

Notation Index: In Order of Appearance

- $(\mathcal{M}, \langle \cdot, \cdot \rangle)$ – Minkowski space, 5
- $\langle \cdot, \cdot \rangle$ – Minkowski metric, 5
- g_{jk} – Minkowski metric in components, 5
- L – light cone, 6
- I – interior light cone, 6
- J – closed light cone, 6
- $J^\vee, J^\wedge$ – closed future and past light cone, 6
- $I^\vee, I^\wedge$ – open future and past light cone, 6
- $\square$ – scalar wave operator, 7
- γ^j – Dirac matrix, 8
- $\not{A}, \not{B}$ – Feynman dagger, 8
- $\langle \cdot | \cdot \rangle, \langle \cdot | \cdot \rangle_x$ – spin inner product, 9, 70, 111
- $(\cdot | \cdot), (\cdot | \cdot)_m$ – scalar product on Dirac solutions in Minkowski space, 10, 280
- B – external potential, 10
- H – Dirac Hamiltonian, 11, 87
- $\langle \cdot | \cdot \rangle$ – spacetime inner product, 11, 78, 259
- Γ – pseudo-scalar matrix, 13, 72
- ϵ_{jklm} – totally anti-symmetric symbol, 13, 178
- χ_L, χ_R – chiral projectors, 13
- $S\mathcal{M}$ – spinor bundle in Minkowski space, 14
- $C_{\text{sc}}^\infty(\mathcal{M}, S\mathcal{M})$ – spatially compact spinorial wave functions, 14, 87, 297
- $C_0^\infty(\mathcal{M}, S\mathcal{M})$ – spinorial wave functions of compact support, 15
- $d(x, y)$ – metric, 21
- $\mathcal{O}$ – topology, 22, 23
- $\bar{A}$ – closure of A , 23
- $\overset{\circ}{A}$ – interior of A , 23
- $\|\cdot\|$ – norm, 24
- $\langle \cdot | \cdot \rangle$ – complex scalar product, 25
- δ_{ij} – Kronecker delta, 26
- $(L(V, W), \|\cdot\|)$ – Banach space of linear bounded operators, 27
- $\|\cdot\|, \|\cdot\|_{\mathcal{H}}$ – sup-norm, operator norm, 27, 107
- A^* – adjoint of operator, 28
- $I^\perp$ – orthogonal complement of $I \subset \mathcal{H}$, 29
- $\mathfrak{M}$ – σ -algebra of measurable sets, 30
- $L^p(\mathcal{F}, d\rho)$ – L^p -spaces with $1 \leq p \leq \infty$, 32
- $\langle \cdot | \cdot \rangle_{L^2(\mathcal{F}, d\rho)}$ – L^2 -scalar product, 32
- χ_A – characteristic function of A , 32
- $\|f\|_{p,q}$ – Schwartz norm of f , 36
- $\mathcal{S}(\mathbb{R}^n)$ – Schwartz space, 37
- $\mathcal{S}'(\mathbb{R}^n)$ – space of tempered distributions, 37
- δ – (Dirac's) δ distribution, 38
- $\mathcal{F}, \mathcal{F}^*$ – Fourier transform, 39
- $f * g$ – convolution of Schwartz functions, 41
- $f * T$ – convolution of Schwartz function with tempered distribution, 42
- $\mathcal{D}(\mathbb{R}^n)$ – space of test functions, 42
- $\mathcal{D}'(\mathbb{R}^n)$ – space of distributions, 42
- $T_p\mathcal{M}$ – tangent space of $\mathcal{M}$ at p , 45
- $T\mathcal{M}$ – tangent bundle of $\mathcal{M}$, 47
- Θ – Heaviside function, 54
- $(\mathcal{M}, g)$ – Lorentzian manifold, 68
- $T_p\mathcal{M}$ – tangent space at p , 68
- $U(2, 2)$ – group of isometries of the spin inner product, 70
- ∇ – Levi-Civita connection, 73
- D – spin derivative, 75
- $(\cdot | \cdot)_{\mathcal{N}}$ – scalar product on Dirac solutions in curved spacetime, 78
- $S\mathcal{M}$ – spinor bundle, 84
- $(\mathcal{H}, \langle \cdot | \cdot \rangle_{\mathcal{H}})$ – Hilbert space of causal fermion system, 102
- n – spin dimension, 102
- $\mathcal{F}$ – set of operators of causal fermion system, 102
- ρ – measure on $\mathcal{F}$ of causal fermion system, 102
- $(\mathcal{H}, \mathcal{F}, \rho)$ – causal fermion system, 102
- M – spacetime of causal fermion system, 102
- $\mathfrak{R}_\varepsilon$ – regularization operator on scale ε , 104, 367
- $F^\varepsilon(x)$ – local correlation operator at $x \in \mathcal{M}$ with regularization on scale ε , 105
- $F_*\mu$ – push-forward measure, 105
- $\mathcal{L}(x, y)$ – causal Lagrangian, 106
- $S(\rho)$ – causal action, 106
- $\mathcal{T}(\rho)$ – functional in boundedness constraint, 106
- tr – trace of linear operator, 106
- $\lambda_1^{xy}, \dots, \lambda_{2n}^{xy}$ – nontrivial eigenvalues of xy , 108
- $\mathcal{C}$ – time direction functional, 108

