Transits of Venus: New Views of the Solar System and Galaxy
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Transits of Venus and Mercury: Patterns of occurrence, and near-resonance phenomena

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Abstract. Transits of Venus occur in pairs 8 yr apart, the pairs separated by intervals of either 112 or 130 yr, because of the pattern of approximate orbital resonances. Transits of Mercury show a different pattern, partly because of the more eccentric orbit, and also because the structure of the approximate resonances is different. The near-resonance in Venus' axial rotation strongly suggests a tidal link with the Earth.

1. Introduction

Here are the dates of transits of Venus, over a period of about a millennium, all given in the Gregorian calendar:

1631 December 7
1639 December 4
1761 June 6
1769 June 3
1874 December 9
1882 December 6
2004 June 8
2012 June 5
2117 December 11
2125 December 8
2247 June 11
2255 June 9
2360 December 12
2368 December 10
2490 June 12
2498 June 9
2603 December 15

All these transits are in pairs 8 yr apart, the pairs separated by either 112 or 130 yr. A transit can only occur when an inferior conjunction of Venus with the Earth happens when Venus is very nearly in the plane of the Earth's orbit round the Sun, that is, when it is very near to one of the two nodes of its orbit. This list of dates shows clearly that one of these node crossings occurs in December (in fact at the ascending node), and the other in June. To understand more about these patterns, it is necessary to examine more closely the arithmetic of the orbits. (See the website prepared by H.M. Nautical Almanac Office (HMNAO 2004) for a longer list of transit dates, detailed predictions, and maps showing regions of visibility.)

Here is a sequence of dates of transits of Mercury, over about two centuries:

1802 November 8
1815 November 11
1822 November 4
1832 May 5
1835 November 7
1845 May 8
1848 November 9
1861 November 11
1868 November 4
1878 May 6
1881 November 7
1891 May 9
1894 November 10
1907 November 12
1914 November 6
1924 May 7
1927 November 8
1940 November 12
1953 November 13
1957 May 5
1960 November 6
1970 May 9
1973 November 9
1986 November 12
1999 November 14
2003 May 7

This shows a more intricate pattern, though it is plain that the corresponding node crossings are in November (in fact at the ascending node) and in May, and that those in November are more frequent.

2. Transit limit for Venus

It is necessary to be more precise about how near to a node the planet must be at conjunction for a transit actually to be seen from the Earth. We may define the "transit limit" (by analogy with "eclipse limit" for eclipses of the Sun or Moon) as the greatest distance of the planet in orbital longitude from the node, at conjunction, which is consistent with a transit being observable. Let us use the method given by Chauvenet (1863) for calculating the eclipse limits, as appropriately amended for transits. We need to note that Venus' orbit is inclined to the ecliptic at $3^{\circ}\!.395$, that the mean angular motion of Venus in its orbit is about $1.6255\times$ that of the Earth, that the semi-diameter of the Sun, as seen from the Earth, is about $16'$, and that, since the major semi-axis of Venus' orbit is about $0.723\times$ that of the Earth, the apparent latitude of Venus as seen from the Earth, at conjunction, is about $2.614\times$ that seen from the Sun. (The orbits are sufficiently nearly circular for us to be able to neglect the variation in this to the precision we seek.) From these, the transit limit is found to be about $1^{\circ}\!.743$.

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3. Approximate resonances, and the occurrence of transits of Venus

Let us now consider what governs the time needed to elapse, after one transit, before the circumstances occur leading to the next. For the next transit at the same node, we need the two planets to return approximately to the same direction from the Sun. This requires an approximate commensurability of orbital period, so let us seek these. The mean angular motion of Venus in its orbit referred to the equinox of date is (Seidelmann, Doggett & Deluccia 1974) $1\overset{\circ}{.}602\,130\,477\,119\ldots\,\mathrm{d}^{-1}$, and that of the Earth is $0\overset{\circ}{.}985\,609\,114\,533\ldots\,\mathrm{d}^{-1}$. Hence the ratio of the mean motion of Venus to that of the Earth is, also giving it in continued fraction form,

$$1.625\,523\,195\,144\ldots = 1 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{29+}\,\frac{1}{2+}\,\frac{1}{16+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{2+}\ldots \tag{3.1}$$

Approximations ("convergents") to this ratio as rational fractions may be found by taking a finite number of terms of this continued fraction. Successive convergents are alternately greater than, and less than, the true value.

