THE GLIDE DIRECTION IN ICE*

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ABSTRACT. The failure to detect experimentally a glide direction in the ice crystal is satisfactorily explained by assuming that the crystal glides simultaneously in three symmetry-equivalent directions with a response to the shear stress component in each direction that is the same as that observed for the crystal as a whole or for polycrystalline aggregates—the typical non-linear, power-type flow law. A hexagonal crystal responding to stress by this type of "non-linear crystal viscosity" behaves very differently from a tetragonal one. For a tetragonal crystal, the glide directions are well defined in the response of the crystal if the power-flow-law exponent n exceeds $n \sim 1 \cdot 5$, whereas for a hexagonal crystal a well-defined glide direction can be observed only if $n > c. 5$. The response of a hexagonal crystal is entirely independent of a-axis orientation if $n = 3$ exactly. For $3 < n < c. 5$ the true glide direction should be weakly apparent, whereas for $1 < n < 3$ the crystal should show a response weakly suggestive of preferred glide in a direction perpendicular to the true glide direction. In the observed range of n values for ice, $2 < n < 4$, the expected response to simultaneous glide differs so slightly from the hitherto-postulated a-axis-independent, "non-crystallographic" glide as to be practically undetectable experimentally. This circumstance makes it possible to identify $\langle 11\bar{2}0 \rangle$ as the glide direction, from structural considerations alone, and to accommodate the plastic properties of the ice crystal into the modern concepts of crystal plasticity. It may be expected that hexagonal close packed and face-centred cubic metals at high temperatures, in steady state creep, will show translation gliding without well-defined glide directions.

RÉSUMÉ. Le fait que l'on ne peut déterminer expérimentalement une direction de glissement de la glace peut s'expliquer en supposant que le cristal glisse simultanément dans trois directions symétriquement équivalentes avec une réponse à la composante de la contrainte de cisaillement dans chaque direction qui est la même que ce que l'on observe pour l'ensemble d'un cristal ou pour un aggregat polycristallin—la loi d'écoulement typique nonlinéaire. Un cristal hexagonal se déformant sous contrainte par ce type de "viscosité cristallographique non linéaire" se comporte bien différemment qu'un cristal tétragonal. Pour un cristal tétragonal, les directions de glissement sont bien definies pendant sa déformation si l'exposant n de la puissance de la loi d'écoulement dépasse $n \sim 1,5$, alors que pour un cristal hexagonal une direction de glissement bien définie ne peut s'observé que si $n > c. 5$. La déformation d'un cristal hexagonal est entièrement indépendant de l'orientation de l'axe a si $n = 3$ exactement. Pour $3 < n < c. 5$, la véritable direction de glissement n'apparait que faiblement, alors que pour $1 < n < 3$ le cristal montrerait une déformation peu suggestive de glissement préférentiel dans une direction perpendiculaire à la vraie direction de glissement. Pour la glace, dans l'intervalle observé de $2 < n < 4$, la déformation attendue de glissement simultané diffère si peu du glissement "non cristallographique", jusqu'ici supposé indépendant de l'axe a, qu'elle n'est practiquement pas décelable par l'experience. Ce fait permet d'identifier $\langle 11\bar{2}0 \rangle$ comme une direction de glissement, uniquement à partir de considérations structurales, et d'adapter les propriétés plastiques du cristal de glace aux conceptions modernes de la plasticité cristallographique. On peut s'attendre à ce que des métaux hexagonaux serrés et des métaux cubiques a faces centrées montreront à de hautes températures et sous fluage stationnaire un glissement de translation sans directions de glissement bien définies.

