

## CORRESPONDENCE.

To the Editor of the *Mathematical Gazette*.

SIR,—When professional mathematicians differ, the smaller fry of school teachers and the like must generally be content to await the issue ; but when the difference bears on points which touch or lie close to their work, teachers may perhaps be allowed to ask questions.

Dr. Robb's article on "Partial Failure of Euclid I, 4" in your July number is a case in point.

He presents the matter as one arising out of Time-space theory, but it appears to be in essence a mere question of geometry—a geometry, if I understand rightly, which was developed long before it found the Time-space application which has made it important.

The received doctrine appears to be that in this geometry there are two classes of lines, distinguished from one another by the fact that the lengths of those of one class, it does not matter which, contain the factor  $i$ . The two classes are separated by a pair of special lines, called I believe "isotropic," directed to  $I, J$  the circular points at infinity which in this geometry are real.

These are, as I understand, the lines which Dr. Robb calls "optical lines," and it is in regard to these that my perplexity arises.

The received doctrine says that the lengths of all lines lying along isotropic lines are zero, as indeed seems natural, seeing they belong to both the classes, or to neither.

But Dr. Robb asserts that such lengths are not zero, though he gives no reason for his assertion.

Again, referring to the figure on p. 475, I understand that the received doctrine regards the right-angled triangles  $OQP, OQ'P'$  as congruent, their third sides  $OP, OP'$  being of zero length. Dr. Robb says they are not.

As to the triangles  $RQP, R'Q'P'$ , which again I should have understood to be congruent, Dr. Robb says on p. 476 that they are not—that  $QP, Q'P'$  are in the ratio of  $x_0$  to  $x_0'$ ; while on p. 475 when he was considering the same lines as elements of  $OQP, OQ'P'$  he apparently regards them as equal.

It is very perplexing! May we have some elucidation?—I am, Sir, Your obedient servant,

25th July, 1929.

W. C. FLETCHER.

SIR,—I am sorry that Mr. Fletcher should have found difficulties in my article, but I hope that he may not find these insuperable.

The substance of my statement was that: to take lengths measured along optical lines as being zero, gives a wrong idea of what actually occurs: since lengths can be compared along the same or along parallel optical lines in just the same way as they can along the same or along parallel lines of either of the other types.

By means of a simple construction of parallelograms on the same base and between the same parallels, it is possible in this, as in ordinary Euclidean geometry, to divide any given line into equal parts and, if we select a unit segment, we may construct any multiple of that segment.

This construction applies to optical lines equally well with either of the other types of line and so, to this extent, an optical line has a property similar to the latter, and this property enables us to make use of Euclid's criterion of proportion. The difficulty arises when we try to compare lengths along lines which are not co-directional. In the case of any two inertia lines or any two separation lines, it is possible to give constructions for doing so; and these will be found in my *Theory of Time and Space*; but in the case of optical lines there is no analogous construction: so that the only true congruence in the case of optical lines is co-directional.

As regards Mr. Fletcher's reference to the triangles  $OQP$  and  $OQ'P'$ ; it is evident that the points  $O$ ,  $P$  and  $P'$  lie in one line and, as the figure is drawn,  $P$  is between  $O$  and  $P'$ .

Thus  $OP$  is a part of  $OP'$  and so, unless one is prepared to give up the axiom that "the whole is greater than its part," we must conclude that  $OP'$  is greater than  $OP$ .

Mr. Fletcher's final difficulty arises, so far as I can see, from his confusing the pair of triangles  $ORP$  and  $OR'P'$  with the pair  $QRP$  and  $Q'R'P'$ . It is the former of these pairs of triangles, (not the latter), which have each got two sides equal to  $b$  and  $c$ , while the included hyperbolic angles are equal to  $\log b/c$ .

The third sides of these triangles, that is to say  $OP$  and  $OP'$ , are in the ratio of the corresponding values of  $x_0$ , and I think that, if Mr. Fletcher will examine my paper again, he will find that this is what I assert.

1st August, 1929.

ALFRED A. ROBB.

SIR,—Writing away from home I have not Dr. Robb's article at hand, but, of course, I accept his comment on the last paragraph of my letter and can only express my regret for my careless misreading.

But this does not touch the essential point, viz. that  $OP$ ,  $OP'$  are as I understand of zero length, and as to this—or rather the contradiction between Dr. Robb's and the accepted doctrine—his rejoinder leaves me unenlightened.

8th August, 1929.

W. C. FLETCHER.

#### ERRATA.

P. 430, l. 11. *For* to Marlborough *read* of Marlborough.

P. 464, *Gleaning* 679, l. 1. *For* brining *read* bringing.

P. 510, l. 2 up. *For* Lynnersley *read* Kynnersley.

P. 532, *Gleaning* 687, l. 5. *For*  $\frac{333}{188}$  *read*  $\frac{333}{188}$ .

P. 533, ,, l. 17. *For* "eatablished" *read* "established."

#### NOTICE.

The Editor will be glad to receive *short* passages to add to the collection of *Gleanings* in the Gazette.

Will Mr. Arnold J. W. Keppel be kind enough to send his address?