6 Grand unification

One of the troubling features of the Standard Model is the plethora of coupling constants; overall there are 18, counting θ. It seems puzzling that a theory which purports to be a fundamental theory should have so many parameters. Another is the puzzle of charge quantization: why are the hypercharges all rational multiplets of one another (and, as a result, the electric charges rational multiples of one another)? Finally, the gauge group itself is rather puzzling. Why is it semi-simple rather than simple?

Georgi and Glashow put forward the *grand unification* proposal which answers some of these questions. They suggested that the underlying gauge symmetry of nature is a simple group, broken at some high-energy scale down to the gauge group of the Standard Model. The Standard Model gauge group has rank 4 (there are four commuting generators); $SU(N)$ groups have rank $N-1$. So the simplest group among the $SU(N)$ groups which might incorporate the Standard Model is $SU(5)$. Without any fancy group theory, it is easy to see how to embed $SU(3) \times SU(2) \times U(1)$ in $SU(5)$. Consider the gauge bosons. These are in the adjoint representation of the group. Written as matrices, under infinitesimal space–time independent gauge transformations we have

$$\delta A_\mu = i\omega^a [T^a, A_\mu]. \tag{6.1}$$

The T_as are 5×5 traceless Hermitian matrices; altogether, there are 24 of them. We can then break up the gauge generators in the following way. Writing indices on T^a as $(T^a)_i^j$, the T^as act on the fundamental five-dimensional representation ("the 5") as

$$(T^a)_i^j 5_j. \tag{6.2}$$

So, if we think of the 5 as

$$5 = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ L_1 \\ L_2 \end{pmatrix} \tag{6.3}$$

then the T^as can be broken up into a set of $SU(3)$ generators and a set of $SU(2)$ generators:

$$T^a = \begin{pmatrix} \lambda^a/2 & 0 \\ 0 & 0 \end{pmatrix}, \quad T^i = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^i/2 \end{pmatrix}. \tag{6.4}$$

Here the λ^as are Gell-Mann's $SU(3)$ matrices and the σ^is are the Pauli matrices. There are three commuting matrices among these. The remaining, diagonal, matrix can be taken to be

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$$\tilde{Y} = \frac{1}{\sqrt{60}} \begin{pmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}. \tag{6.5}$$

Finally, there are 12 off-diagonal matrices:

$$\left(X_a^i\right)_j^b = \delta_j^i \delta_a^b \tag{6.6}$$

where $a, b = 1, 2, 3$; $i, j = 1, 2$. These are not Hermitian; they are analogous to the raising and lowering operators in $SU(2)$. One can readily form Hermitian linear combinations. The associated vector mesons must be very heavy; they mediate B-violating processes, as in Fig. 6.1. These can lead, for example, to $p \to \pi^0 e^+$.

We want to claim that $\tilde{Y}$ is proportional to the ordinary hypercharge and determine the proportionality constant. To do this, we consider, not the 5 but the $\bar{5}$ and make the identification

$$\bar{5} = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ L_1 \\ L_2 \end{pmatrix}. \tag{6.7}$$

Now, the generators of $SU(5)$ acting on the $\bar{5}$ are $-T^{aT}$. So we can read off immediately that $Y = \sqrt{60}\tilde{Y}/3$. Since the gauge groups are unified in a single group, the gauge couplings are all the same, so we can compute the Weinberg angle. Calling g the $SU(5)$ coupling,

$$g\tilde{Y} = \frac{g'}{2} Y, \tag{6.8}$$

where g' is the hypercharge coupling of the Standard Model. From this, $g^2 = (5/3)g'^2$. The Weinberg angle is given by

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8}. \tag{6.9}$$

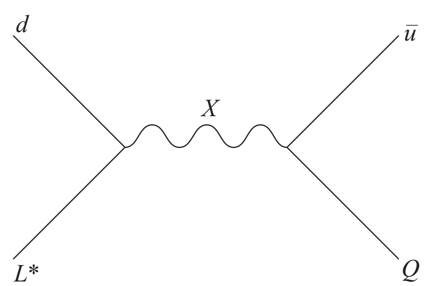

Fig. 6.1 The exchange of heavy vector particles in GUTs violates B and L. It can lead to processes such as $p \to \pi^0 e^+$.

