CHAOS AND TURBULENCE IN SOLAR WIND

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Abstract. Large amplitude waves as well as turbulence has been observed in the interplanetary medium. This turbulence is not understood to the extent that one would like to. By means of techniques of nonlinear dynamical systems, attempts are being made to properly understand the turbulence in the solar wind, which is essentially a nonuniform streaming plasma consisting of hydrogen and a fraction of helium. We demonstrate that the observed large amplitude waves can generate solitary waves, which in turn, because of some propagating solar disturbance, can produce chaos in the medium. The chaotic fields thus generated can lead to anomalously large plasma heating and acceleration.

Unlike the solitary waves in uniform plasmas, in nonuniform plasmas we get accelerated solitary waves, which lead to electromagnetic as well as electrostatic (e.g. ion acoustic) radiations. The latter can be a very efficient source of plasma heating.

1. Introduction

There are plenty of observations of turbulence as well as a variety of low and high frequency waves e.g., Langmuir, ion - acoustic, Alfvén, lower - hybrid and recently detected proton cyclotron waves (Tsurutani et al., 1994) in the solar wind and solar corona. Some of these waves are large amplitude waves. We do understand the observed turbulence to some extent but not to the extent that we would like to.For example, most often, we do not know the source of the turbulence. Voyager 1 (Gurnett et al., 1981) data showed the presence of Langmuir wave packets corresponding to solitary Langmuir waves upstream of Jupiter bow shock. Recently Fast Envelope Sampler of Unified Radio and Plasma Wave (URAP) experiment on Ulysses has also indicated Langmuir wave collapsing events (Kellog et al., 1992). Thiessen and Kellog (1993) claim that the number of collapsing events is much less than the expected ones. However, Buti et al. (1986) had earlier shown that the random density inhomogeneities of the interplanetary medium can be deterrant to the Langmuir wave collapse. Accordingly, it is not surprising that the Ulysses data do not show many collapsing events.

Unlike the fluid turbulence which quickly turns subsonic (from supersonic) away from the source, in the solar wind the Mach number increases with the distance and the large scale fluctuations remain supersonic upto large heliocentric distances. Moreover the Helios mission showed that between 0.3 and 1 AU, the energy spectra of the turbulence evolve with the heliocentric distance (Bavassano et al., 1982) ; the high frequencies decay more rapidly

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with increasing heliocentric distance than the low frequencies. These observations clearly indicate that the nonlinear processes are operational but the question, which specific nonlinear processes, is not yet clear. Ovenden et al. (1983) showed that the nonlinear Alfvén waves in the solar wind can become modulationally unstable ; this leads to Alfvén solitons. They considered a collection of Alfvén solitons and calculated the power spectrum which showed a break point. The calculated radial dependence of this break point was found to be in qualitative agreement with the observations by Bavassano et al. (1982).

Burlaga (1991, 1993 and references therein) has done the detailed analysis of the solar wind data obtained from various space crafts ; he found some signatures of chaos e.g., multifractals and intermittent turbulence in the solar wind.Anomalous heating e.g., solar coronal heating, preferential heating of α particles ($T_\alpha > T_p$; α , p corresponding to α particles and protons respectively) in the solar wind and anomalous acceleration of heavy ions, namely preferential acceleration of α particles over protons ($V_\alpha > V_p$) are the other features which still need to be interpreted properly. In the following sections we will make an attempt to see how chaotic plasma processes in the solar wind can help us in this regard. As an illustrative example, we will consider Alfvén waves since large amplitude Alfvén waves have been observed in the solar wind (Belcher et al., 1969 ;Belcher and Davis, 1971 ; Scarf et al., 1988). Even the Alfvénic turbulence has been observed in the solar wind (Burlaga, 1983 ; Bavassano et al., 1982).

