Impact of Gravitational Lensing on Cosmology
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Dark Energy and CMB Bispectrum

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Abstract. We consider the CMB bispectrum signal induced by structure formation through the correlation between the Integrated Sachs-Wolfe and the weak lensing effect. We investigate how the bispectrum knowledge can improve our knowledge of the most important cosmological parameters, focusing on the dark energy ones. The bispectrum signal arises at intermediate redshifts, being null at present and infinity, and is characterized by a large scale regime (dominated by linear dynamics of cosmological perturbation) and a small scale one (dominated by density perturbations in a non-linear regime); on the other hand, the effect induced by dark energy on the power spectrum is mostly geometrical and imprinted at redshift close to the present. Because of this, the knowledge of power spectrum and bispectrum yield two complementary informations at very different cosmological epochs, particularly suitable to extract informations about the onset of the cosmic acceleration and dark energy properties that provide it. In order to quantify how much the bispectrum can help the power spectrum in constraining the dark energy parameters, we choose a fiducial model on a three-dimensional space including the following dark energy parameters: dark energy density Ω_V; dark energy equation of state today w_0 and dark energy equation of state in the past w_∞ ($w_\infty - w_0$ is related to the first derivative of equation of state). Then we simulate a likelihood analysis showing how contour levels become narrower when bispectrum is included. Preliminary results suggest a consistent improvement on the estimation of dark energy abundance and on dynamical properties of the equation of state. This indicates that the knowledge of the bispectrum in future high resolution and high sensitivity CMB observations could yield a substantial improvement with respect to the traditional analysis based on the power spectrum only.

1. Introduction

The combination of several independent cosmological datasets, namely type Ia supernovae (Riess *et al.* 1998, Perlmutter *et al.* 1999), CMB (Spergel *et al.* 2003) and large scale structure (see e.g. Dodelson *et al.* 2002) indicate that the universe is presently accelerating. The dark energy responsible for the acceleration appears to be a fraction of about 73% of the cosmic energy density today. The main properties of the dark energy are described in terms of its equation of state $w = p/\rho$, i.e. the ratio between pressure and energy density; its value, for a pure cosmological constant, is -1. A current experiments indicate that the present value of w should be in the range $w_0 < -0.78$ (Spergel *et al.* 2003). The cosmological constant as an explanation to cosmic acceleration has two well-known problems: the coincidence problem (why cosmological constant density and matter density are comparable today?) and the fine tuning problem (i.e. why cosmological constant is 123 orders of magnitude less than Planck scale?). To solve the latter problem, a dynamical scalar field, known as Quintessence (Peebles & Ratra 2003), has been introduced as a minimal extension of the cosmological constant; the dark energy equation of state gets dynamical alleviating the fine tuning problem. In most models the dark energy equation of state can be easily parameterized with only two parameters: w_0 and its first derivative with respect to the scale factor (Linder 2003). The next challenge

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in cosmology is to constrain the time evolution of w; this can be done with future SNIa observations and future CMB experiments like Planck (Balbi *et al.* 2003). As we shall see in the next section, the CMB power spectrum alone is limited to constrain dark energy. Here we study the improvement which might be achieved by taking into account the non-Gaussian distortion induced by the correlation between weak lensing and Integrated Sachs-Wolfe effect (ISW). Such signal is suitable studied in the higher order statistic of CMB anisotropies. We choose here the CMB bispectrum as an estimator of third order statistics (see e. g. Giovi, Baccigalupi & Perrotta 2003). The weak lensing effect on CMB anisotropies has been studied (see e. g. Komatsu & Spergel 2001 and references therein) and the third order statistics, the bispectrum, has been used to constrain the effective dark energy equation of state by Verde & Spergel (2002).