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So we have two dramatic predictions, if we assume that the Standard Model is unified in this way:

1. the $SU(3)$ and $SU(2)$ gauge couplings are equal;
2. the Weinberg angle satisfies $\sin^2\theta_W = 3/8$.

Before assessing these predictions, let us first figure out where we would put the rest of the quarks and leptons. In a single generation of the Standard Model, there are 15 fields. The group $SU(5)$ has a ten-dimensional representation, the antisymmetric product of two 5s. It can be written as an antisymmetric matrix, 10_{ij}. If i and j are both $SU(3)$ indices, we obtain a $(\bar{3}, 1)_{-4/3}$ of $SU(3)$. If one is an $SU(3)$ and one an $SU(2)$ index, we obtain a $(3, 2)_{1/3}$. If both are $SU(2)$ indices, we obtain a $(1, 1)_2$. Here the subscripts denote the ordinary hypercharge, related to $\tilde{Y}$ as above. These are just the quantum numbers of the quark doublet Q, of $\bar{u}$ and of $\bar{e}$. As a matrix,

$$10 = \begin{pmatrix} 0 & \bar{u}^3 & -\bar{u}^2 & Q_1^1 & Q_1^2 \\ -\bar{u}^3 & 0 & \bar{u}^1 & Q_2^1 & Q_2^2 \\ \bar{u}^2 & -\bar{u}^1 & 0 & Q_3^1 & Q_3^2 \\ -Q_1^1 & -Q_2^1 & -Q_3^1 & 0 & \bar{e} \\ -Q_1^2 & -Q_2^2 & -Q_3^2 & -\bar{e} & 0 \end{pmatrix}. \tag{6.10}$$

So, a single generation of quarks and leptons fits neatly into a $\bar{5}$ and 10 of $SU(5)$.

6.1 Cancelation of anomalies

An anomaly in a gauge symmetry would represent a breakdown of gauge invariance. The consistency of gauge symmetries rests, however, on gauge invariance. For example, to demonstrate that such theories are both unitary and Lorentz invariant we have used different gauges. The cancelation of anomalies is crucial, and the absence of anomalies in the Standard Model is surely no accident.

It is not hard to check that in $SU(5)$ the anomaly of the $\bar{5}$ cancels that of the 10. In general, the anomalies in a gauge theory are proportional to d_{abc}, where

$$\{T_a, T_b\} = d_{abc} T_c. \tag{6.11}$$

One can organize the anticommutator above in terms of the various types of generator, for example $SU(3)$, $SU(2)$, $U(1)$, and the off-diagonal generators, which transform as $(3, 2)$ of $SU(3) \times SU(2)$, and then check each class. We leave the details for the exercises.

6.2 Renormalization of couplings

If we are going to describe the Standard Model, $SU(5)$ must break at some high-energy scale to $SU(3) \times SU(2) \times U(1)$. Above this scale, the full $SU(5)$ symmetry holds to a

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good approximation, and all couplings renormalize in the same way. Below this scale the couplings renormalize differently. We can write down the equations for the renormalization of the three separate couplings:

$$\alpha_i^{-1}(\mu) = \alpha_{\text{gut}}^{-1}(M_{\text{gut}}) + \frac{b_0^i}{4\pi} \ln \frac{\mu}{M_{\text{gut}}}. \tag{6.12}$$

We can calculate the beta functions at one loop starting with the usual formula:

$$b_0 = \frac{11}{3} C_A - \frac{4}{3} c_{\text{f}}^{(i)} N_{\text{f}}^{(i)} - \frac{1}{3} c_\phi^{(i)} N_\phi^{(i)}, \tag{6.13}$$

where $N_{\text{f}}^{(i)}$ is the number of fermions in the ith representation; $N_\phi^{(i)}$ is the number of scalars. For $SU(N)$ $C_{\text{A}} = N$ and, for fermions or scalars in the fundamental representation, $c_{\text{f}} = c_\phi = 1/2$.