The third convergent is

$$1 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1} = \frac{5}{3} = 1.\dot{6}. \tag{3.2}$$

The fourth convergent is

$$1 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{2} = \frac{13}{8} = 1.625. \tag{3.3}$$

This shows that 8 yr is close to thirteen Venus orbital periods and five synodic periods, so that, after five conjunctions, Venus and the Earth will have moved almost to the same radial line from the Sun. The conjunction line will in fact have moved through an integral number of revolutions, less $2\overset{\circ}{.}408866735\ldots$. This is less than twice the "transit limit", so that two successive such conjunctions can occur, each within the transit limit of the node. But, at the corresponding conjunction a further 8 yr later, the conjunction line will have moved out of the transit limit, so no transit will occur. Thus there will usually be a pair of transits, at the same node, 8 yr apart. (During 8 yr, the node of Venus' orbit moves backwards through about $0\overset{\circ}{.}002\,338\ldots$, which does not affect this conclusion. In fact, in 1388 there was a "near miss" at the end of a sequence of transits, the conjunction line having been moving gradually towards the edge of the transit region during the sequence, to be followed by the beginning of a new sequence 8 yr later.)

This near resonance may also be expressed by saying that linear combination $8L_V - 13L_E$ (where L_V is the mean longitude of Venus, and L_E is the mean longitude of the Earth), moves backwards through $0\overset{\circ}{.}004125328\ldots\ \mathrm{d}^{-1}$, and so completes a revolution in 238.9258 ... yr. Thus, in the expressions for the perturbations of the orbit of Venus by the Earth, and the perturbations of the Earth by Venus, terms which involve sines or cosines of this combination of mean longitudes will have a period of about 238.9258 yr, and will be of enlarged amplitude because of the small denominator arising on integration with respect to time of the expressions for the time rate of change of the perturbations.

Airy (1828, 1832) predicted the occurrence of such terms from this cause. He showed that this term in the longitude of the Earth round the Sun (and so in the apparent motion of the Sun as seen from the Earth) had an amplitude of $2\overset{\prime\prime}{.}6$, and noted that the period of these terms is a larger multiple of the orbital periods of the planets concerned than for any other terms needing to be taken into account in the theory of perturbations in the solar system. (The "great inequalities" in the perturbations in the motions of Jupiter

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and Saturn, arising from the near 5:2 commensurability between the periods of these two planets, are of course of longer period – about 890 yr – but considerably less than 240× the periods of these planets.)

The fifth convergent is

$$1 + \frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{29} = \frac{382}{235} = 1.6255319\ldots. \tag{3.4}$$

This shows that 235 yr is close to 382 Venus orbital periods, and 147 synodic periods, so that, after 147 conjunctions, Venus and the Earth will also be close to the same radial line through the Sun. The conjunction line will in that time have moved through an integral number of revolutions, plus $1\overset{\circ}{.}179\,318\,08\ldots$. This is less than the "transit limit", so, 235 yr after the second of a pair of transits 8 yr apart, the conjunction line will have moved forwards by this angle, so the conjunction will again be within the transit limit, and another transit can be expected. (During 235 yr, the node of Venus' orbit moves backwards through about $0\overset{\circ}{.}068\,671\,56\ldots$, which, again, does not affect the conclusion.) A pair of transits at the other node, of course also 8 yr apart, will usually occur very nearly midway between successive pairs of transits at the first node, since the two nodes are of course diametrically opposed, and the orbits are very nearly circular.

Let us now set out the dates of transits of Venus, putting those at the two nodes separately, and noting the interval in years between successive transits at the same node. First, those in December:

1631 December	7	8 yr
1639 December	4	235 yr
1874 December	9	8 yr
1882 December	6	235 yr
2117 December	11	8 yr
2125 December	8	235 yr
2360 December	12	8 yr
2368 December	10	235 yr
2603 December	15	

now those in June:

1761 June	6	8 yr
1769 June	3	235 yr
2004 June	8	8 yr
2012 June	5	235 yr
2247 June	11	8 yr
2255 June	9	235 yr
2490 June	12	8 yr
2498 June	9	

showing, in each case, the expected alternation between 8-yr and 235-yr intervals.