2. Removing the distance degeneracy with CMB bispectrum

The main problem of the CMB power spectrum in studying the dark energy is the degeneracy that affects the distance of last scattering surface with respect the dark energy abundance and, in particular, its equation of state. Its variation produces a change to the distance at last scattering surface (see e. g. Baccigalupi *et al.* 2002); unfortunately such distance is degenerate with respect the main dark energy parameters. This can be easily understood writing the formula of the comoving distance to the last scattering surface (we restrict our analysis to the flat case and we neglect the radiation contribution):

$$r(z_{lss}) = \frac{c}{H_0}\int_0^{z_{lss}} \frac{dz}{\sqrt{\Omega_{0M}(1+z)^3 + \Omega_V e^{f(z)}}}. \tag{2.1}$$

In the previous equation c is the speed of light, H_0 is the Hubble constant today, z_{lss} is the redshift of last scattering surface, Ω_{0M} is the matter density today, $\Omega_V = 1 - \Omega_{0M}$ (because we consider only the flat case) is the dark energy density and $f(z)$ depends on the equation of state of dark energy $w(z)$ and is defined as

$$f(z) = 3\int_0^{z'} dz' \frac{1+w(z')}{1+z'}. \tag{2.2}$$

Analyzing eq. (2.1) and (2.2), we can see that the time dependence on equation of state is washed out by two redshift integrations. In most models the dark energy equation of state can be parameterized with the following relation (Linder 2003)

$$w(z) = w_0 + (w_\infty - w_0)\frac{z}{1+z}, \tag{2.3}$$

where w_0 and w_∞ are respectively its present and the asymptotic values. The difference $(w_\infty - w_0)$ represents the time-variation of dark energy equation of state.

Several different combinations of these dark energy parameters can produce the same comoving distance to the last scattering surface; for example a comoving distance of about 13900 Mpc can be obtained from these three sets of dark energy parameters: $(\Omega_V = 0.73, w_0 = -1, w_\infty = -1)$; $(\Omega_V = 0.735, w_0 = -0.93, w_\infty = -0.89)$ and $(\Omega_V = 0.74, w_0 = -1, w_\infty = -0.59)$. The net effect of this distance degeneracy is reflected in the CMB power spectrum; different cosmological models with the same $r_{z_{lss}}$, will produce very similar power spectra (see figure 1, right panel). Together with degeneracy in the projection effect, there is another physical motivation for which the CMB power spectrum is limited in its capability to constrain the dark energy: the power spectrum is mostly injected at decoupling, and at that time the dark energy density was negligible with respect to the matter density and couldn't produce remarkable signatures in the CMB.

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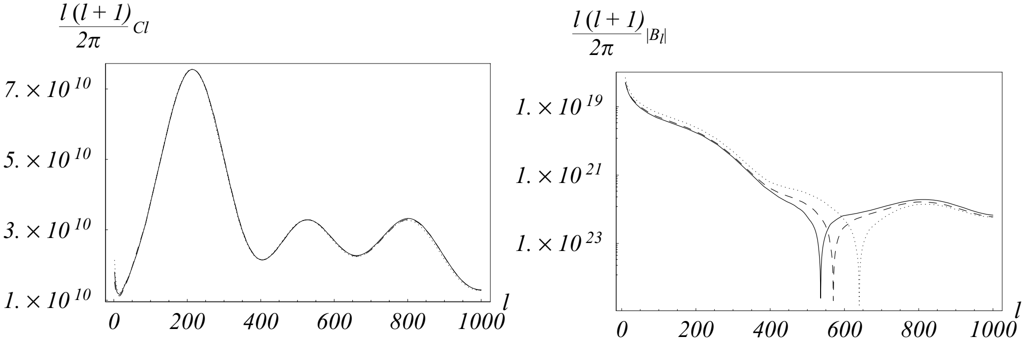

Figure 1. CMB power spectra (left panel) and absolute value of equilateral bispectra (right panel) for models with same comoving distance of last scattering surface but different values of dark energy parameters. Solid line: $(\Omega_V = 0.73, w_0 = -1, w_\infty = -1)$. Dashed line: $(\Omega_V = 0.735, w_0 = -0.93, w_\infty = -0.89)$. Dotted line: $(\Omega_V = 0.74, w_0 = -1, w_\infty = -0.59)$. Notice that the three models are fully degenerate with the power spectrum while the degeneracy is removed with the equilateral bispectrum.

