open forward contract to buy \$100 000 in 1 year at forward rate $F_{€\$}$

Using first strategy:

- buy $\$100\,000/1.04 = \$96\,154$ in spot market
- costs $\$96\,154 \times 0.714 = €68\,654$ today
- gives $\$100\,000$ in 1 year

Using second strategy

- costs nothing today
  forward rate set such that value today is zero
- pay $100\,000 \times$forward rate $F_{€\$}$ in 1 year
- also gives $\$100\,000$ in 1 year

Same payoff 1 year $\Rightarrow$ same value today
$€68\,654 = PV(100\,000 \times F_{€\$})$
$€68\,654 = (100\,000 \times F_{€\$})/1.08$
$F_{€\$} = 0.741$

Any other rate gives arbitrage possibility

- called *covered interest arbitrage*
- exploits difference interest rates/forward-spot rate

Suppose forward rate is too high, say $F_{€\$} = 0.775$
As always, arbitrage means

- buy what is cheap
  - here: dollars in spot market
- sell what is expensive
  - here: dollars in forward market

1	borrow amount for 1 year, e.g.	€100 000
2	exchange spot for \$: $100\,000/0.714 =$	\$140 056
3	invest risk free, after 1 year: $\$140\,056 \times 1.04 =$	\$145 658
4	sell forward for euros = after 1 yr $145\,658 \times 0.775 =$	€112 885
5	use euros to pay back loan of: $100\,000 \times 1.08 =$	€108 000
6	difference is arbitrage profit: €112 885 - €108 000 =	€4 885
7	repeat 1-6 until arbitrage opportunity has disappeared	

Why is this covered?

long position dollars hedged by short forwards

How can you take out arbitrage profit today?

borrow PV(€112 885)=€112 885/1.08 = €104 523

proceed with step 2

Covered interest arbitrage:

- borrows money
- to buy an asset ($)
- and store it
- to replicate forward contract

Same as cash-and-carry relation
To demonstrate, write cash-and-carry relation:

$$F = [S_0 - PV(dividends)](1 + r_f)^T$$

in terms of currency forward ($F_{€\$}, S_{€\$}$ is forward, spot ex. rate):

$$F_{€\$} = [S_{€\$} - PV(interest)](1 + r_{€})$$

$$F_{€\$} = [S_{€\$} - PV(interest)](1 + r_{€})$$

interest is: \$amount×\$interest rate, i.e. $S_{€\$} \times r_{\$}$
PV(interest) is: interest discounted at 1+\$interest rate

$$F_{€\$} = \left[S_{€\$} - \frac{r_{\$} S_{€\$}}{(1 + r_{\$})}\right](1 + r_{€}) = \left[\frac{S_{€\$}}{(1 + r_{\$})}\right](1 + r_{€})$$

Filling in numbers reproduces forward rate:

$$F_{€\$} = \left[\frac{S_{€\$}}{(1 + r_{\$})}\right](1 + r_{€}) = \left[\frac{0.714}{1.04}\right](1.08) = 0.741$$

Cash-and-carry relation:

$$F_{€\$} = \left[\frac{S_{€\$}}{(1 + r_{\$})} \right] (1 + r_{€})$$

can be written as:

$$\frac{F_{€\$}}{S_{€\$}} = \frac{1 + r_{€}}{1 + r_{\$}}$$

Known in international finance as *interest rate parity*

- links Forex markets to international money markets
- in equilibrium, parity relation holds
- is 'enforced' by covered interest arbitrage
- holds very well in practice

Second parity relation is *Fisher effect*:

$$\frac{1 + r_{€}}{1 + r_{\$}} = \frac{E(1 + i_{€})}{E(1 + i_{\$})}$$

- *i* stands for inflation rate
- links interest rates to expected inflation
- says *real* interest rates, corrected for inflation, are the same
- again: in equilibrium parity relation holds
- Fisher effect contains expectations, difficult to verify empirically
- on an ex post basis, holds well for short term debt
- weaker for long term debt

Parity relations imply:

- no benefit in investing in country with high interest rate
- nonsense to borrow in country with low interest rate
- still advertised by 'financial advisers'

Intuition should be clear:

- what you win on the interest rate
- equals your loss on the exchange rate
- and the other way around

Such advertisements are now illegal in many countries

Example: domestic vs foreign investment

American investor has \$100 to invest for a year. She can:

- put it in American bank
    - earn dollar interest rate of 4%
    - gives \$104 after a year
- Alternative: invest in Europe
    - exchange dollars for $100 \times 0.714 =$ €71.4
    - put them in European bank
    - earn double interest rate of 8%
    - gives $71.4 \times 1.08 =$ €77.112 after a year
- But: selling these euros forward for dollars gives:
    - $77.112/0.741 = \$104$
    - same as the dollar investment
- *Has to be* the same, or cov. int. arbitrage is money machine

Interest rate parity can be used to construct synthetic forward

- interest rates are quoted for longer periods than currency forwards
- multi-period notation for interest rate parity is:

$$\frac{F_{€\$}^{T}}{S_{€\$}^{T}} = \frac{\left(1 + r_{€}^{T}\right)^{T}}{\left(1 + r_{\$}^{T}\right)^{T}}$$

- interest rates time superscripted
- allows different rates for different maturities
- currency forwards available for up to ten years
- but longer maturities can be required for e.g. international investment decisions