2. Soliton Turbulence

Nonlinear waves (e.g., Langmuir, ion - acoustic, Alfvén waves) under certain conditions have been shown to be modulationally unstable and the modulational instability leads to a variety of solitons which represent coherent turbulence. In uniform plasmas these solitons move with constant speeds but in nonuniform plasmas, like solar wind, the solitons are accelerated or decelerated depending upon whether they are propagating towards increasing or decreasing density gradients (Buti, 1991). However in a nonuniform plasma with drifts (say U) one observes another interesting feature : one finds that for $U > U_c$, the solitons which were accelerated due to density gradients, start getting decelerated and vice versa (Buti, 1992a). Moreover when these accelerated solitons interact with another coexisting mode, they produce very intense coherent radiation (Lakhina et al., 1990). Recently Callebaut and Tsintsadze (1994) have shown the emission of ion - sound waves from the accelerated Alfvén solitons.

Large amplitude Langmuir waves, in realistic systems of more than one dimension, can lead to collapsing solitons i.e., to wave packets with widths of a few Debye lengths. Such wave packets have been observed by Voyager 1 (Gurnett et al., 1981) and Ulysses (Kellog et al., 1992). Ion - acoustic waves, in the presence of strong Langmuir turbulence represented by an assembly of Langmuir solitons, get heavily damped (Yu and Spatschek, 1976). The damping rate increases with the turbulent energy density. The maximum damping rate has been shown to be two orders of magnitude larger compared to the linear Landau damping. This anomalous damping, in turn, can lead to anomalous heating of plasmas.

3. Chaotic Waves

Starting from the multifluid equations, we have derived the nonlinear evolution equations for Alfvén (Buti, 1992b; Verheest and Buti, 1992) as well as lower - hybrid waves (Buti and Lakhina, 1987). By using the Hamiltonion formulation, in this section, we will show how and under what conditions these waves can become chaotic. We will first consider the Alfvén waves and later the lower - hybrid waves.

3.1. Chaotic Alfvén Waves

Let us consider a plasma, consisting of electrons and a variety of ions, which is embedded in a magnetic field $\mathbf{B}_o$ which we take along the x - axis. The evolution equation for the finite amplitude Alfvén waves in such a plasma, in normalised form is given by (Buti, 1992b; Verheest and Buti, 1992) :

$$\frac{\partial \mathbf{b}_\perp}{\partial t} + \alpha \frac{\partial}{\partial x}\left(\mathbf{b}_\perp \mid \mathbf{b}_\perp \mid^2\right) + \mu\left(\hat{e}_x \times \frac{\partial^2 \mathbf{b}_\perp}{\partial x^2}\right) = 0, \quad (1)$$

where $\mathbf{b}_\perp = (b_y, b_z)$,

$$\alpha = \frac{1}{4} \sum_s \frac{\rho_{so}^2(1 - Z_s\delta)}{(\rho_{so} - \gamma_s P_s)},$$

$$\mu = \frac{1}{2}\left[\sum_s \frac{\rho_{so}}{Z_s}\right],$$

$$\delta = \left[\sum_s \frac{\rho_{so} Z_s \gamma_s P_s}{(\rho_{so} - \gamma_s P_s)}\right]\left[\left[\sum_s \frac{Z_s^2 \rho_{so}^2}{(\rho_{so} - \gamma_s P_s)}\right]^{-1}\right],$$

with subscript s representing the different species and o corresponding to equilibrium quantities. For hydrogen plasma, Eq.(1) reduces to the equation obtained by Kennel et. al (1988) with $\mu = 1/2$, $\alpha = 1/4\,(1-\beta)$ and β as the plasma β i.e., the ratio of the kinetic pressure to the magnetic pressure. Eq.

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(1) governs the elliptically polarized Alfvén waves; for circularly polarized waves this reduces to,

$$\frac{\partial b_\pm}{\partial t} + \alpha \frac{\partial}{\partial x}\left(b_\pm \mid b_\pm \mid^2\right) \pm i\mu \frac{\partial^2 b_\pm}{\partial x^2} = 0 \tag{2}$$

where $b_\pm = b_y \pm i b_z$; b_+ for left hand polarized mode and b_- for right hand mode. Throughout this paper we have used the normalised units (Buti, 1992b). Eq.(2) is the Derivative Nonlinear Schrodinger (DNLS) equation, which can be solved exactly (Verheest and Buti, 1992).