ZUSAMMENFASSUNG. Das Fehlen einer experimentell feststellbaren Gleitrichtung im Eiskristall wird befriedigend erklärt durch die Annahme, dass der Kristall simultan in drei symmetrisch gleichwertigen Richtungen gleitet, wobei die Gleitgeschwindigkeit in jeder dieser Richtungen unter der zugehörigen Scherspannungskomponente dem gleichen Fliessgesetz folgt, wie es auch für den Kristall gesamthaft oder für polykristallines Eis gilt—nämlich dem typischen Potenzgesetz. Ein hexagonaler Kristall verhält sich bei dieser Art "nichtlinearer Kristallviskosität" ganz anders als ein tetragonaler. Beim tetragonalen Kristall sind bei der Verformung des Kristalls die Gleitrichtungen ausgeprägt, wenn der Exponent n im Fliessgesetz den Wert $n \sim 1,5$ überschreitet; beim hexagonalen Kristall hingegen werden die Gleitrichtungen erst ausgeprägt, wenn $n > c. 5$. Das Gleiten eines hexagonalen Kristalles geht völlig unabhängig von der Orientierung der a-Achsen vor sich, wenn genau $n = 3$. Im Bereich $3 < n < c. 5$ sollte die wahre Gleitrichtung schwach zum Vorschein kommen; innerhalb $1 < n < 3$ sollte hingegen das Verhalten des Kristalls eine schwach bevorzugte Gleitbewegung andeuten, die senkrecht zur wahren Gleitrichtung steht. Innerhalb des beim Eiskristall beobachteten Bereiches $2 < n < 4$ weicht der Vorgang des simultanen Gleitens so wenig von dem ab, was bisher als a-Achsen unabhängiges, "nicht-kristallographisches" Gleiten angenommen wurde, dass die zwei Bewegungsformen kaum experimentell zu unterscheiden sind. Dieser Umstand gestattet es, aus rein strukturellen Rücksichten $\langle 11\bar{2}0 \rangle$ als Gleitrichtung zu erklären, und dadurch die Plastizitätseigenschaften des Eiskristalles in Einklang mit modernen Ansichten über die Kristallplastizität zu bringen. Es ist zu erwarten, dass bei hohen Temperaturen hexagonale Metalle und auch andere Kristalle unter stetigem Kriechen Gleitung ohne ausgeprägte Gleitrichtung aufweisen werden.

INTRODUCTION

The plastic properties of the ice crystal are now rather well understood.[1] Glen and Perutz [2] and Steinemann [3] have shown that ice deforms plastically only by translation gliding on the

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basal plane (0001), confirming the original observations of McConnel [4] and Mügge.[5] The geometry of the deformation produced by this type of gliding has been studied in detail in bending experiments by Nakaya.[6] The creep behavior and flow law of ice, both single crystals and in polycrystalline aggregates, has been extensively studied experimentally (Glen,[7, 8] Griggs and Coles,[9] Steinemann,[3, 10] Butkovich and Landauer [11]), and the creep law for ice in glacier flow has been the subject of numerous studies (reviewed by Meier [12]). The influence of superimposed hydrostatic pressure on the flow of ice single crystals has been investigated by Rigsby.[13] Although Steinemann (Ref. 10, p. 31–32), in contrast to Butkovich and Landauer,[11] was unable to verify the applicability to different states of stress of the formulation of the flow law for polycrystalline ice given by Nye,[14] it is possible that a more general treatment (Glen [15]) may prove successful. The causes of certain disagreements among the results of the various investigators are not yet clear, and the relation between the plastic properties of the ice single crystal and that of polycrystalline aggregates is not yet understood in detail, but nevertheless the salient features of ice plasticity seem to be known and can be explained roughly in terms of dislocation models of the creep process (Weertman [16, 17, 18]).

The only outstanding exception to the similarity of ice plasticity and that of metals and other crystals [19, 20] is in the matter of the *glide direction*, and this exception represents the only feature of ice plasticity (except perhaps also for the strain softening shown by ice [1]) that cannot up to now be accommodated within the modern concepts of crystal plasticity.[20] Unlike other plastic crystals, ice seems to show no preferential glide direction. The translation gliding on (0001) apparently takes place simply in the direction of the shear-stress vector acting across (0001), without regard to the orientation of the a-axes in this plane. This fact was first noted by Mügge.[5] Glen and Perutz [2] attempted to determine the glide direction directly by the standard method of tensile tests on single crystals. Steinemann [3] retried Mügge's method,[5] in which the gliding rate is measured as a function of a-axis orientation in a single crystal sheared parallel to (0001) under fixed load in a Bausch shear apparatus. In none of these experiments was there any definite indication of a preferred glide direction, within the experimental uncertainties.

It was pointed out by Glen and Perutz [2] that if the ice single crystal were to glide simultaneously in symmetry-equivalent directions in the (0001) plane, and if the gliding in each direction were to take place at a rate proportional to the component of the stress vector in that direction, the resultant effect would be a motion in the direction of the shear stress vector acting across (0001), as observed. But this linear response for the individual glide directions would correspond to a linear flow law for the crystal as a whole, whereas a distinctly non-linear stress–rate-of-strain relation is observed. With a non-linear flow law for the individual glide directions, one would expect the resultant motion to tend strongly in the direction of that glide direction which is most nearly aligned with the shear stress vector, and the more strongly so the larger the value of the exponent n in the characteristic power flow law.[7–12] This is what is observed in metal and ionic crystals, which show typical non-linear plastic behavior and have well-defined glide directions.

Consequently Glen and Perutz rejected the mechanism of simultaneous glide in symmetry-equivalent glide directions, and concluded reluctantly that "ice near the melting point does not slip along definite crystallographic directions, possibly because many bonds are broken."