A way to include the CMB sensitivity on the dark energy is to consider the signatures in the CMB produced by the correlation between the Integrated Sachs-Wolfe effect and the weak lensing on the CMB. The ISW takes into account the Rees-Sciama effect that arises when the non linear growth of structures is included. The ISW effect affects the CMB photons with a reddening; the photon acquires a blueshift when it falls down into the potential well of growing structures and it acquires a redshift when it climbs out, but these two contributions are not balanced because the perturbations change in time. The gravitational lensing effect is the well-known deflection of light due to the gravitational potential of clump of matter. Both effects arise at the epoch of structure formation, thus we can try to exploit them to alleviate the distance degeneracy. The correlation between ISW and weak lensing induce a non vanishing power in the CMB higher order statistics, which we study it considering the CMB bispectrum. The latter is the harmonic transform of three point correlation function and it is null, within cosmic variance, if the CMB anisotropies are Gaussian. Since the ISW and the weak lensing are produced by the same physical entity (growing perturbations in the matter distribution), these secondary anisotropies are correlated and induce a non-vanishing bispectrum with exceeds the cosmic variance.

3. Integrated Sachs-Wolfe and weak lensing induced CMB bispectrum

When we consider the ISW and weak lensing effect, the CMB anisotropies in a direction $\hat{n}$ in the sky can be decomposed as

$$\Theta(\hat{n} + \vec{\alpha}) \simeq \Theta(\hat{n}) + \vec{\nabla}\Theta \cdot \vec{\alpha} \tag{3.1}$$

where Θ includes the primordial and ISW anisotropy contributions, and the last term is the contribution from the weak lensing re-mapping; $\vec{\alpha}$ is the deflection angle. Following Verde & Spergel (2002) and expanding eq. (3.1) in spherical harmonics, from the general bispectrum definition $b^{m_1 m_2 m_3}_{l_1 l_2 l_3} = a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$, we can build the quantity

$$B_{l_1 l_2 l_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} b^{m_1 m_2 m_3}_{l_1 l_2 l_3}$$

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Table 1. Position and shift of multipole l_0 of bispectrum zero-crossing varying the main cosmological parameters.

Parameters	Lower	Higher	Δl_0
$-1 \leqslant w_0 \leqslant -0.8$	418	482	+66
$0.60 \leqslant \Omega_V \leqslant 0.86$	324	>1000	>+676
$0.020 \leqslant \Omega_b h^2 \leqslant 0.028$	404	420	+16
$0.64 \leqslant h \leqslant 0.80$	492	360	-132
$0.80 \leqslant n_s \leqslant 1.12$	446	388	-58

$$\simeq \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \cdot \qquad (3.2)$$
$$\cdot \; \frac{l_1(l_1+1) - l_2(l_2+1) + l_3(l_3+1)}{2} C_{l_1}^P Q(l_3) + 5P. \; ,$$

where $5P.$ indicates the permutations over the three multipoles, the parenthesis are the Wigner's 3J symbols, C_l^P is the primordial CMB power spectrum, and

$$Q(l) \equiv < (a_{lm}^{lens})^* a_{lm}^{ISW} > \simeq 2 \int_0^{z_{lss}} dz \frac{r(z_{lss}) - r(z)}{r(z_{lss}) r^3(z)} \left[\frac{\partial P_\Psi(k,z)}{\partial z} \right]_{k=\frac{l}{r(z)}} , \qquad (3.3)$$

is the correlation between lensing and ISW. In eq. (3.3) $P_\Psi(k,z)$ is the gravitational potential power spectrum; to evaluate the non linear contribution to density power spectrum we have used the existing semi-analytical approach (Ma *et al.* 1999).

The asymptotic redshift behavior of the integrand of eq. (3.3), for $z \to 0$ and $z \to \infty$, is vanishing (Giovi, Baccigalupi & Perrotta 2003); in fact, fixing the multipole, at low redshift the gravitational potential power spectrum probes infinite wavenumbers where the power is vanishing, while at high redshift the gravitational potential is constant since the universe approach the standard CDM. Therefore, the bispectrum signal is acquired at intermediate redshifts only, and is expected to reflect the cosmological expansion rate at that epoch. The dark energy domination occurs approximately at the same time, and therefore the bispectrum can be used as a tool to investigate the dark energy properties and to alleviate the distance degeneracy (Giovi, Baccigalupi & Perrotta 2003). This can be seen clearly in figure 1 where we compare the CMB power spectrum with the equilateral bispectrum ($l_1 = l_2 = l_3$) for three dark energy models with the same comoving distance at last scattering surface: the degeneracy is removed at the bispectrum level; we describe in the next section the shape of bispectrum curves.

