IV. CALIBRATION OF THE ^{14}C TIME SCALE

A. Data Records Other Than ^{14}C

B. Wiggle-Matching and Floating Chronologies

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PROBABILITY AND DATING

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ABSTRACT. Statistical analysis is becoming much more widely used in conjunction with radiocarbon dating. In this paper I discuss the impact of Bayesian analysis (using computer programs such as OxCal) on archaeological research. In addition to simple analysis, the method has implications for the planning of dating projects and the assessment of the reliability of dates in their context.

A new formalism for describing chronological models is introduced here: the Chronological Query Language (CQL), an extension of the model definitions found in the program OxCal.

New methods of Bayesian analysis can be used to overcome some of the inherent biases in the uncertainty estimates of scientific dating methods. Most of these methods, including ^{14}C, uranium series and thermoluminescence (TL), tend to favor some calendar dates over others. ^{14}C calibration overcomes the problem where this is possible, but a Bayesian approach can be used more generally.

INTRODUCTION

With large numbers of archaeological and environmental sites being dated in some detail, the way in which the scientific dating information is used to understand chronology is becoming increasingly important. The multiple factors underlying chronological information make this almost impossible to do by intuition, especially with calibrated ^{14}C dates that give multiple ranges and complicated probability distributions. For these reasons, new statistical methods were developed (*e.g.*, Buck *et al.* 1991; Litton and Buck 1995) to allow information about sequences and phases to be used with ^{14}C evidence to arrive at quantifiable conclusions with known probabilities. In order to make such techniques more widely applicable, the computer program OxCal (Bronk Ramsey 1995a) was developed. One of the most important new aspects of this program was the method employed to allow chronological models of all kinds (incorporating sequences, phases, wiggle-matched sequences, *etc.*) to be specified in a fairly simple but nonetheless rigorous way.

The overall impact of this approach has been considerable (*e.g.*, Bronk Ramsey and Allen 1995; Bayliss, Bronk Ramsey and McCormac 1997). To take the large dating program of English Heritage as an example, OxCal has been used for many sites, significantly improving overall precision of chronology. Equally important, the specification of the chronological relationships has been a helpful exercise in itself.

As well as allowing the analysis of whole sites with stratigraphic information, the methods themselves are also useful for very specific cases such as tree ring sequences. They could also be applied to a number of other, slightly more complex, cases such as sedimentary deposits and dated material related in some way to horizons (either destruction layers or overlying deposits).

Another area of interest is how to deal with ^{14}C dates close to background level. Quite apart from the problems of calibration in this time range, there is a tendency for the raw dating information to give misleading estimates of the uncertainty. This tendency can be seen as a hidden bias in the technique. The kinds of model used for periods that are more recent turn out to be unsatisfactory for this purpose, so a new approach is needed.

FORMAL DESCRIPTION OF MODELS

Of equal importance to the statistical methods needed to analyze chronological information is the formalism necessary to express the models in a clear, unambiguous way. Such model formulation

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requires similar skills to those employed by archaeologists and earth scientists when studying site stratigraphy. Indeed, in many cases stratigraphy (and its formal description by Harris Matrices; see Harris 1989) will form the basis of the model. However, it must be stressed that a chronological model is based on events that occurred in the past rather than on objects. A method of formal description must include elements for dealing with isolated events, groupings and relative orderings of those events as well as specific distributions of events. Since the purpose of description is to allow analysis to take place, it should also be possible to include queries within, or associated with, the description.

Events

Events form the building blocks of any chronology. An event is by definition short when compared to the resolution of the measurement techniques employed. We may have direct dating evidence for such events or they may be related in some way to other events; a minimum requirement is that each event in the model should have a unique name. In some cases, the information available might be comprehensive, with a number of direct dating measurements.

Groupings and Sequences

The main element of most models will be the way in which individual events are grouped and the relationships between them. We clearly need to be able to specify events as belonging to groups where there is no constraint on their relative order and to specify sequences of both individual events and whole groups of them. Most possible models can be built using these two basic building blocks.

Specific Models

Often, more closely specified models are useful although many of these must to some extent be based on assumptions that are impossible to prove. One that has been widely employed is the concept of a "uniform" phase, within which the dated events are evenly distributed. The main reason for employing a model of this kind is that it overcomes the inherent tendency of scientific dating methods to produce dates that are scattered because of limited precision. It almost certainly gives us a more realistic interpretation of the given information although we have made an extra (and possibly difficult to substantiate) assumption. To use no model at all is in fact to assume that all of the events are truly independent; this is in effect a model in itself, and in many cases a very unreasonable one.