This conclusion seems, however, to be in contradiction with the high degree of lattice perfection and the absence of stacking faults parallel to the basal plane in ice single crystals, as shown by X-rays (Owston and Lonsdale [21]), and with the strong control exerted by the a-axes on other properties of the crystal, such as growth [2] and Tyndall-figure formation.[3] Moreover, it cannot be reconciled with modern concepts of crystal plasticity,[20] according to which crystal deformation is made possible by the motion of dislocations having specific Burgers vectors and having therefore necessarily well-defined glide directions, even in cases where the glide plane is not well defined (Ref. 20, p. 4; Ref. 29, p. 9).

In the present paper it is shown that, although simultaneous translation gliding with non-linear creep law leads in tetragonal crystals to the effects envisaged by Glen and Perutz, in hexagonal crystals quite different effects occur, which explain in a satisfactory way the apparent absence of a glide direction in the ice crystal.

TETRAGONAL CRYSTALS

To fix our ideas on the effects of simultaneous glide, and to show the difference in behavior of crystals of different symmetry, we consider first the case of a tetragonal crystal with glide plane (001) and with two perpendicular, symmetry-equivalent glide directions that we may take to be [100] and [010]. The situation during simultaneous glides is shown in Figure 1. The shear stress vector τ acts at an angle θ to the a-axis. If the gliding motion v_i parallel to each of the glide directions (designated by i) takes place according to the characteristic power-law relation
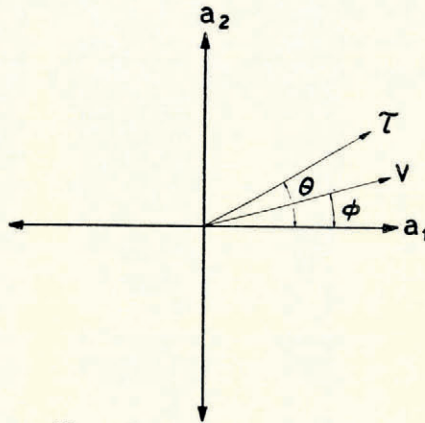
$$v_i = k\tau_i^n, \tag{1}$$

Fig. 1. Diagram illustrating simultaneous glide parallel to the basal plane (001) in a tetragonal crystal. Direction of the shear stress vector acting across (001) is shown by τ, and the direction of the resultant gliding motion by $\mathbf{v}$. The glide directions are a_1 and a_2

where τ_i is the component of the stress vector parallel to glide direction i, then the resultant gliding motion $\mathbf{v}$ (Fig. 1) evidently takes place in the direction defined by an angle ϕ given by

$$\frac{v_2}{v_1} = \tan \phi = \tan^n \theta. \tag{2}$$

The result of equation (2) is shown in Figure 2, where the azimuth ϕ of the gliding direction $\mathbf{v}$ is plotted as a function of the azimuth θ of the applied stress τ, for values n of 1 through 4. For $n = 1$ evidently $\phi = \theta$, so that no glide direction is apparent, but for $n > 1$ the motion strongly concentrates in the glide direction most nearly aligned with the applied stress. The deviation between directions of applied stress and response should be easily detectable experimentally even for $n = 2$, as should doubtless also be the variation in magnitude $|\mathbf{v}|$ of the gliding velocity as a function of stress orientation.

HEXAGONAL CRYSTALS

For crystals of hexagonal symmetry gliding on (0001), Figure 3 applies, the glide directions being arbitrarily designated a_1, a_2, and a_3. In the case of dihexagonal symmetry six separate glide directions instead of only three are possible; this case could be treated by superimposing two diagrams like Figure 3, rotated by a specific amount with respect to one another. The dihexagonal case reduces to Figure 3 alone when the glide direction is either a primary or secondary a-axis, as is most likely in ice.