For the $SU(3)$ and $SU(2)$ couplings the beta function coefficients b_0^i are readily computed. For $U(1)$, we need to remember the relative normalization computed above:

$$b_0^2 = \frac{181}{6}, \quad b_0^3 = 7, \quad b_0^1 = \frac{61}{15}. \tag{6.14}$$

We can run these equations backwards. The $SU(2)$ and $U(1)$ couplings are the best measured, so it makes sense to start with these and run them up to the unification scale. This determines α_{gut} and M_{gut}. We can then predict the value of the $SU(3)$ coupling at, say, M_Z. One finds that the unification scale, M_{gut}, is about 10^{15} GeV and that α_3 is off by about seven standard deviations. In the exercises you will have the opportunity to perform this calculation in detail. We will see later that low-energy supersymmetry greatly improves this.

6.3 Breaking to $SU(3) \times SU(2) \times U(1)$

In $SU(5)$, it is relatively easy to introduce a set of Higgs fields which break the gauge symmetry down to $SU(3) \times SU(2) \times U(1)$. Consider a Hermitian scalar field Φ in the adjoint representation. Writing Φ as a matrix, we have the transformation law

$$\delta\Phi = \omega^a [T^a, \Phi]. \tag{6.15}$$

Suppose that the minimum of the Φ potential lies at a point where

$$\Phi = v\tilde{Y}. \tag{6.16}$$

Then the $SU(3)$, $SU(2)$ and $U(1)$ generators all commute with $\langle\Phi\rangle$, but those for the X bosons do not.

Consider the most general $SU(5)$-invariant potential:

$$V = -m^2 \operatorname{Tr} \Phi^2 + \frac{\lambda}{4} \operatorname{Tr} \Phi^4 + \frac{\lambda'}{4} (\operatorname{Tr} \Phi^2)^2. \tag{6.17}$$

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One can find the minimum of this potential by first using an $SU(5)$ transformation to diagonalize Φ, obtaining

$$\Phi = \text{diag}(a_1, a_2, a_3, a_4, a_5). \tag{6.18}$$

The potential is a function of the a_is, which one wants to minimize subject to the constraint of vanishing trace. This can be done by using a Lagrange multiplier.

To establish that one has a local minimum of the form Eq. (6.16), one can proceed more simply. Write the potential as a function of v:

$$V = -\frac{1}{2}m^2 v^2 + \frac{a\lambda}{4} + \frac{b\lambda'}{4}v^4, \tag{6.19}$$

where $a = 7/120,\ b = 1/4$. Then the extremum with respect to v occurs for

$$v = \frac{\mu}{\sqrt{a\lambda + b\lambda'}}. \tag{6.20}$$

To establish that this is a local minimum, we need to show that the eigenvalues of the scalar mass-squared matrix are all positive. We can investigate this by considering small fluctuations about the stationary point. This point preserves $SU(3)\times SU(2)\times U(1)$. Writing $\Phi = \langle\Phi\rangle + \delta\Phi$, $\delta\Phi$ can be decomposed under $SU(3)\times SU(2)\times U(1)$ as follows:

$$\delta\Phi = (1,1) + (8,1) + (1,3) + (3,2) + (\bar{3},2). \tag{6.21}$$

The point (6.20) is certainly stationary; because of the symmetry, only the $(1,1)$ term can appear linearly in the potential, and it is this piece whose minimum we have just found. To establish that the point (6.20) is in fact a local minimum, one needs to show that the quadratic terms in the fluctuations are all positive. This is done in the exercises.

6.4 $SU(2)\times U(1)$ breaking

In addition to the adjoint, it is necessary to include a 5 representations of the Higgs H in order to break $SU(2)\times U(1)$ down to the $U(1)$ of electromagnetism and to give mass to the quarks and leptons. The Higgs has the form

$$H = \begin{pmatrix} H_\mathrm{c} \\ H_\mathrm{d} \end{pmatrix}, \tag{6.22}$$

where H_c is a color triplet of scalars and H_d is the ordinary Higgs doublet. For H one might have been tempted to write a potential of the form

$$V(H) = -\mu^2|H|^2 + \frac{\lambda}{4}|H|^4. \tag{6.23}$$

However, this would lead to a number of difficulties. Perhaps the most important is that, when included in the larger theory with the adjoint field Φ, this potential has too much symmetry; there is an extra $SU(5)$ which would lead to an assortment of unwanted Goldstone bosons. At the same time the scale μ must be of order the scale of electroweak symmetry breaking (as long as λ is not too much larger than unity). So, the Higgs triplets

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will have masses of order the weak scale. But if the doublet couples to quarks and leptons, the triplet will have *baryon- and lepton-number-violating* couplings to the quarks and leptons. So the triplet must be very massive.

