

IN MEMORIAM: MATI RUBIN  
1946–2017

Matatyahu (Mati) Rubin passed away on February 6, 2017, at the age of 70, after a struggle with lung cancer. He left his widow, Chaya, four children, six grand children, and an impressive mathematical legacy. Born on September 24, 1946, in Tel-Aviv (Israel) to Puah and Hanan Rubin, a socialist leader who served in the Kneset (the Israeli parliament, 1949–1962). From his parents Mati absorbed a deep knowledge of the political history of Europe, and his love of classical music in which he grew to become a true *connoisseur*.

Rubin's mathematical work included contributions to set theory and model theory, to the study of ordered algebraic structures, and to general topology, but most significantly to the theory of Boolean algebras and to the study of reconstruction problems. His role in the study of the problem of reconstructing various structures from groups of their automorphisms, is a fundamental part of the theory and a standard tool in the study of the problem.

Mati received his academic education at the Hebrew University of Jerusalem. Being an extremely thorough and assiduous scholar, Mati decided that in order to truly understand first order logic, he had to study at depth a specific example. In his M.Sc. thesis, under the supervision of H. Gaifman, he investigated the theory of linear orders. Despite being a well-known example already studied by central figures in the field, Mati obtained significant new results. His first published article "*Theories of linear order*" (1974, 52 pages), based on this work, is still a standard reference in the subject, and a canonical example of classical model theoretic analysis, in the spirit of A. Tarski and A. Robinson.

Following Gaifman's advice, Mati started his Ph.D. as S. Shelah's first student upon the latter's arrival to the Hebrew University. His dissertation "On Boolean Algebras and Their Automorphism Groups" already touched upon the main themes that will occupy Mati throughout his career: Boolean algebras, automorphism groups, and reconstruction problems. The problem of reconstructing a Boolean algebra from its group of automorphism was formulated by J. D. Monk in 1975, with rudimentary answers due to Monk and McKenzie (around 1977). Mati's work greatly extended the subject in several directions.

In view of McKenzie's (and independently Shelah's) proof that Boolean algebras, in general, cannot be reconstructed from their automorphism groups, Mati embarked on a long term project of finding the most general homogeneity conditions assuring the existence of reconstruction theorems. Realising that, in most cases, the reconstruction results he obtained did not depend on the group  $G$  he worked with to be the full group of automorphisms, Mati also started studying sufficient conditions for groups of automorphisms from which homogeneous enough Boolean algebras could be reconstructed. It is one of Mati's most striking results that the most important of those conditions is first order expressible in the language of groups (i.e., not even in the language of permutation groups). Mati's work on the subject continued during his 1977 assistant professorship in Boulder, Colorado, and upon his return in 1978 to Israel, as a lecturer at Ben Gurion University, in the southern city of Be'er Sheva. This work, culminated in a fairly complete solution of Monk's problem in his 1989 article "*On the reconstruction of Boolean algebras from their automorphism groups*" which appeared in the Handbook of Boolean algebras.

Mati's character as a meticulous, hard working, and independent researcher manifested itself already as a student. During his Ph.D. he deliberately steered away from directions suggested to him by Shelah, in order to maintain his independence as a researcher, and pave his own path in mathematics. When faced with tough problems, others would have set aside to come back to at a later time, Mati never gave up. He chain smoked his way through long days and nights of hard work, until the problem was solved.

After a decade or so of work around the reconstruction of Boolean algebras, Rubin realised that its solution could help in many other reconstruction problems. For a structure  $X$  let  $H(X)$  denote the group of automorphisms of  $X$  and  $G$  a subgroup of  $H(X)$ . A class  $\mathcal{K}$  of *space-group* pairs  $(X, G)$  is *faithful* if, for every  $(X_1, G_1), (X_2, G_2) \in \mathcal{K}$  and any isomorphism  $\phi : G_1 \rightarrow G_2$ , there is an automorphism  $\pi : X_1 \rightarrow X_2$  such that  $\phi(g) = \pi \circ g \circ \pi^{-1}$  for every  $g \in G_1$ . For instance the faithfulness of a singleton  $\{(X, G)\}$  means that every automorphism of  $G$  is a conjugation by a member of  $H(X)$ . In those terms, Mati embarked on a project of proving faithfulness results for a great variety of such structure-group pairs.

