A Method of Dividing the Circumference of a Circle into 360 equal parts.

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This Graduation of the Circumference of a Circle is effected by the aid of an instrument called a trisector which I contrived with the view of trisecting an angle by its assistance, but subsequently perceived that it could help to divide an angle into 5 equal angles, and recently discovered that it could contribute towards dividing the circumference of a circle into 360 equal degrees or arcs.

That instrument is outlined in the diagram (Fig. 32). It consists, as was formerly explained to this Society, of two flat or nearly flat pieces BEFGD and AHJ of peculiar forms; and may be made of metal or other material. Those two pieces are conjoined, as the blades of a pair of scissors are, by a small cylindrical pin inserted in a small cylindrical hole at C in each of them, which it fits, and round which they can be moved in either direction. A similar hole is made through the piece BEFGD at B. The centres of those holes at B and C, and also the points E and D are all in the same straight line BECD. An arm EFG projects from one of the sides of this piece. The edge EF of that arm is straight, and is perpendicular to the line BC which joins the centres of the holes at B and C. It also bisects the line BC in the point E. The line BE is therefore equal to the line EC.

The point A of the other piece AHJ, the centre of the hole at C, and its edge IJ, are all in the straight line ACIJ. The line AC, drawn from the point A to the centre at C, is exactly equal to the line BC which connects the centres at B and C. The triangle ACB is therefore isosceles.

The circumference of the circle, which it is proposed to graduate, may in the first instance be divided, as is easily done, into 3 equal arcs of 120°. Let MS be such an arc of 120°, B the centre of its circle, and BM and BS radii of the circle drawn to the extremities M and S of the arc. Also, let KL be a perpendicular to the radius BS drawn from the point K in it, K being at the same distance

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from the centre at B as the point E is from the centre of the hole at B. The lines EB and KB are therefore equal.

In employing the instrument, with a view to the graduation of the circumference of the circle, a sharp cylindrical pin which fits, but not tightly, the hole at B, is inserted through that hole into the centre B of the circle, and at right angles to its plane. The point A of the piece AHJ is then moved along the radius BM until its edge IJ meets the edge EF of the arm EFG at a point, which may be termed O, on the perpendicular KL. Next, through O draw the radius BOQ, and bisect the angle CBO or NBQ with the radius BP and the angle OBS or QBS with the radius BR; and produce the line BECD till it meets the arc MS at N.

Now, as by construction, the sides BE and CE of the right-angled triangles BOE and COE are equal, and as EO is a corresponding side of both those triangles, so their corresponding angles CBO or NBQ and BCO are equal. But, as ACB is an isosceles triangle, so its angle ABC or MBN is equal to half of its exterior angle BCO. The angle ABC or MBN is therefore equal to each of the angles NBP and PBQ, they being halves of the angle NBQ, which has just been shown to be equal to the angle BCO. Thus the 3 angles MBN, NBP and PBQ are all equal.

Again, the sides BK and BE of the right-angled triangles BOK and BOE are, by construction, equal to each other; and, as these triangles have the same hypotenuse BO, so their corresponding angles OBK and OBE or NBQ are also equal to each other; and therefore their halves RBS, QBR, PBQ and NBP are equal to one another and to the angle MBN. Consequently the 5 arcs RS, QR, PQ, NP and MN by which those 5 equal angles are subtended, are all equal to each other. Each of those arcs is therefore a fifth of the arc MS of 120°, and is accordingly an arc of 24°.

Further, each of those 5 arcs may be divided into 3 equal arcs of 8°. To do this the trisector is moved round the pin at B, and the point A of the piece AHJ is moved along the radius BR until, when it is at a point A′, the edges IJ and EF, which may now be denoted by I′J′ and E′F′, meet at a point which may be termed O′ on the radius BS. Now, through the centre at C′ in the piece A′H′J′ draw the radius BC′N′, and bisect the angle N′BS with the radius BT. The arc RS is thereby divided into 3 equal arcs RN′, N′T, and TS. For, as it was proved that the angle ABC or

MBN is equal to each of the angles NBP and PBQ, so it may similarly be shown that the angle A′BC′ or RBN′ is equal to each of the angles N′BT and TBS. Consequently the 3 arcs RN′, N′T and TS, of which the arc RS of 24° consists, and which subtend those equal angles, are equal to each other. Therefore each of them is an arc of 8°.

The other 4 arcs MN, NP, PQ and QR of 24° each, may each be similarly divided into 3 arcs of 8°, and thereby the entire arc MS of 120° would be divided into 15 arcs of 8°. Each of those 15 arcs may then be easily divided into 2 arcs of 4°, which may be divided into 4 arcs of 2°, which may be further divided into 8 arcs of 1°. The entire arc MS would thereby be divided into 120 equal arcs of 1°.

The other 2 arcs of 120° may each be similarly divided into 120 arcs of 1°. In other words, the entire circumference of the circle may be thus divided or graduated into 360 equal arcs of 1°.

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