Both problems can be solved if we couple Φ to H. The allowed couplings include:

$$V_{\Phi H} = \Gamma H^* \Phi H + \lambda' H^* H \,\mathrm{Tr}\, \Phi^2 + \lambda'' H^* \Phi^2 H. \tag{6.24}$$

If we carefully adjust the constants Γ, λ', λ'' and μ^2, we can arrange that the doublets are light and the triplets are heavy. For example, if we choose $\lambda = \lambda' = 0$ and $\mu^2 = -3(\Gamma/\sqrt{60})v - \epsilon$ then the Higgs doublets have mass-squared $-\epsilon$ in the Lagrangian, while the triplets have mass of order $M_{\rm gut}$. This tuning of parameters, which must be performed in each order of perturbation theory, provides an explicit realization of the hierarchy problem.

Turning to the fermion masses, we are led to an interesting realization: not only does grand unification make predictions for the gauge couplings, it can predict relations among fermion masses as well. The gange group $SU(5)$ permits the following couplings:

$$\mathcal{L}_y = y_1 \epsilon_{ijklm} H^i 10^{jk} 10^{lm} + y_2 H_i^* \bar{5}_j 10^{ij}. \tag{6.25}$$

Here the ys are matrices in the space of generations. When H acquires an expectation value, it gives mass to the quarks and leptons. The first coupling gives mass to the up-type quarks. The second coupling gives mass to both the down-type quarks and the leptons. If we consider only the heaviest generation, we then have the tree level prediction

$$m_b = m_\tau. \tag{6.26}$$

This prediction is off by a factor 3 but, like the prediction of the coupling constant, it can be corrected by renormalization to roughly the observed amount. For the lightest quarks and leptons the prediction fails. However, unlike the unification of gauge couplings, such predictions can be modified if there are additional Higgs fields in other representations. In addition, for the lightest fermions, higher-dimensional operators, suppressed by powers of the Planck mass, can make significant contributions to masses. In supersymmetric grand unified theories, the ratio of the GUT scale to the Planck scale is about 10^{-2}, whereas the lightest quarks and leptons have masses four orders of magnitude below the weak scale. We will postpone a numerical study of these corrections since the simplest $SU(5)$ theory does not correctly predict the values of the coupling constants, and will return to this subject when we discuss supersymmetric grand unified theories, which do successfully predict the observed values of the couplings.

6.5 Charge quantization and magnetic monopoles

While we must postpone success with the calculation of the unified couplings to our chapters on supersymmetry, we should pause and note two triumphs. First, we have a possible explanation for one of physics' greatest mysteries: why is electric charge quantized? Here it is automatic; electric charge, an $SU(5)$ generator, is quantized, just as color and isospin are quantized.

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However, Dirac long ago offered another explanation of electric charge quantization: magnetic monopoles. He realized that the consistency of quantum mechanics demands that if even a single monopole exists in the universe, electric charges must all be integer multiples of a fundamental charge. So we might suspect that magnetic monopoles are hidden somewhere in this story. Indeed they are; this are discussed in Chapter 7.

6.6 Proton decay

We have discussed the dimension-six operators which can arise in the Standard Model and violate baryon number. Exchanges of the X bosons generate operators such as

$$\frac{g^2}{M_X^2} Q\sigma_\mu \bar{u}^* Q\sigma^\mu \bar{e}^*. \tag{6.27}$$

This leads to the decay $p \to \pi^0 e^+$. In this model, one predicts a proton lifetime of order 10^{28} years if $M_{\rm gut} \approx 10^{15}$ GeV. The current limit on this decay mode is 5×10^{33} years. We will discuss the situation for supersymmetric models later.