Expanding significantly the seminal work of Whittaker (1963) on the reconstruction of Euclidean manifolds, Mati introduced the notion of systems of locally moving automorphisms and proved a reconstruction theorem for such systems. These results lie at the core of most of Rubin's topologically flavoured faithfulness theorems. Namely, those reconstruction results where  $X$  was a topological space and  $G$  a subgroup of its group of auto-homeomorphisms.

In his formidable article "*On the reconstruction of topological spaces from their groups of homeomorphisms*" (1989, 51 pages) Mati applied his new technique to reprove basically all known results in the area and obtained many spectacular new results, e.g., concerning groups of differentiable and Lipschitz homeomorphisms of differentiable manifolds, groups of measure-preserving automorphisms of measure algebras,

and groups of automorphisms of certain linear orderings and Boolean algebras.

Expanding the scope of his techniques even further Mati addressed reconstruction problems for many different structures and spaces: In a joint article with A. Leiderman, “*On the reconstruction of locally convex spaces from their groups of homeomorphisms*”, (1999, 31 pages), faithfulness results were proved for manifolds over a class of locally convex spaces (expanding the class of all normable spaces). This was later generalised further, with V. Fonf in “*A reconstruction theorem for locally convex metrizable spaces, homeomorphisms groups without small sets, semigroups of nonshrinking functions of a normed spaces*”, (appearing only in 2016, 35 pages) to all metrizable locally convex spaces. Combined with ideas arising from his large research monograph “*Reconstruction of manifolds and subsets of normed spaces from subgroups of their homeomorphism groups*” (2005, 246 pages, written in collaboration with Y. Yomdin) these results were applied to reconstruction problems for (among others) the group of bi-Lipschitz homeomorphisms of  $\mathbb{U}$ , the universal Urysohn space (2010, 29 pages, with W. Kubiś).

But his reconstruction techniques were also applied in less topological contexts. In his monumental monograph (1993, 260 pages) *The reconstruction of trees from their automorphism groups*, he proved sufficient (and close to necessary) conditions for the faithfulness of complete  $U$ -trees, completely solving the problem in the case of  $\aleph_0$ -categorical  $U$ -trees. Using his mastery of set theoretic techniques Rubin was also able to give strong reconstruction results for a large class of  $\aleph_0$ -categorical structures, “*On the reconstruction of  $\aleph_0$ -categorical structures from their automorphism groups.*” (1994, 24 pages).

As author, Rubin was always sensitive to his readers. He dedicated extensive amounts of time to polishing, clarifying and perfecting his exposition—making it accessible to as wide a readership as possible. Mati’s attitude as a speaker was similar: he was willing to take on board anyone interested, not shying from extensive explanations and numerous repetitions, always checking that no one in the audience ever was lost. This attitude, combined with his natural social awareness also made Mati very popular with undergraduate students. He created no distance between himself and the students who overflowed his office hours. Mati was not only keen to disseminate his works and ideas. He was also an avid listener: whenever he was in the audience, he would ask questions, making sure that he—and others—kept track of the lecture, repeating delicate arguments, and adding clarifications for the benefit of students, who may have been too shy to ask questions.

Rubin’s mathematical contributions were not restricted to reconstruction problems. Though his interest in Boolean algebras was initiated by the study of reconstruction problems, he made several significant contributions to that field. One of his best results on the subject appeared as “*A Boolean algebra with few subalgebras, interval Boolean algebras and retractiveness*” (1983, 25 pages) where Mati showed that every subalgebra of an interval algebra is retractive. Applying yet again his set theoretic knowledge and abilities, he constructed, using  $\diamond_{\omega_1}$ , a retractive Boolean algebra  $B$  (of cardinality  $\aleph_1$ ) which is not embeddable in an interval algebra. This algebra has only

$\aleph_1$  subalgebras and every nowhere dense subset of  $B$  is countable. The reconstruction of Boolean algebras with such properties refuted conjectures of J. D. Monk, R. McKenzie, B. Rotman, and others. To construct this example, Mati developed the notion of “concentrated Boolean algebra”. These techniques were used, among others, in his 2011 article, “*A thin-tall Boolean algebra which is isomorphic to each of its uncountable subalgebras*” (23 pages, with R. Bonnet), solving a conjecture of J. Roitman.