Unlike equation (2), Eq.(1) cannot be solved analytically. For any further analysis of Eq. (2) and for investigating the possibility of chaotic behaviour of the system governed by this equation, it is useful to use the *Hamiltonian formulation.*

In a stationary frame of reference, namely,

$$\xi = (x - Vt),$$

Eq.(1) can be rewritten as a set of following two equations:

$$\frac{db_y}{d\xi} = \frac{\partial H}{\partial b_z}, \tag{3a}$$

$$\frac{db_z}{d\xi} = -\frac{\partial H}{\partial b_y}, \tag{3b}$$

where

$$H = \left[\frac{1}{4}\,\bar{\alpha}\,\bar{\mu}\,(b^2 - 1)^2 - \frac{\Lambda}{2}\,(\mathbf{b} - \hat{e}_y)^2\right],$$

is the Hamiltonion of the system and

$$\bar{\alpha} = \alpha/\alpha_o\ ;\ \alpha_o = 1/4\,(1-\beta)\ ,$$

$$\bar{\mu} = \mu_o/\mu\ ;\ \mu_o = 1/2\ ,$$

$$\Lambda = \bar{\mu}\ \left[-\bar{\alpha} + V/\left(\alpha_o\, b_o^2\right)\right],$$

with b_o as a constant (Buti, 1992b). We immediately recognize these equations as Hamilton's equations with b_y and b_z as canonical co-ordinates. Numerical solution of Eq. (3) gives dark, bright and mixed solitons (Buti,1990; Hada et. al 1990) ; these are shown in Fig. 1a . For $\beta < 1$, dark (bright) solitons have right (left) hand polarization. These solitons do not interact with each other and hence there is absolutely no possibility of having a chaos.

From Fig.1a, it is very apparent that the solitary (localized stationary nonlinear) Alfvén waves, left to themselves, will propagate as such for ever but this is somewhat an ideal situation. We would like to find out what happens to these solitary waves when there is some local disturbance e.g., if there is a possibility of having another plasma wave generation or there is some solar activity (say solar flare) which could propagate and interact with the Alfvén waves. To explore this, we will first consider the external driver as a plane wave. In order to make the problem tractable i.e., to be able to reduce the coupled partial differential equations (given by Eq.(1)), modified by the driver, to a set of ordinary differential equations like Eq.(3), we have to further restrict ourselves to the driver which is stationary in the frame of reference of the stationary Alfvén waves. This is indeed a very big restriction; in the latter part of this lecture we will remove this restriction. Eq.(3), in the presence of such a driver, is replaced by (Buti, 1992b),

$$\frac{db_y}{d\xi} = \frac{\partial H}{\partial b_z} + A\,\bar{\mu}\,\cos\theta \tag{4a}$$

$$\frac{db_z}{d\xi} = -\frac{\partial H}{\partial b_y} + A\,\bar{\mu}\,\sin\theta \tag{4b}$$

$$\frac{\partial\theta}{\partial\xi} = \Omega \tag{4c}$$

In Eqs.(4), A is the amplitude of the driver and Ω is its frequency. We will now show that for $A \neq 0$, the dark, bright and the mixed solitons start interacting among themselves and when the driver is sufficiently strong i.e., A is sufficiently large, this interaction can lead to chaos. The reason for the appearance of chaos is rather transparent. In the absence of the driver, our system has two degrees of freedom (cf. Eqs. (3)) but in the presence of the driver, it has three degrees of freedom (cf. Eqs. (4)) and from our knowledge of nonlinear dynamics, we know that the minimum number of degrees of freedom required, for a system to be chaotic, is three and hence the possibility of chaos with the driver.