For the purpose of calculating the resultant motion $\mathbf{v}$ from simultaneous glide on a_1, a_2,
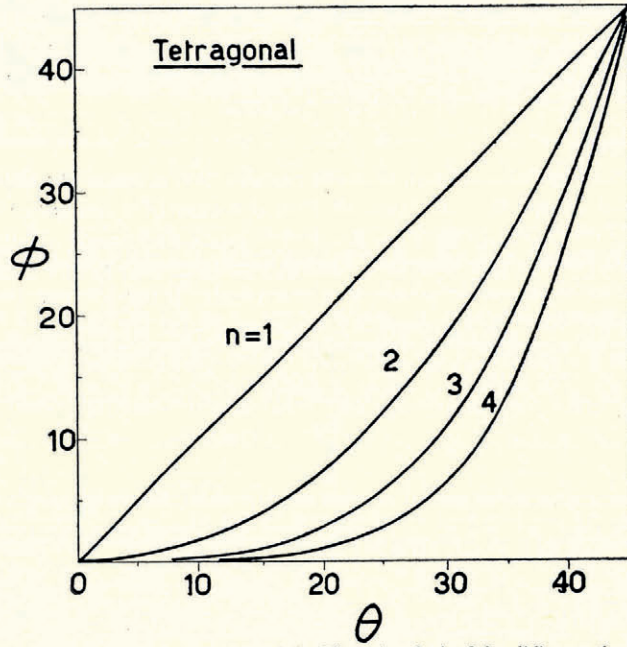
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Fig. 2. *Response of a tetragonal crystal to simultaneous glide. The azimuth ϕ of the gliding motion is plotted (in degrees) as a function of the azimuth θ of the applied shear stress for four values of the power-law exponent n. It is seen that for $n > 1$ the motion tends strongly in the glide direction, which lies at azimuth $0°$*
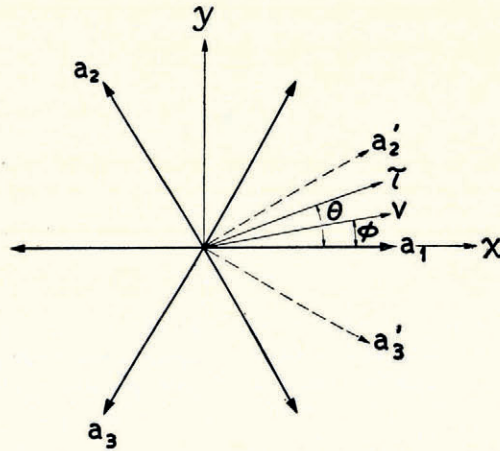

Fig. 3. *Diagram illustrating simultaneous glide parallel to the basal plane (0001) in a hexagonal crystal. The glide directions are a_1, a_2 and a_3, while the "secondary axes" a_1', a_2' and a_3' are perpendicular to the directions a_1, a_2 and a_3 respectively. τ is the direction of the shear stress vector acting across (0001) and $\mathbf{v}$ is the direction of the resultant gliding motion. The x and y cartesian axes are parallel respectively to a_1 and a_1'.*

and a_3 we introduce cartesian coordinates x, y as shown in Figure 3. Application of the gliding law (1), and resolution of the resultant velocities along the x and y axes gives

$$\frac{v_x}{k\tau^n} = \cos^n\theta + \frac{1}{2}\left|\cos^n\left(\frac{\pi}{3}-\theta\right)\right|\operatorname{Sgn}\left\{\cos\left(\frac{\pi}{3}-\theta\right)\right\} + \frac{1}{2}\left|\cos^n\left(\frac{\pi}{3}+\theta\right)\right|\operatorname{Sgn}\left\{\cos\left(\frac{\pi}{3}+\theta\right)\right\},$$

$$\frac{v_y}{k\tau^n} = \frac{\sqrt{3}}{2}\left[\left|\cos^n\left(\frac{\pi}{3}-\theta\right)\right|\operatorname{Sgn}\left\{\cos\left(\frac{\pi}{3}-\theta\right)\right\} - \left|\cos^n\left(\frac{\pi}{3}+\theta\right)\right|\operatorname{Sgn}\left\{\cos\left(\frac{\pi}{3}+\theta\right)\right\}\right], \quad (3)$$

where "Sgn x" means "sign of x." If we restrict θ to the range $-\frac{\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$, i.e. restrict τ to lie between the a_2' and a_3' axes in Figure 3, then the "Sgn" functions and absolute value signs in equations (3) can be omitted.

For the case $n = 1$, equations (3) reduce simply to

$$v_x = \tfrac{3}{2} k\tau \cos \theta, \qquad v_y = \tfrac{3}{2} k\tau \sin \theta, \tag{4}$$

so that $\theta = \phi$, and $|\mathbf{v}|$ is constant independent of θ. Thus for linear "crystal viscosity" the hexagonal crystal is indistinguishable from the tetragonal.

