

JAMES, I. M., *Fibrewise topology* (Cambridge Tracts in Mathematics 91, Cambridge University Press, Cambridge, 1989), x+198 pp., 0 521 36090 0, £27.50.

The author gives a careful and detailed analysis of basic concepts in topology in a fibrewise setting. The category of fibrewise topological spaces over a fixed base space  $B$  has as objects  $\{(X, p, B)\}$  where  $X$  is a space and  $p: X \rightarrow B$  is a continuous map; the definition of a morphism is clear. Initially one seeks analogues of the definitions of the separation axioms, compactness, quotient topology, etc. which specialize to the usual concepts when  $B$  becomes a point and lead to fibrewise versions of standard theorems. It is the interaction between the topologies of  $X$  and  $B$  which provides the mathematics with its particular flavour.

There are five chapters. The first is entitled "Basic fibrewise topology". The terms fibrewise open, fibrewise closed, fibrewise discrete, several fibrewise separation conditions and fibrewise compactness and local compactness are defined. For example, a fibrewise topological space  $X$  over  $B$  is compact if and only if the projection  $p$  is proper. One can then prove that if  $\{X_\alpha\}$  is a family of fibrewise compact spaces over  $B$ , then the fibrewise topological product is fibrewise compact. There follows a discussion of fibrewise quotient spaces and fibrewise pointed topological spaces, the latter being fibrewise spaces over  $B$  with sections. The opening chapter ends with a discussion of  $G$ -spaces where  $G$  is a topological group, which is one of the areas providing the strongest motivation for developing the general fibrewise topological theory. Filters appear briefly in Chapter 1 and are used in the second chapter to define Wallman and Alexandroff fibrewise compactifications and to determine conditions which ensure that they are fibrewise Hausdorff. The fibrewise compact-open topology is introduced to study fibrewise mapping-spaces and leads in both the free and based theories to analogues of the well-known theorems on continuity and exponentiation which hold for topological spaces. The chapter ends with a discussion of fibrewise compactly-generated spaces.

One can now read Chapter 3 or 4. Chapter 3 develops a theory of fibrewise uniform spaces and Chapter 4 discusses fibrewise homotopy theory. They are likely to appeal to different groups of mathematicians. Algebraic topologists will find Chapter 4 useful for although fibrewise homotopy theory has been in general use for some time, it is difficult to find elsewhere in the literature rigorous treatments of some of the fundamental results. The final chapter is entitled "Miscellaneous topics" and begins with a discussion of how fibre bundle theory fits into the more general framework of fibrewise topological spaces. It ends with results on numerable coverings and a brief consideration of fibrewise connectedness.

Both the mathematics and the writing are elegant. The background knowledge in basic topology needed to understand the text is nicely judged; those who normally work in areas of mathematics with a geometric or categorical underlay should experience few problems. Even the most elementary parts of the text are interesting; in seeking to generalize fundamental concepts of general topology, there may be several plausible definitions and insight is needed to choose a good definition for later developments. The reaction to this tract of those who work in topos theory will be interesting. Certainly Professor James's book will be found of lasting value to analytic topologists, global analysts and others, but perhaps particularly to algebraic topologists.

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