The solar wind, however, is composed of electrons, protons and α-particles (helium). To study the chaotic processes in solar wind, we derived the evolution equation by using 3 fluid equations instead (Buti, 1992b). For solar wind parameters with 5% helium in abundance, Poincare maps for the left hand polarized driver i.e., $\Omega = -2$, for three-species plasmas are shown in Figs. 1b - 1d. The corresponding results for hydrogen plasma are shown in Figs. 2b - 2d. For $A = 0$, the entire set of Poincare points, originating from a given initial point, remains on the potential contour containing that initial point. For $A = .01$ i.e., for a weak driver, one of the sets of Poincare points, near the bright soliton separatrix, starts to scatter in a limited region of phase space. This leads to the onset of chaos for this set of Poincare points. For somewhat stronger driver e.g., for $A = 0.3$, the two - species plasma is almost chaotic except for the region inside the dark soliton. For both Figs. 1

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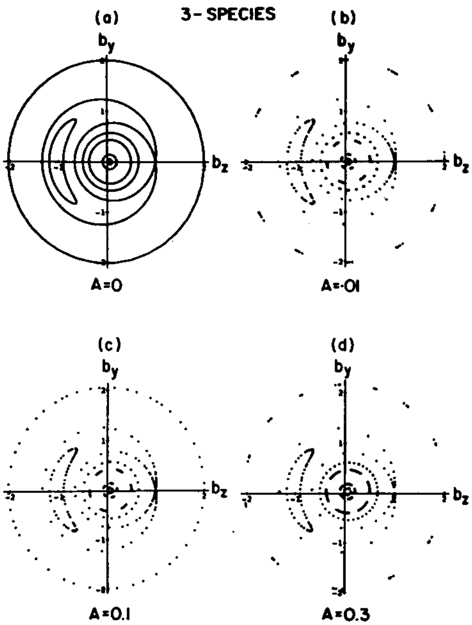
Fig. 1. Poincare maps for driven Hamiltonian system for solar wind at 1 AU, with three species (electrons, protons and α particles) and with $N_\alpha/N_p = .05$. Driver in this case is left hand polarized.



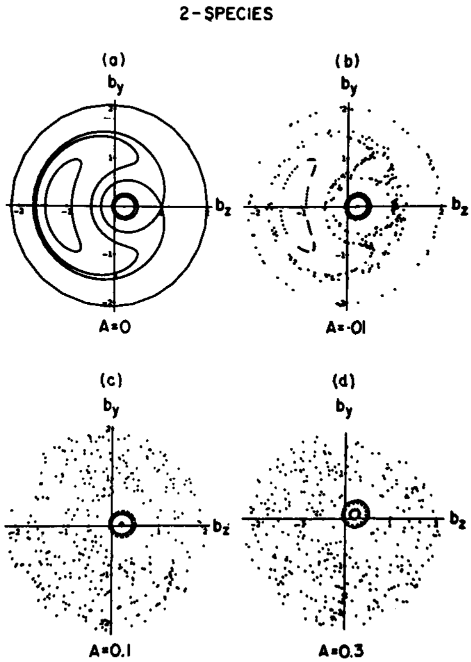
and 2, the driver is the left hand driver and since all the parameters for these figures are the same, Fig. 1 shows the effect of α particles. From comparison of these two figures, it is evident that the chaos is reduced due to the presence of helium - in other words, the threshold for chaos goes up because of heavy ions. Physically this could be interpreted as the inertial stabilisation due to heavy ions.