For $n > 1$ (we will not be interested in the case $n < 1$) this similarity disappears. The behavior of a hexagonal crystal for which $n = 3$ is particularly noteworthy. We have from equations (3), for $n = 3$ and $-\frac{\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$,

$$\begin{aligned}
\frac{v_x}{k\tau^3} &= \cos^3\theta + \frac{1}{16}(\cos\theta + \sqrt{3}\sin\theta)^3 + \frac{1}{16}(\cos\theta - \sqrt{3}\sin\theta)^3 \\
&= \frac{9}{8}\cos^3\theta + \frac{9}{8}\cos\theta\sin^2\theta = \frac{9}{8}\cos\theta, \\
\frac{v_y}{k\tau^3} &= \frac{\sqrt{3}}{16}[(\cos\theta + \sqrt{3}\sin\theta)^3 - (\cos\theta - \sqrt{3}\sin\theta)^3] \\
&= \frac{\sqrt{3}}{8}(3\sqrt{3}\cos^2\theta\sin\theta + 3\sqrt{3}\sin^3\theta) = \frac{9}{8}\sin\theta.
\end{aligned} \tag{5}$$

Thus, remarkably, for $n = 3$ equations (3) reduce to the same result as for $n = 1$, except for a proportionality factor. Hence for a hexagonal crystal deforming by simultaneous translation gliding with power-law flow relation having exponent $n = 3$, the gliding response takes place exactly in the direction of the applied shear stress ($\phi = \theta$) and the gliding rate is independent of the a-axis orientation with respect to the applied stress.

For the gliding behavior of hexagonal crystals it is useful to distinguish the flow law exponent ranges $1 < n < 3$, $3 < n < n_0$ and $n > n_0$, where n_0 is a characteristic but not well-defined value of n whose significance will appear later. The different type of behavior in the ranges $1 < n < 3$ and $3 < n < n_0$ can be illustrated by considering the special cases $n = 2$ and $n = 4$. We examine first the gliding direction $\mathbf{v}$ as a function of direction of applied shear stress τ. From equations (3) we find for $n = 2$ and $-\frac{\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$

$$\frac{v_y}{v_x} = \tan\phi = \frac{3\sin 2\theta}{4 + \cos 2\theta}, \tag{6}$$

and for $n = 4$ in the same interval of θ

$$\tan\phi = \frac{6\sin 2\theta\,(2 - \cos 2\theta)}{8\cos^4\theta + 9} \tag{7}$$

From equations (6) and (7) we can calculate the angular deviation $\delta = \theta - \phi$ between the shear stress vector and the resultant gliding motion. The result is shown in Figure 4. It is seen that for $n = 4$ the deviation of the motion is toward the nearest glide direction ($\delta > 0$), whereas for $n = 2$ the deviation is *away* from the nearest glide direction ($\delta < 0$), an unexpected result. This distinction is valid for the exponent ranges mentioned:

For $1 < n < 3$, $\delta \leqslant 0$.
For $3 < n < n_0$, $\delta \geqslant 0$.

This can be exhibited for example by evaluating the angular deviation δ for small values of θ, from equations (3):

$$\delta = \left(1 - \frac{3n}{2^n + 1}\right)\theta \tag{8}$$

or for values of θ near $\dfrac{\pi}{6}$:

$$\delta = \left(\frac{n}{3}-1\right)\left(\frac{\pi}{6}-\theta\right)+\frac{2^n}{3^{(n+1)/2}}\left(\frac{\pi}{6}-\theta\right)^n. \tag{9}$$

(In equation (9) the non-linear term must be retained because it contributes significantly as $n \to 1$.) The (linear term) coefficients in (8) and (9) change sign at $n = 3$.

Another striking feature of the results shown in Figure 4 is the small size of the angular deviations. The largest deviation for $n = 2$ is $(-)2\cdot1°$. On the basis of equation (8), the deviation-curve departing furthest from $\delta = 0$ in the range $1 \leqslant n \leqslant 3$ is the curve for
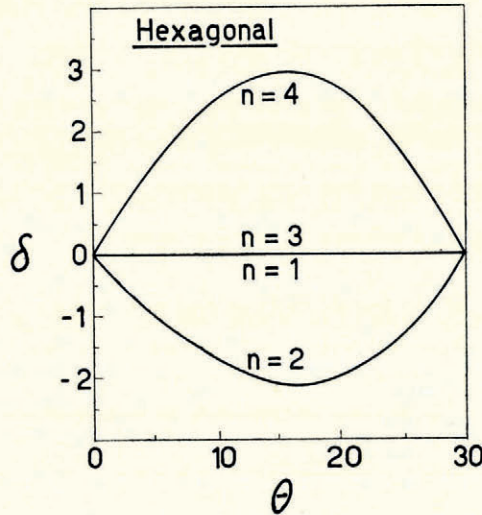

Fig. 4. Response of a hexagonal crystal to simultaneous glide on (0001). The deviation $\delta = \theta - \phi$ between the azimuth θ of the applied shear stress and the azimuth ϕ of the resulting motion is plotted, in degrees, as a function of θ for four values of the power-law exponent n. Positive δ represents deviation toward the nearest glide direction, negative δ deviation away from it

$n = 1\cdot84$; this curve probably does not differ sensibly from the curve for $n = 2$. The curves decrease in height continuously from $n = 2$ to $n = 3$. The largest angular deviation for $n = 4$ is $(+)2\cdot9°$, and the deviations for curves with n in the range $3 \leqslant n < 4$ are of course smaller. By contrast the deviations in the tetragonal case are much larger; for example, for $n = 4$ deviations up to $24°$ occur.

