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On the existence of orthogonal designs

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An orthogonal design of type $(s_1, s_2, \ldots, s_u)$ and order n on the commuting variables $x_1, x_2, \ldots, x_u$, is an $n \times n$ matrix A with entries from $\{0, \pm x_1, \pm x_2, \ldots, \pm x_u\}$ such that

$$AA^t = \left(\sum_{i=1}^{u} s_i x_i^2\right) I .$$

The existence question for orthogonal designs stems from many problems originating in fields as diverse as algebraic topology and coding theory. A brief history of the existence question is included in the introduction.

Wolfe [2], and Shapiro [1] have recently found effective necessary conditions for the existence of orthogonal designs in terms of rational matrices. Subsequent research has been directed mainly toward the question of determining precisely when these necessary conditions suffice for existence.

This question is answered in many particular cases by the direct construction of orthogonal designs in Chapter 2. A method which searches for an orthogonal design of given parameters is presented. This method has been implemented by hand and by computer to construct a large number of previously unknown orthogonal designs. Some related techniques are used to construct infinite families of orthogonal designs.

In Chapter 3 two different asymptotic existence results are proved. Firstly, it is shown that if all of $n, s_1, s_2, \ldots, s_u$, are sufficiently divisible by 2, then often the existence of an orthogonal design of type

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$(s_1, s_2, \ldots, s_u)$ and order n can be deduced. Secondly, the Wolfe-Shapiro necessary conditions are shown to be often sufficient for the existence of orthogonal designs with few nonzero entries.

A kind of integral analogue to the Wolfe-Shapiro theory is presented in Chapter 4. As a consequence, it is shown that the Wolfe-Shapiro necessary conditions suffice for the existence of an $n \times n$ matrix A with entries from $\{mx_i : 1 \le i \le u, m \in \mathbb{Z}\}$ such that $AA^t = \left(\sum_{i=1}^{u} s_i x_i^2\right) I$. This is important because such a matrix resembles an orthogonal design.

In Chapter 5 the power of the results in previous chapters, especially Chapter 2, is illustrated by the tabulation of numerical results.

Many of the results in this thesis can be found in the published papers of the author.

References

[1] D. Shapiro, "Similarities, quadratic forms and Clifford algebras" (PhD thesis, University of California, Berkeley, 1974).

[2] Warren W. Wolfe, "Rational quadratic forms and orthogonal designs" (Queen's Math. Preprints, No. 1975-22. Queen's University, Kingston, 1975); *J. Number Theory* (to appear).

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