Many of Mati’s contributions were written with colleagues and students who—almost invariably—were also (or became) his friends. For instance, in collaboration with R. Bonnet, he introduced the class of *well-generated [Boolean] algebras* (2000, 50 pages), i.e., algebras having a well-founded generating set. In particular, such an algebra is superatomic (i.e., every subalgebra is atomic). Already the existence of a superatomic algebra which is not well-generated is not trivial. Those algebras, related to a work of A. Dow and S. Watson were studied by Rubin throughout 2000–2009 in collaboration with U. Abraham, R. Bonnet, W. Kubiś, and others.

Later on, collaborating with U. Abraham, R. Bonnet, and W. Kubiś, Mati developed yet another class of Boolean algebras, the so called *[free] Poset algebras*, (2003, 26 pages). In this work they show that for a poset  $P$  the following are equivalent. (i):  $P$  is scattered and satisfies the finite antichain condition, that is:  $P$  does not contain a subset isomorphic to the chain of rational numbers and  $P$  does not contain an infinite set of pairwise incomparable elements and (ii): the poset algebra  $B(P)$  is well-generated and thus superatomic. This improved a result of M. Pouzet, announced in 1981, stating that Property (i) implies that  $B(P)$  is superatomic.

Though Mati mainly used his strong background in set theory in applications to problems in other fields, he also contributed to set theory. His most important work in that field, obtained in collaboration with U. Abraham and S. Shelah, was published as “*On the consistency of some partition’s theorems for continuous colorings, and the structure of  $\aleph_1$ -dense real order types*” (1985, 84 pages). In this work they introduce the Open Coloring Axiom (OCA) showing that MA plus OCA imply  $2^{\aleph_0} = \aleph_2$ ; and they continue the work of J. Baumgartner on the axiom BA (stating that any two  $\aleph_1$ -dense subsets of the real line without endpoints are isomorphic). Their axiom OCA was further extended and refined by S. Todorcević in 1989.

When, in June 1988, Mati was diagnosed with stomach cancer, he fought the disease fiercely, never stopping to work (or to smoke), coming to his office even on the day he received chemotherapy treatments. According to legend, when—due to these treatments—Mati’s weakened immune system required that he be hospitalised in a solitary clean room, Mati convinced his friends to “smuggle” cigarettes for him. Throughout these hardships, Mati continued working with S. Shelah and R. Bonnet: on a draft “*On HCO spaces. An uncountable compact  $T_2$  space, different from  $\aleph_1 + 1$ , which is homeomorphic to each of its uncountable closed subspaces*” (1993, 44 pages). Mati refused to be added as a co-author of this work because, he was not—in his view—able to sufficiently improve the results obtained originally by Bonnet and Shelah.

About a year after his retirement in 2015, Rubin was diagnosed with cancer for the second time. As in the first time, he fought the disease, he kept working, and remained optimistic. Just a few days before he passed away Rubin attended a prize ceremony in memory of his colleague H. Gauchman, sharing his old memories of his friend. Little did we know that this was the last time most of us would meet him. The disease and the harsh treatments overcame him just a few days after the ceremony. Mati Rubin was a remarkable, devoted, mathematician and a unique person. He will be greatly missed by his colleagues at Ben Gurion University, his many co-authors, with whom he was always keen to cooperate and share his knowledge, his friends and his family: his widow, children, and seven grand-children (one of whom he has never met).

A meeting in memory of Mati Rubin will be held in Eilat (Israel) on 22–26 April 2018.

ASSAF HASSON and ROBERT BONNET