Another widely employed model is the sequence with defined gaps between the events. This is useful for the wiggle matching of tree-ring sequences for ^{14}C dating.

Several new models would be widely applicable and work is underway to find general mathematical methods for their analysis (see, *e.g.*, Christen, Clymo and Litton 1995). Two of these are special kinds of sequence widely found in environmental sites. In the first, we are constrained by some uniform process (usually growth or sedimentation rate). As in wiggle matching, the gaps between the events are specified, but in this case these only define the relative, not the absolute, chronological intervals. The statistical analysis should then yield probability distributions for the events concerned and a distribution for the growth/sedimentation rate. Similar to this would be the case where the growth/sedimentation rate is not uniform but is used to weight the intervals. A second model widely applicable to archaeological sites is the exponential model in which events either build up to or decay away from some defining event. A good example of this would be assorted finds under a destruction layer. These will tend to cluster close to the destruction itself but may well include items that were of some antiquity at the time of destruction.

Queries

Clearly, since the purpose of any analysis is to elicit new information, it is important that any formal description should allow one to interrogate the chronological model. We might, for example wish to know the probable relative order of events, when a phase started or finished, or the time scale spanned by a series of events.

Chronological query language (CQL)

One possible method for formal definition is that developed for OxCal. This will be further developed here to make it useful for a wider range of dating methods and to introduce some new models such as exponential distributions. The name "Chronological Query Language" (CQL) will be used for this development of the formal description language. Inevitably, not all details can be given here, but Table 1 gives a list of the elements of the language so that readers can gain an impression of the scope of the method.

Elements can be split up into three different categories: events; groupings and sequences; and specific model definitions.

Events

An event can be described in a number of different ways. We can just give it a name with the **event** statement or we can define its age by ^{14}C, TL, U-series or other dating methods using **r_date, l_date, th_u_date, pa_u_date** and **c_date**. If several measurements are available, the statements **r_comb, l_comb** and **c_comb** can be used to combine them, or if they are U-series or of different kinds, the more general **comb**. The most general information of all is simply a prior probability distribution defined by a **prior** statement. Examples of these types are given in Table 1 along with the details of how ^{14}C calibration curves, paleodose estimates, *etc.*, are defined.

Groupings and Sequences

The most general group is a **phase** (not in the specific archaeological sense), which imposes no internal constraints; the second most useful is the **sequence**, which constrains the elements within it to be in chronological order. These *groups* contain a list of *elements* each of which can be either an *event* or another *group*. Supposing, for example, we have a site with sequential phases and during phase II we have a well-stratified sequence of dates from "site x", we can describe this chronological model as:

```
sequence {
  phase "I" { r_date 3310 40; .... };
  phase "II" { r_date 3200 50; sequence "site x"{ r_date 3220 40; r_date ... }; ...};
  phase "III" {r_date 3110 30; .... };};
```

The use of these elements, along with cross references (**x_reference**) to events constrained within more than one phase or sequence, allows the description of any chronology derived, *e.g.,* from Harris matrices (Harris 1989).

Specific Model Definitions

One of the most widely used specific models is the "uniform phase" (see, *e.g.,* Bronk Ramsey and Allen 1995). In the formalism of CQL this is described by giving phases definite boundaries, so in the above example we could treat the phases as uniform in the following way:

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TABLE 1. Summary of Chronological Query Language (CQL) Elements