The realization that baryon-number violation is likely in any more fundamental theory opens up a vista on a fundamental question about nature: why is there more matter than antimatter in the universe? If, at some very early time, there were equal amounts of matter and antimatter then, if baryon number is violated, one has the possibility of producing an excess. Other conditions must be satisfied as well; we will describe this in the chapter on cosmology.

6.7 Other groups

While $SU(5)$ may in some respects be the simplest group for unification, once one has set off in this direction there are many possibilities. Perhaps the next simplest is unification in the group $O(10)$. As $O(10)$ has rank 5, there is one extra commuting generator; presumably this symmetry must be broken at some scale. More interesting, though, is the fact that a single generation fits neatly into an irreducible representation: the 16. The group $O(10)$ has an $SU(5)$ subgroup, under which the 16 decomposes as a $10 + \bar{5} + 1$. The singlet has precisely the right Standard Model quantum numbers – none – to play the role of the right-handed neutrino in the seesaw mechanism; see below Eq. (4.17).

We will not review the group theory of O groups in detail, but we can describe some of the important features. We will focus specifically on $O(10)$, but much of the discussion here is easily generalized to other groups. The generators of $O(10)$ are 10×10 antisymmetric matrices. There are 45 of these. We are particularly interested in how they transform under the Standard Model group. The embedding of the Standard Model in $SU(5)$, as we have learned, is very simple, so a useful way to proceed to understand $O(10)$ is to find its $SU(5)$ subgroup.

One way to think of $O(10)$ is as the group of rotations of ten-dimensional vectors. Call the components of such a vector x^A, $A = 1, \ldots, 10$. Transformations in $SU(5)$ are "rotations" of complex five-dimensional vectors z^i. So, we define

$$z^1 = x^1 + ix^2, \quad z^2 = x^3 + ix^4, \quad z^3 = x^5 + ix^6 \tag{6.28}$$

and so on. With this correspondence it is easy to see that there is a subgroup of $O(10)$ transformations that preserves the product $z \cdot z'^*$. This is the $SU(5)$ subgroup of $O(10)$.

From our construction, it follows that the 10 of $O(10)$ transforms as a $5 + \bar{5}$ of $SU(5)$. We can determine the decomposition of the adjoint by writing

$$A^{AB} = A^{i\bar{i}} + A^{ij} + A^{\bar{i}\bar{j}}. \tag{6.29}$$

The labeling here is meant to indicate the types of complex index that the matrix A can carry. The first term is just the 24-dimensional representation of $SU(5)$, plus an additional singlet. This singlet is associated with a $U(1)$ subgroup of $O(10)$, which rotates all the objects with i-type indices by one phase and all those with $\bar{i}$ type indices by the opposite phase. Note that A^{ij} is antisymmetric in its indices; in our study of $SU(5)$ we learned that this is the 10 representation. We can take it to carry charge 2 under the $U(1)$ subgroup. Then $A^{\bar{i}\bar{j}}$ corresponds to the $\overline{10}$ representation, with charge -2. This accounts for all 45 fields.

But where is the 16-dimensional representation? We are familiar, from our experience with ordinary rotations in three and (Euclidean four) dimensions as well as from the Lorentz group, with the fact that O groups may have spinor representations. To construct these we need to introduce the equivalent of the Dirac gamma matrices Γ, satisfying

$$\{\Gamma^I, \Gamma^J\} = 2\delta^{IJ}. \tag{6.30}$$

It is not hard to construct explicit matrices which satisfy these anticommutation relations but there is a simpler approach, which also makes the $SU(5)$ embedding clear. The anticommutation relations are similar to the relations for fermion creation and annihilation operators. So, define

$$a^1 = \frac{1}{2}(\Gamma^1 + i\Gamma^2), \quad a^2 = \frac{1}{2}(\Gamma^3 + i\Gamma^4) \tag{6.31}$$

and so on, and similarly for their complex conjugates. Note that the a^is form a 5 of $SU(5)$, with charge $+1$ under the $U(1)$. These operators satisfy the algebra

$$\{a^i, a^{\bar{j}}\} = \delta^{i\bar{j}}. \tag{6.32}$$

These are the anticommutation relations for five pairs of fermion creation–annihilation operators. We know how to construct the corresponding "states", i.e. the representations of the algebra. We define a state $|0\rangle$ annihilated by the a^is. Then there are five states created by the action of $a^{\bar{i}}$ on this state:

$$\bar{5}_{-1} = a^{\bar{i}}|0\rangle. \tag{6.33}$$

The main symbol $\bar{5}$ indicates the $SU(5)$ representation and the subscript indicates the $U(1)$ charge. We could now construct the states obtained with two creation operators, but let us