3.2. Chaotic Lower - Hybrid Waves

Let us consider a three - species (electrons, protons, heavy ions e.g., helium in solar wind) plasma. Lower - hybrid waves, propagating almost transverse to the magnetic field, have been shown to be unstable due to relative streaming between the protons and the heavy ions (Buti and Lakhina, 1987). Moreover Karney and Bers (1977) had shown that the threshold for chaotic lower - hybrid waves is given by :

$$E_t = \left[B_o\, \omega_{LH}^{2/3}\, \Omega_i^{1/3} \,/\, (4ck)\right], \tag{5}$$

Fig. 2. Same as Fig. 1 but for solar wind with only two species i.e., electrons and protons.



where ω_{LH} is the lower - hybrid frequency and Ω_i is the ion cyclotron frequency. If the saturated electric field associated with the unstable lower - hybrid wave exceeds the threshold given by Eq. (5), then these waves can be chaotic and can lead to preferential acceleration and heating of heavy ions (Buti and Lakhina, 1987). They showed that the maximum energy picked up by the heavy ions is proportional to $M_i^{5/3}$; M_i being the mass of the ion. Part of this energy would preferentially accelerate the heavy ions and the rest would be consumed in heating.

4. Anomalous Heating and Particle Acceleration

Alfvén waves, since long, have been considered as a source of plasma heating in general and in particular of solar coronal heating but the heating produced by linear Alfvén waves is insufficient to account for the observed coronal heating. Here we will briefly discuss how chaotic fields can act as very efficient sources of particle acceleration and plasma heating (Buti and Lakhina, 1987; Buti, 1988: Lakhina and Buti. 1995). Lakhina and Buti have shown that the pick-up cometary ions (1987) as well as the hot electrons injected from the solar coronal loops (1995) can lead to unstable lower -

hybrid waves. The saturated electric fields associated with these lower - hybrid waves, in both the cases, exceed the threshold for the generation of chaotic fields. They also showed that these chaotic fields preferentially accelerate the heavier ions. The maximum energy gained by the particles, from the chaotic fields, is proportional to the $5/3rd$ power of the mass of the particle. A part of the maximum energy gained is utilised in anomously heating the plasmas. We are now investigating the contributions of chaotic Alfvén waves, discussed in this paper, towards the anomalous heating of solar corona and anomalous acceleration of solar wind ions. These will be reported in a forthcoming paper.

In a weakly inhomogeneous plasma, like the solar wind, large amplitude Alfvén waves evolve into accelerated solitons. The latter while nonlinearly interacting with Langmuir waves, lead to very intense coherent electromagnetic radiations (Lakhina et al., 1990). These accelerated solitons have recently been shown to emit ion acoustic waves which act as a strong source of heating (Callebaut and Tsintsadze, 1994).Yu and Spatschek (1976) had also looked into the problem of wave - soliton interaction but they had cosidered strong Langmuir turbulence as a collection of Langmuir solitons and had shown that the ion - acoustic waves in the presence of Langmuir turbulence get very heavily damped. This damping, which could be two orders of magnitude larger compared to Landau damping, can very effectively heat the plasma. Efforts are on to determine the quantitative estimates of solar wind heating and acceleration by the turbulent and chaotic fields generated by various large amplitude waves.

5. Conclusions

The techniques, discussed in this paper for the study of chaotic Alfvén and lower - hybrid waves, can be used for the study of chaotic phenomena in any other system. Here we have discussed only the stationary waves with a very special kind of driver. However, this restriction on the driver can be relaxed; we can consider any other driver e.g., a beam representing solar wind or a pulse representing solar burst or a wave packet or any other type but then we cannot reduce our partial differential equation (cf. Eq.(1)) to ordinary differential equations and we have to solve partial differential equations (PDE). The solution of PDE's gives us the time series (time evolution) for b_y and b_z. From the time series, like in the case of observed time series, we have calculated the energy spectrum, correlation dimension and Lyapunov exponents which characterise the phenomena of chaos (Nocera and Buti, 1995). The latter calculations for a variety of drivers are under progress. The different spectra obtained for different drivers will be compared with the observed spectrum to finally conclude about the source of observed Alfvénic turbulence. As mentioned earlier, here we have considered Alfvén and lower - hybrid waves but the procedures outlined here are applicable to any other mode.

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