- $S_x := x(\mathcal{H})$ – spin space, 109
- $(S_x M, \prec, \cdot, \succ_x)$ – spin space, 109, 111
- π_x – orthogonal projection on spin space, 109
- $P(x, y)$ – kernel of fermionic projector, 109
- Tr_{S_x} – trace on spin space, 110
- A_{xy} – closed chain, 110
- $\lambda_1^{xy}, \dots, \lambda_{2n}^{xy}$ – nontrivial eigenvalues of xy , 111
- ψ^u – physical wave function of $u \in \mathcal{H}$, 111
- $\Psi(x)$ – wave evaluation operator at $x \in M$, 112
- $C^0(M, SM)$ – continuous wave functions in spacetime M , 131
- $\mathcal{L}_\kappa$ – κ -Lagrangian, 144
- $\mathcal{F}^{\text{reg}}$ – regular points of $\mathcal{F}$, 144
- ℓ – integrated Lagrangian, 148
- s – Lagrange parameter in Euler–Lagrange equations, 148
- ∇_u – jet derivative in the direction of u , 151
- $\mathfrak{J}$ – space of smooth jets, 151
- $\mathfrak{M}^{\mathcal{G}}$ – equivariant measures under symmetry group $\mathcal{G}$, 152
- Δ – linearized field operator, 159
- div – divergence in smooth spacetime, 163
- Γ – space of smooth jets without scalar component, 166
- Φ_τ – variation of spacetime, 173
- γ_ρ^Ω – conserved one-form, 176
- I_{k+1} – general conserved surface layer integral, 177
- $\mu(\mathbf{v}, x)$ – boundary measure induced by inner solution $\mathbf{v}$, 178
- $\langle \cdot, \cdot \rangle_\rho^\Omega$ – commutator inner product, 180
- $Q^{\text{dyn}}(x, y)$ – kernel in the dynamical wave equation, 181
- $\sigma_\rho^\Omega(\cdot, \cdot)$ – symplectic form on linearized solutions, 182
- $(\cdot, \cdot)_\rho^\Omega$ – surface layer inner product, 183
- $\gamma^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho)$ – nonlinear surface layer integral, 184
- ν – correlation measure, 185
- $Z^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho)$ – partition function, 186, 386
- $Q(x, y)$ – kernel describing the first variation of causal Lagrangian, 188
- L_ρ – positive operator arising from second scalar variations, 198
- s_x – Euclidean sign operator, 202
- T_x – tangent space of causal fermion system, 202
- D_x – spin connection of causal fermion system, 205
- ∇_x – metric connection of causal fermion system, 205
- $\ll$ – absolute continuity of measures, 226
- $\mathfrak{m}^{(\ell)}$ – ℓ th moment measure, 229
- C_x – tangent cone at $x \in M$, 239
- μ_x – tangent cone measure at $x \in M$, 239
- $\tilde{k}_m$ – causal fundamental solution, 258
- $\tilde{s}_m^\vee, \tilde{s}_m^\wedge$ – causal Green’s operators, 258, 335
- $(\eta_t)_{t \in I}$ – local foliation inside $U \subset M$, 268