For n sufficiently large, the motion tends to align more and more with the glide directions in a hexagonal crystal as well as in a tetragonal one. The reason for distinguishing the exponent ranges $3 < n < n_0$ and $n > n_0$ is to make this evident. We may choose n_0 so that for $n < n_0$ the gliding response is more nearly parallel to the shear stress vector than to the nearest glide direction, whereas for $n > n_0$ the reverse is true. It is clear that the greatest restriction on the upper limit of the range $3 < n < n_0$ is placed by considering small values of θ (equation 8), because as $\theta \to 30°$ the deviation $\delta \to 0$ no matter how large the value of n. We find $n_0 = 4\cdot8$ by setting the numerical coefficient in equation (8) equal to $\frac{1}{2}$, and we may therefore take $n_0 \sim 5$ as the upper limit of the range over which a hexagonal crystal in steady creep does not display a well marked glide direction. For a tetragonal crystal the upper limit, by a similar argument, is at $n_0 \sim 1\cdot5$.

I have not calculated profiles of velocity $|\mathbf{v}|$ as function of θ from equations (3), but the general effect of orientation on rate of gliding can be judged by comparing the response

$\mathbf{v}\,(0°)$ parallel to the glide direction $(\theta = 0°)$ with the response $\mathbf{v}\,(30°)$ midway between neighboring glide directions $(\theta = 30°)$. It is easily shown that

$$\frac{|\mathbf{v}(0°)|}{|\mathbf{v}(30°)|} = \frac{2^n+1}{3^{\frac{1}{2}(n+1)}}. \qquad (10)$$

The ratio $|\mathbf{v}(0°)/\mathbf{v}(30°)|$ from equation (10) is shown for a succession of n values in Table I.

TABLE I

n	1	2	2·5	3	3·5	4	5
$\dfrac{|\mathbf{v}\,(0°)|}{|\mathbf{v}\,(30°)|}$	1·00	0·96	0·98	1·00	1·04	1·09	1·22

It is seen that for n in the range $1 < n < 3$, the response parallel to the glide direction is actually *less* than the response between the glide directions. Instead of being the direction of maximum response, the glide direction turns out in this case, surprisingly, to be the direction of minimum response. Only for $n > 3$ is the glide direction the direction of maximum response. Throughout the whole range $1 \leqslant n \leqslant 4$, the variation in gliding rate as a function of a-axis orientation under a given shear stress is rather slight.

ICE

Most observations of ice deformation in steady-state creep have been rationalized with a bulk flow law of the form

$$\dot{\gamma} = k\tau^n. \qquad (11)$$

For polycrystalline aggregates τ and $\dot{\gamma}$ refer respectively to shear stress and shear strain-rate in simple shear, whereas for single crystals they refer to shear stress and shear strain-rate across the (0001) plane. Flow law studies of ice single crystals are less numerous and less conclusive than equivalent studies of polycrystalline ice, probably because of the greater difficulties of preparing and handling single crystals, and because of the greater variation in properties from specimen to specimen and during the course of deformation of a given specimen. Steinemann[3, 10] found values of n ranging from 2·3 to 3·9 in "secondary flow" and the value 1·5 for "tertiary flow" of single crystals is a Bausch shear apparatus at $-2·3°$ C. The compression experiments of Griggs and Coles[9] on single crystal cylinders showed progressively accelerating creep without a final steady state, which Glen[1] interprets as indicating that the duration of the experiments was too short for the final steady state creep to set in. Griggs and Coles results cannot be compared directly with the steady-state law (11), but at any given time they indicate a stress-dependence of the flow rate corresponding to an n value of 2·0. For single crystals in "easy glide," Butkovich and Landauer[11] found a flow law of the type (11) with $n = 2·5$; since the total strain in their experiments was less than 10 per cent, presumably their results represent "secondary flow."[3] Recent experiments of my own on torsion of hollow single-crystal ice cylinders deformed several hundred per cent at $-2·9°$ C. indicate the value $n = 2·5$.

