(3) The strong cuts I of a countable nonstandard model $\mathcal{M}$ of $\mathrm{I}\Sigma_{n+1}$ which are Σ_{n+1}-elementary submodel of $\mathcal{M}$ are precisely those cuts of $\mathcal{M}$ for which there is a proper Σ_n-elementary (initial) self-embedding j of $\mathcal{M}$ such that I is the set of all fixed points of j.

(4) The standard cut $\mathbb{N}$ is strong in a countable nonstandard model $\mathcal{M}$ of $I\Sigma_{n+1}$ iff there is a proper Σ_n-elementary (initial) self-embedding of $\mathcal{M}$ which moves all Σ_{n+1}-undefinable elements of $\mathcal{M}$.

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Lukas Daniel Klausner, ***Creatures and Cardinals***, Technische Universität Wien, Austria, 2018. Supervised by Martin Goldstern. MSC: Primary 03E17, Secondary 03E35, 03E40. Keywords: cardinal characteristics of the continuum, continuum hypothesis, localisation cardinals, anti-localisation cardinals, creature forcing, Yorioka ideals, Cichoń's diagram, splitting number, reaping number, independence number.

Abstract

This thesis collects several related results on cardinal characteristics of the continuum, all of which employ the method of creature forcing.

In Chapter A, we use a countable support product of lim sup creature forcing posets to show that consistently, for uncountably many different functions the associated Yorioka ideals' uniformity numbers can be pairwise different. (For an introduction on Yorioka ideals, see [1].) In addition, we show that, in the same forcing extension, for two other types of simple cardinal characteristics parametrised by reals (localisation and antilocalisation cardinals), for uncountably many parameters the corresponding cardinals are pairwise different. The proofs are based on standard creature forcing methods and Tukey connections.

In Chapter B, we disassemble, recombine, and reimplement the creature forcing construction used by Fischer/Goldstern/Kellner/Shelah [2] to separate Cichoń's diagram into five cardinals as a countable support product with more easily understandable internal structure. Using the fact that it is of countable support, we augment the construction by adding uncountably many additional cardinal characteristics, namely, localisation cardinals. The proofs use both creature forcing and combinatorial methods.

In Chapter C, we introduce nine cardinal characteristics related to the splitting number $\mathfrak{s}$, the reaping number $\mathfrak{r}$ and the independence number $\mathfrak{i}$ by using the notion of asymptotic density to characterise various intersection properties of infinite subsets of ω. We prove several bounds and consistency results, e. g. the consistency of $\mathfrak{s} < \mathfrak{s}_{1/2}$ and $\mathfrak{s}_{1/2} < \mathrm{non}(\mathcal{N})$, as well as several results about possible values of $\mathfrak{i}_{1/2}$. Most proofs are of a combinatorial nature; one of the more sophisticated proofs utilises a creature forcing poset already introduced in Chapter B.

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Ari Meir Brodsky, ***A Theory of Stationary Trees and the Balanced Baumgartner–Hajnal–Todorcevic Theorem for Trees***, University of Toronto, Canada, 2014. Supervised by Stevo Todorcevic. MSC: Primary 03E02, Secondary 03C62, 05C05, 05D10, 06A07. Keywords: combinatorial set theory, nonspecial trees, stationary trees, stationary subtrees, partial orders, diagonal union, regressive function, normal ideal, Pressing-Down Lemma, balanced partition relation, partition calculus, Erdős–Rado Theorem, Baumgartner–Hajnal–Todorcevic Theorem, elementary submodels, nonreflecting ideals, very nice collections.

Abstract

Building on early work by Stevo Todorcevic, we develop a theory of stationary subtrees of trees of successor-cardinal height. We define the diagonal union of subsets of a tree, as well as normal ideals on a tree, and we characterize arbitrary subsets of a nonspecial tree as being either stationary or nonstationary.

We then use this theory to prove the following partition relation for trees:

Main Theorem. *Let κ be any infinite regular cardinal, let ξ be any ordinal such that $2^{|\xi|} < \kappa$, and let k be any natural number. Then*

$$\textit{non-}(2^{<\kappa})\textit{-special tree} \to (\kappa + \xi)^2_k .$$

This is a generalization to trees of the Balanced Baumgartner–Hajnal–Todorcevic Theorem, which we recover by applying the above to the cardinal $(2^{<\kappa})^+$, the simplest example of a non-$(2^{<\kappa})$-special tree.

An additional tool that we develop in the course of proving the Main Theorem is a generalization to trees of the technique of nonreflecting ideals determined by collections of elementary submodels.

As a corollary of the Main Theorem, we obtain a general result for partially ordered sets:

Theorem. *Let κ be any infinite regular cardinal, let ξ be any ordinal such that $2^{|\xi|} < \kappa$, and let k be any natural number. Let P be a partially ordered set such that $P \to (2^{<\kappa})^1_{2^{<\kappa}}$. Then*

$$P \to (\kappa + \xi)^2_k .$$

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Aaron Thomas-Bolduc, ***New Directions for Neo-logicism***, University of Calgary, Canada, 2018. Supervised by Richard Zach. MSC: 00A30, 03A99. Keywords: neo-logicism, philosophy of mathematics, arithmetic, analysis.