- $(\cdot, \cdot)^t$ – softened surface layer inner product, 270
- $\sigma^t(\cdot, \cdot)$ – softened symplectic form, 270
- $\mathfrak{J}^{\text{vary}}$ – space of jets used for varying the measure, 271, 353
- $\mathfrak{J}_x, \Gamma_x$ – jets at spacetime point $x \in M$, 272
- $\|\cdot\|^t$ – norm corresponding to softened surface layer inner product, 272
- $\langle \cdot, \cdot \rangle_{L^2(L)} - L^2$ – scalar product in lens-shaped region, 273
- $\bar{\mathfrak{J}}, \mathfrak{J}$ – jet spaces vanishing in future or past, 274
- $\mathcal{S}$ – fermionic signature operator, 281
- $C_{\text{sc}, 0}^\infty(\mathcal{M} \times I, S\mathcal{M})$ – space of families of spinorial wave functions, 283
- $(\cdot, \cdot)_I$ – scalar product on families of Dirac solutions, 283
- $\mathcal{H}^\infty$ – domain for mass oscillation property, 283
- T – operator of multiplication by mass parameter, 283
- $\mathfrak{p}$ – operator of integration over mass parameter, 284
- $s_m(x, y)$ – Dirac Green’s operator in Minkowski space, 296
- $s_m^\vee, s_m^\wedge$ – causal Green’s operators in the vacuum, 297, 333, 335
- k_m – causal fundamental solution, 297
- $U^{t', t}$ – time evolution operator, 298
- ϵ – sign function, 301
- $\Pi_\pm(\vec{k})$ – projections to Dirac solutions in momentum space, 301
- Pexp – ordered exponential, 318
- R_λ – resolvent of causal fundamental solution, 324
- $T_{m^2}(x, y)$ – Fourier transform of lower mass shell, 329
- $\mathcal{O}((y - x)^{2p})$ – order on the light cone, 332
- $T^{(n)}(x, y)$ – singular factors in light-cone expansion, 333
- $S_{m^2}^\vee, S_{m^2}^\wedge$ – causal Green’s operators of Klein–Gordon equation, 334
- $B = \mathcal{B} - m$ – external potential combined with the mass, 335
- $S^{(l)}$ – mass expansion of S_a , 335
- $\text{WF}(\phi)$ – wave front set of ϕ , 339
- $\mathfrak{J}^{\text{diff}}, \Gamma^{\text{diff}}$ – Space of jets for which ℓ is differentiable, 353
- $\mathfrak{J}^{\text{test}}$ – space of test jets, 353
- $\xi_{[p]}^{(n)}$ – ultraviolet regularized factor ξ , 368
- $P^\varepsilon(x, y)$ – regularized kernel of fermionic projector, 368
- $z_{[p]}^{(n)}$ – abbreviation for $(\xi_{[p]}^{(n)})^2$, 369
- $T_{[p]}^{(n)}$ – ultraviolet regularized $T^{(n)}$, 369

$T_o^{(n)}$ – stands for $T_{\{p\}}^{(n)}$ or $T_{[p]}^{(n)}$, 369
 deg – degree on light cone, 369
 L – degree of simple fraction, 370
 c_{reg} – regularization parameter,
 370

∇ – derivation on the light cone, 370
 ℓ_{macro} – length scale of macroscopic physics,
 371
 $\Psi(x)$ – wave evaluation operator at $x \in M$,
 382