For polycrystalline ice the values found by Steinemann (Ref. 10, p. 25) range mostly from 2·0 to 4·0. The value found by Glen[8] is 3·17 or 4·2, depending on how the transient creep is taken into account. The numerous flow data of Butkovich and Landauer[11] on polycrystalline ice of several different kinds are fit best by a flow law of type (11) with $n = 2·96$. Higashi's experiments[22] on collapse of hollow polycrystalline ice cylinders suggested a stress-dependent n value varying in the range $n \sim 2$ to $n \sim 4$. It is not known exactly to what extent the flow law for the polycrystalline aggregate reflects the flow law of the single crystal, or to what extent it is influenced by inter-grain relationships. Steinemann[3] expects that the exponent values should be comparable for single crystals and polycrystals, even though the k values will differ, whereas Butkovich and Landauer[11] think that the rate-limiting process in poly-crystalline ice deformation is not the basal gliding of the individual ice crystals. Nevertheless, the overall similarity of results of the two types of experiment is good. The experiments of Brill[23] and Jellinek and Brill,[24] showing a quite different type of rheology, were carried out

for strains much smaller than those involved in the other creep experiments considered here. The creep laws derived theoretically by Weertman [16, 17, 18] have n values of $4 \cdot 0$, $4 \cdot 5$, $3 \cdot 0$ and $2 \cdot 5$, depending on the rate-limiting mechanism in the creep process.

In view of the range $2 < n < 4$ of exponent values observed in the non-linear flow law of ice, the general considerations given above for hexagonal crystals provide a natural explanation for the failure to detect experimentally a glide direction in the ice crystal. In tensile tests of the type carried out by Glen and Perutz,[2] one can expect to observe a deviation of at most only about $2°$, depending upon the pertinent n value under the conditions of the experiment, between the shear stress vector and the direction of the gliding motion. Of the four tensile tests for which data are given (Ref. 2, Fig. 13), three show small deviations ($0°$ to $4°$) toward $[10\bar{1}0]$, whereas the fourth seems to indicate gliding only in the direction $[11\bar{2}0]$, and indicates a large deviation ($\sim 15°$) between shear stress vector and gliding motion; the reason for the discrepancy with the other three tests is not known. For Steinemann's method the expected maximum variation in gliding rate as a function of a-axis orientation is less than 4 per cent over most of the likely range of n values. The 10 per cent variation in flow rate actually observed in one of Steinemann's experiments [3] was considered by him to lie within the experimental uncertainties. Nakaya (Ref. 6, p. 43) has hinted at differences in creep curves as a function of a-axis orientation in certain beam-bending experiments, but no details have yet been published.

Even without regard to experimental uncertainties, there is a fundamental difficulty in determining the glide direction in ice, because the response of the crystal has opposite sense according as the flow law exponent n lies in the range $1 < n < 3$ or in $3 < n < n_0$. An observed small deviation between applied shear stress and gliding motion can be interpreted either as indicating a particular glide direction, with $n > 3$, or as indicating a glide direction rotated $30°$ with respect to the first, with $n < 3$. Steinemann's observation in one experiment (Ref. 3, Fig. 8) of a 10 per cent greater creep rate parallel to $[11\bar{2}0]$ than parallel to $[10\bar{1}0]$ could, if it were experimentally significant as believed by Glen,[1] be interpreted as $[11\bar{2}0]$ glide if $n > 3$ at the time of the experiments or as $[10\bar{1}0]$ glide if $n < 3$. The experiments were carried out on a crystal in "tertiary flow," for which Steinemann reports $n = 1 \cdot 5$, hence they indicate if anything a glide direction $[10\bar{1}0]$, rather than $[11\bar{2}0]$ as thought by Glen.[1] If the flow law is not known under the conditions of an experiment in search of the glide direction, as in the experiments of Glen and Perutz, the result may be ambiguous and cannot be readily interpreted; variations in flow law from crystal to crystal may explain the inconsistent results reported.

In principle, the glide direction could be distinguished, and thus n and the glide direction determined simultaneously, by the asymmetry of the angular-deviation curves in Figure 4. Thus for $n = 2$ the maximum value of $|\delta|$ occurs at $\theta = 16 \cdot 6°$, and for $n = 4$ the maximum occurs also at a θ value greater than $15°$. However, the asymmetry of the curves is so slight that it is quite unlikely to be detectable experimentally.

The glide direction in ice can, however, be identified *a priori* as $\langle 11\bar{2}0 \rangle$ from structural considerations alone, without resort to experiment. In accordance with the modern concept of crystal plasticity, it can be assumed that the ice crystal deforms by the motion of dislocations lying in the (0001) plane and having Burgers vectors (Ref. 20, p. 15) parallel to this plane. The shortest lattice vector in this plane is the a-axis translation, and this vector will therefore be the Burgers vector of the most stable dislocations of the type considered; a dislocation with a Burgers vector corresponding to any other lattice vector in this plane will dissociate (Ref. 20, p. 70) spontaneously into separate dislocations having Burgers vectors of the type $a\langle 11\bar{2}0 \rangle$.* For example the energy of a dislocation (Ref. 20, p. 37) with Burgers vector $\sqrt{3}a[10\bar{1}0]$, the second shortest possible Burgers vector, is 50 per cent greater than the combined energy of