4. Bispectrum features and likelihood analysis

The bispectrum has some peculiar features that can be used to discriminate between different cosmological models. Plotting the absolute value of bispectrum we have a *cusp*, for which the bispectrum is vanishing, and its position depends on cosmological model (see right panel in figure 1). The position of this zero-crossing point is the value of the multipole l_0 for which the integrand of $Q(l)$ is null as shown Verde & Spergel (2002): the positive contribution (due to linear growth) balances exactly the negative one (due to non linear growth). Another relevant feature that must be taken into account is the amplitude of the bispectrum in the linear part ($l < l_0$): different cosmological models produce different amplitudes in the linear regime. The linear power affect l_0: the higher is the power in the linear regime ($l < l_0$), the higher is the value of l_0. In table 1 we analyze the variation of l_0 with respect to the main cosmological parameters: dark energy

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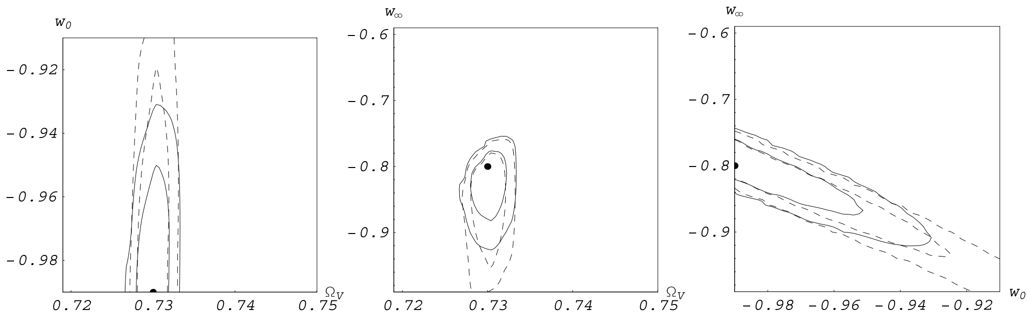

Figure 2. Likelihoods confidence levels at 68% (innermost contours) and 95%(outermost contours) for power spectrum only (dashed line) and for power spectrum and bispectrum (solid line). From left to right: marginalization over w_∞, marginalization over w_0, marginalization over Ω_V. The filled dot is our fiducial model.

equation of state today w_0, dark energy density Ω_V, baryons energy density $\Omega_b h^2$, Hubble constant h and spectral index of primordial fluctuations n_s. The values of l_0 and its shift are evaluated when we vary each parameter, fixing the others to a reference model. At first sight l_0 is more sensitive to parameters which affect geometrically distances (Ω_V, h), with the exception of w_0 at least in the interval considered. Unfortunately the bispectrum is much more noisy than the CMB power spectrum already at the level of cosmic variance, since it is a higher order effect. We can increase the signal to noise ratio including all triangles configurations in l-space (i.e. considering all possible multipoles triplets) in our analysis. We have tested the improvement that the bispectrum can bring to the power spectrum alone building a three-dimensional likelihood on dark energy parameters $(\Omega_V, w_0, w_\infty)$ and choosing a fiducial model for an ideal (cosmic variance limited) experiment. In figure 2 we show our results marginalizing each time over one dark energy parameter; as we can see in all cases the contours of the joint likelihoods are narrower than the contours of the power spectrum likelihoods alone. This first, and very preliminary result, is encouraging: a gain is expected on the estimation of dark energy equation of state at the beginning of structure formation.

5. Conclusions

We have discussed how to alleviate the distance degeneracy in the CMB power spectrum using the ISW and weak lensing induced CMB bispectrum; we have shown some preliminary results about the sensitivity of the bispectrum with respect to the main cosmological parameters. We have simulated a joint likelihood of power spectrum and bispectrum, limited to only dark energy parameters and fixing a fiducial cosmological model. Our preliminary results indicate that adding the bispectrum likelihood to the power spectrum one, the contour levels are narrower and a gain in the estimation of dynamics of dark energy equation of state is expected. Further study is needed to assess the magnitude of this improvement.

Future precision cosmology data from high resolution experiments like Planck and future measures of tridimensional matter power spectrum with Ly-α forest (Viel, Heahnelt & Springel 2004) and cosmic shear (Bacon *et al.* 2004) will help in the use of the bispectrum since they will allow on a better knowledge of the matter power spectrum affecting the bispectrum signal.

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