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construct the states built using an odd number:

$$10_{-3} = a^{\bar{i}} a^{\bar{j}} a^{\bar{k}} |0\rangle, \quad 1_{-5} = a^{\bar{1}} a^{\bar{2}} a^{\bar{3}} a^{\bar{4}} a^{\bar{5}} |0\rangle. \tag{6.34}$$

We have indicated that the first representation transforms like a 10 of $SU(5)$, while the second transforms like a singlet.

The states which involve even numbers of creation operators transform like a 5, a $\overline{10}$, and a singlet. Why do we distinguish these two sets? Remember, the goal of this construction is to obtain *irreducible* representations of the group $O(10)$. As in the Dirac theory, we can construct the symmetry generators from the Dirac matrices,

$$S^{IJ} = \frac{i}{4}[\Gamma^I, \Gamma^J]. \tag{6.35}$$

These, too, can be decomposed on a complex basis, like A^{IJ}. But, as for the usual Dirac matrices, there is another Γ matrix that we can construct, which is the analog of Γ^5: Γ^{11}. This matrix anticommutes with all the Γs, and so with the a^is. Thus the states with even numbers of creation operators are eigenstates with eigenvalue $+1$ under Γ^{11}, while those with odd numbers are eigenstates with eigenvalue -1. Since Γ^{11} commutes with the symmetry generators, these two representations are irreducible.

A similar construction works for other groups. When we come to discuss string theories in ten dimensions, we will be especially interested in the representations of $O(8)$. Here the same construction yields two eight-dimensional representations, denoted 8 and $8'$.

The embedding of the states of the Standard Model in $O(10)$ is clear, since we already know how to embed them in a $\bar{5}+10$ of $SU(5)$. But what of the other state in the 16? This is a Standard Model singlet. We do not yet have a candidate in the particle data book for this. However, there are two observations we can make. First, the symmetries of the Standard Model do not forbid a mass for this particle. What does forbid a mass is the extra $U(1)$. So, if this symmetry is broken at very high energies, perhaps with the initial breaking of the gauge symmetry, this particle can gain a large mass. We will not explore the possible Higgs fields in $O(10)$ but, as in $SU(5)$, there are many possibilities and the $U(1)$ can readily be broken. Second, this particle has the right quantum numbers to couple to the left-handed neutrino of the Standard Model. So this particle can naturally lead to a "seesaw" neutrino mass. This mass might be expected to be of order some typical Yukawa coupling squared divided by the unification scale. It is also possible that this extra $U(1)$ is broken at some lower scale, yielding a larger value for the neutrino mass.

Suggested reading

There is any number of good books and reviews on the subject of grand unification. The books by Ross (1984), Mohapatra (2003) and Ramond (1999) all treat the topics introduced in this chapter in great detail. The reader will find his or her interest in this topic increases after studying some aspects of supersymmetry.

Exercises

(1) Verify the cancelation of anomalies between the $\bar{5}$ and 10 representations of $SU(5)$.

(2) Establish the conditions for the solution of Eq. (6.16) to be a local minimum of the potential.

(3) Perform the calculation of coupling unification in the $SU(5)$ model. Verify Eqs. (6.14) for the $SU(3)$, $SU(2)$ and $U(1)$ beta functions. Start with the measured values of the $SU(2)$ and $U(1)$ couplings, being careful about the differing normalizations in the Standard Model and in $SU(5)$. Compute the value of the unification scale (the point where these two couplings are equal); then determine the value of α_3 at M_Z. Compare with the value given by the Particle Data Group. You need only study the equations to one-loop order. In practice, two-loop corrections, as well as threshold effects and higher-order corrections to the beta function, are often included.

(4) Add to the Higgs sector of the $SU(5)$ theory a set of scalars in the 45 representation. Show that in this case all the quark masses are free parameters.