4. Transit limits for Mercury

We turn now to the case of Mercury, and begin by calculating the appropriate limits. We first note that the eccentricity of Mercury's orbit, 0.20563, is large enough so that we must take into account the difference between its distances from the Sun at different places in the orbit. The ecliptic longitude of the ascending node is $48\overset{\circ}{.}378\ldots$, so that, at a conjunction here, the ecliptic longitude of the Sun, seen from the Earth, will be about $228\overset{\circ}{.}378$, so such conjunctions will occur in November. Since the ecliptic longitude of the

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apse of Mercury is about 77$^\circ\!.$518, the true anomaly will then be 330$^\circ\!.$860, from which we find, using the fact that the major semi-axis of Mercury's orbit is about 0.3871 a.u., that the radial distance from the Sun will be about 0.3143 a.u., at which distance the apparent latitude of Mercury, at inferior conjunction, seen from the Earth, will be about 0.4583$\times$ that seen from the Sun. Now Mercury's orbit is inclined to the ecliptic at about 7$^\circ\!.$005, and we find from all this that the transit limit; that is, the maximum distance from the node at conjunction consistent with a transit, is about 4$^\circ\!.$832.

The ecliptic longitude of the descending node is about 228$^\circ\!.$378, so that transits at this node will occur in May. The true anomaly is then 150$^\circ\!.$860, from which we find that the radial distance is about 0.4519 a.u., so that the apparent latitude of Mercury, seen from the Earth, at inferior conjunction, will be about 0.8245$\times$ that seen from the Sun. Then we deduce that the transit limit at this distance is about 2$^\circ\!.$686.

The difference in size of the two transit limits leads us to expect more transits of Mercury in November than in May, as we do of course in fact observe.

5. Approximate resonances for Mercury

The mean angular orbital motion of Mercury is 4$^\circ\!.$092 338 785 462 . . . d^{-1}. The ratio of this to that of the Earth, also given as a continued fraction, is

$$4.152091053010\ldots = 4 + \frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{92+}\cdots. \tag{5.1}$$

The second convergent of this is

$$4 + \frac{1}{6+}\frac{1}{1} = \frac{29}{7} = 4.\dot{1}4285\dot{7}, \tag{5.2}$$

which shows that 7 yr is close to 29 Mercury orbital periods, and 22 synodic periods, so that, after 22 conjunctions, Mercury and the Earth will have moved almost to the same radial line from the Sun. The conjunction line will have moved through an integral number of revolutions, less 7$^\circ\!.$382227605 . . . . This is more than twice the transit limit for May transits, so that there will not be transits of Mercury in May separated by 7 yr, but this angle is not far from the transit limit for November transits, so there will be some pairs of transits in November 7 yr apart.

The third convergent is

$$4 + \frac{1}{6+}\frac{1}{1+}\frac{1}{1} = \frac{54}{13} = 4.1538461\ldots, \tag{5.3}$$

showing that 13 yr is close to 54 Mercury orbital periods, and 41 synodic periods, so that, after 41 conjunctions, Mercury and the Earth will again have moved almost to the same radial line from the Sun. The conjunction line will have moved through an integral number of revolutions, plus 2$^\circ\!.$605 855 . . . . This is less than twice either transit limit, so that there will normally be pairs of transits, at either node, 13 yr apart.

The fourth convergent is

$$4 + \frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2} = \frac{137}{33} = 4.\dot{1}\dot{5}, \tag{5.4}$$

showing that 33 yr is close to 137 Mercury orbital periods, and 104 synodic periods, so that, after 104 conjunctions, Mercury and the Earth will have again moved almost to the same radial line from the Sun. The conjunction line will have moved through an integral

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number of revolutions, less $2^\circ\!.170\,530\ldots$. So we will expect there to be pairs of transits, at either node, 33 yr apart.

The fifth convergent is

$$4+\frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1}=\frac{191}{46}=4.1521739\ldots, \tag{5.5}$$

showing that 46 yr is close to 191 Mercury orbital periods, and 145 synodic periods, so that, after 145 conjunctions, Mercury and the Earth will have again moved almost to the same radial line from the Sun. The conjunction line will have moved through an integral number of revolutions, plus $0^\circ\!.43532\ldots$. So we will usually expect the repetition of a transit, at either node, after 46 yr. (During 7 yr, the node of Mercury's orbit moves backwards through about $0^\circ\!.008791$, which does not affect any of these conclusions.)