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* The notation here means "a vector of length equal to the a-axial length and in one of the $\langle 11\bar{2}0 \rangle$ (a-axis) directions in the lattice"; it does not agree exactly with the corresponding notation customarily used for isometric crystals (Ref. 20, p. 16), because of the awkward features of assigning indices to lattice lines in a hexagonal net.

the two dislocations with Burgers vectors respectively $a[2\bar{1}\bar{1}0]$ and $a[11\bar{2}0]$ into which the original dislocation can dissociate. Moreover, dislocations with Burgers vector $a\langle 11\bar{2}0\rangle$ may be expected to have the highest possible mobility. This might be attributed in part to the fact that the "puckered sheets" of water molecules in layers parallel to (0001) have well-defined "grooves" in the direction $\langle 11\bar{2}0\rangle$, as pointed out by Glen and Perutz.[2] A similar feature was considered important by Buerger [25] in the NaCl structure. But in the ice structure the "grooves" do not "interlock" from layer to layer, and it therefore seems unlikely that they exert an important influence on dislocation mobility. The inter-molecular forces that operate during slip of one (0001) layer in ice over another cannot be accurately evaluated, and the clear-cut restrictions placed by charge distribution in ionic crystals do not apply to ice. In terms of bonding there is thus no clear basis for distinguishing the mobility of dislocations having Burgers vectors $a\langle 11\bar{2}0\rangle$ or $a\langle 10\bar{1}0\rangle$, or any other. The limitation on mobility is therefore probably placed by the geometry of the motion itself, which strongly favors the dislocation with the shortest Burgers vector for a given interplanar spacing of the glide planes (Ref. 20, pp. 62–64). There may be other unforseen limitations placed by the nature of the non-linear creep process (for example by the "dislocation climb" mechanism considered by Weertman [16]), but in our present state of knowledge all evident factors point to $a\langle 11\bar{2}0\rangle$ as the likely Burgers vector and therefore glide direction in (0001) translation-gliding of ice. The same conclusion was reached by Glen and Perutz [2] on the basis of a less detailed argument.

The *a priori* identification of $\langle 11\bar{2}0\rangle$ as the glide direction in ice would be most questionable were it not for the fact that simultaneous glide with non-linear flow law accounts for the failure to observe a glide direction experimentally. It thus appears that dislocation motion, governed by non-linear response to stress (Weertman [16, 17, 18]), and resulting in simultaneous glide in the three glide directions $\langle 11\bar{2}0\rangle$, provides a satisfactory description of the basic features of ice single-crystal plasticity.

It would, of course, be desirable to have accurate enough experimental evidence to search for the expected features of simultaneous non-linear glide, as a test of the theory here given, but it would be difficult to obtain the experimental accuracy necessary to detect the small expected deviations from the type of behavior that hitherto has been considered evidence for non-crystallographic gliding without preferred glide direction.

OTHER CRYSTALS

The hexagonal close-packed metals zinc and magnesium, which at room temperature have typical plastic behavior with gliding elements $T(0001)t\langle 11\bar{2}0\rangle$, at high temperature show (in polycrystalline aggregates) non-linear quasi-viscous creep that can be represented approximately with a power-law stress dependence. Creep data for magnesium (Roberts [26]) correspond to n values ranging from about 3 at 170° C. to about 1·5 at 310° C. The data for zinc obtained by Cottrell and Aytekin [27] at temperatures below 120° C. correspond to an n value greater than 7, but lower values might be found at higher temperatures, as suggested by the magnesium data. It may therefore be expected that at high temperatures single crystals of these metals will show gliding behavior similar to that of ice, with lack of a well-defined glide direction. The same can also be expected for face-centered-cubic metals, because the glide directions $\langle 110\rangle$ have hexagonal symmetry in each of the $\{111\}$ glide planes. For aluminium at temperatures above about 300°, a power-law creep with n values ranging from 3·0 to 4·5 has been observed, as reported in the summary by Weertman.[16]

I am not aware of any attempt to determine the gliding elements of any of these metals during steady-state creep at high temperature, which would provide a test of the above prediction. A confirmation of it would not only reinforce the above considerations for ice, but would indicate that for true creep processes, which doubtless predominate over plasticity (*sensu stricto*) in rock deformation as it occurs in nature, Schmid's law of maximum resolved shear stress [19, 20] is not valid. In this case laboratory studies of rock and single crystal plasticity,

in which the law has been verified (see for example Ref. 28), would be applicable only with appropriate modification to the deformation process in nature.

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