Let us now set out the dates of transits of Mercury, putting those at the two nodes separately, and noting the interval in years between successive transits. First, those in November:

1802 November 8	13 yr
1815 November 11	7 yr
1822 November 4	13 yr
1835 November 7	13 yr
1848 November 9	13 yr
1861 November 11	7 yr
1868 November 4	13 yr
1881 November 7	13 yr
1894 November 10	13 yr
1907 November 12	7 yr
1914 November 6	13 yr
1927 November 8	13 yr
1940 November 12	13 yr
1953 November 13	7 yr
1960 November 6	13 yr
1973 November 9	13 yr
1986 November 12	13 yr
1999 November 14	

We notice that the pattern of occurrence of the 7-yr and 13-yr intervals leads to very frequent combinations giving both 33-yr and 46-yr intervals.

Then, the transits in May:

1832 May 5	13 yr
1845 May 8	33 yr
1878 May 6	13 yr
1891 May 9	33 yr
1924 May 7	33 yr
1957 May 5	13 yr
1970 May 9	33 yr
2003 May 7	

This time we notice that the pattern of occurrence of the 13-yr and 33-yr intervals leads to very frequent combinations giving 46-yr intervals.

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6. Near-resonance in the rotation period of Venus

Successive determinations of the rotation period of Venus using radar soundings of formations on the surface, with increasing precision, have converged towards a period of 243.00 d, the rotation being retrograde, that is, in the opposite sense to most rotations and orbital revolutions in the Solar System (see, e.g., Gold & Soter 1969). Now, a period of 243.16 d would correspond to the resonant situation in which Venus presented the same face to the Earth at each successive conjunction. It is very unlikely that this is a pure coincidence, and the slight difference in the periods probably indicates the libration of the system about the exact "locked-in" situation. But such a situation would imply that Venus' departure from exact axial symmetry is great enough for a gravitational torque due to the Earth's attraction to have captured Venus' rotation in this way. Such a torque would of course be very small (though much greater at conjunction than at other times), so this situation is very surprising. There is an extensive literature considering possible ways in which this could have come about.

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H.M. Nautical Almanac Office 2004 *http://www.nao.rl.ac.uk/nao/transit*

Discussion

Don Kurtz: How frequent are transits of Earth as seen from Mars?

Jim Message: I didn't finish that calculation, although I started it!

Don Kurtz: Is there any situation where the orbital periods are such that you would never see a transit of an interior planet from some exterior planet?

Jim Message: It is difficult to see how that could happen, but in principle, it might. [Added in proof:] This would happen if there were a resonance, and a critical angle associated with it were fixed, or oscillating about, an appropriate value. For example, if there were a near 3:1 orbital resonance, and the angle 3× longitude of outer planet − longitude of inner planet − 2× node longitude of inner planet = 180°, then conjunctions could only occur with the inner planet at 90° from the node, which would certainly prevent a transit from happening (unless the orbital inclination were very small, when there would be a transit every time!) If this configuration were in fact dynamically possible, heuristic considerations suggest that it would be a stable one.

Nick Kollerstrom: Between the two Venus transits I gather there are occultations of Venus by the Sun at four year intervals – like 2000, 2008, 2016 – so the sequence is something like 5 occultations, transit, occultations. Are these a bit like lunar eclipses turning up either side of the solar eclipse? Are they more frequent than the transits?

Jim Message: The transit limit will be different, because the angles seen from the Earth will be different as they relate to the orbital parameters. You'll have to do a fresh calculation as to when you would get such an event at the superior conjunction. I haven't done that. It would be very difficult to observe an occultation of Venus by the Sun, so that would be a challenge to the instrumentalist, wouldn't it?

Steven Dick: Just to comment on Don's question. Transits of Earth as seen from Mars: about every 100 years. There was one in 1984, and that's when Arthur C. Clarke wrote his famous short story called "Transit of Earth, " and I know that there is another one in 2084.

Jim Message taking notes at Stonyhurst Observatory

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Mike Marsh, Hannah Worters and Gordon Bromage

Coffee-time discussion: Lena Pitjeva, Paul Marston, John Southworth, Mark Northeast, Kate Bird, Jacqueline and Simon Mitton

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