CQL Statement	OxCal v2.18	Syntax*	Explanation	Example
EVENT DESCRIPTION				
Undated events				
event	--	event *name*;	an undated event in the model	event "Conquest";
	file			
prior	--	prior *name*;	event with a prior probability distribution	prior "post-conquest";
	file	file *name*;		
Any dating method				
c_date	--	c_date *[name] date [error1 [error2]]*;	true calendar date	c_date "Conquest" 1066;
	cal	cal *[name] date [error1]*;		cal "Conquest" 1066;
	asym	asym *[name] date error1 error2*;		asym "Period A" 600 30 60;
c_sim*[ulate]*	--	c_sim *[name] calendar_date error*;	simulated calendar date	c_sim 1066 40;
c_comb	c_comb	c_comb *[name]* {c_date...; c_date...;};	combine calendar dates with a χ^2 test	c_comb "burial" { c_date 910 30; c_date 950 30; c_date 930 40;}
comb	comb	comb *[name]* {*eventlist*};	combine dates of all sorts for a single event	comb "burial" { c_date 910 30; r_date 1010 40;};
year	year	year *measurement_year*;	year of measurement	year 1997;
error	error	error *proportional_error*;	overall uncertainty in age multiplier	error 5%;
factor	factor	factor *multiplier*;	multiplier to obtain calendar age	factor 1.23;
Radiocarbon dating				
r_date	--	r_date *[name] date error*;	^{14}C dated event	r_date "OxA-2000" 3000 30;
	date	date *[name] date error*;		date "OxA-2000" 3000 40;
r_sim*[ulate]*	--	r_sim *[name] calendar_date error*;	simulated ^{14}C date	r_sim 1066 40;
	rand	rand *[name]*		rand 1066 40;
r_comb	r_comb	r_comb *[name]* {r_date...; r_date...;};	combine ^{14}C dates with a χ^2 test	r_comb "burial" { r_date 2910 30; r_date 2950 30; r_date 2930 40;}
curve	curve	curve *curve_file_name*;	calibration curve to be used	curve cal20.dta;
delta_r	delta_r	delta_r *offset error*;	ΔR value for marine curves (Stuiver and Braziunas 1993)	curve marine.14c; delta_r 300 20;
reserv[oir]	reserv	reserv*[oir] reservoir_age error*;	time constant of the reservoir	curve cal20.dta; reserv 100 20;
Luminescence dating				
l_date	--	l_date *paleodose error*;	luminescence date	l_date 1.0 0.2;
	cal	cal d*paleodose* d*error*;		cal d1.0 d0.2;
l_sim*[ulate]*	--	l_sim *[name] calendar_date error*;	simulated luminescence date	l_sim 1066 5%;
l_comb	--	l_comb *[name]* {l_date...; l_date...;};	combine luminescence dates with a χ^2 test	l_comb "burial" { l_date 1.0 0.2; l_date 1.1 0.2; l_date 0.9 0.15;}
	c_comb			
dose	dose	dose *dose_rate*;	estimated dose rate (for the subsequent samples.	year 1995; dose 2.0e-3; error 5%;

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TABLE 1. Summary of Chronological Query Language (CQL) Elements *(Continued)*

CQL Statement	OxCal v2.18	Syntax*	Explanation	Example
Uranium series dating				
Th_U_date	--	th_u_date *230/234 err1 234/238 err2*;	uranium series ($^{230}Th/^{234}U$) date	th_u_date "A" 0.623 0.006 2.82 0.03;
Pa_U_date	--	pa_u_date *231/230 err1 234/238 err2*;	uranium series ($^{231}Pa/^{235}U$) date	pa_u_date "B" 0.042 0.001 1.83 0.04;
Th_U_sim*[ulate]*	--	th_u_sim *[name] date error1 error2*;	simulated ($^{230}Th/^{234}U$) series date	th_u_sim "C" -120000 0.006 0.03;
Pa_U_sim*[ulate]*	--	pa_u_sim *[name] date error1 error2*;	simulated ($^{231}Pa/^{235}U$) series date	pa_u_sim "D" -120000 0.001 0.04;
Event date modifiers				
offset	offset	*event*; offset *offset [error]*;	offset the event from the dating evidence	r_date "bone" 3000 30; offset 20 5;
Cross references				
xref*[erence]*	xref	xref *name*;	cross reference to an event specified elsewhere in the model	xref "bone";
MODEL DEFINITIONS				
General groupings				
phase	phase	phase *[name]* { *elementlist* }	a group of elements (no assumptions)	phase {r_date 3005 40; r_date 3210 40;};
seq*[uence]*	seq	seq*[uence] [name]* { *elementlist* }	a group of elements in chronological order	seq {r_date 3305 40; r_date 3200 40;};
taq	taq	taq *[name]* { *elementlist* }	elements which define a terminus ante quem	seq {r_date 3205 40; taq {r_date 3100 40;}; r_date 3150 40;};
tpq	tpq	tpq *[name]* { *elementlist* }	elements which determine a terminus post quem	seq {r_date 3205 40; tpq {r_date 3400 40;}; r_date 3150 40;};
Specific models				
boundary	bound	seq *[name]* { bound *[name]*; phase *[name]* {*eventlist*}; bound *[name]*;};	a phase of uniformly distributed events between two undated "boundary" events	seq {bound "start"; phase {r_date 3005 40; r_date 3210 40;};bound "end"; };
d_seq	d_seq	d_seq *[name]* { *event*; gap *gap*; *event*; gap *gap*; ... };	a series of events spaced in time with known gaps (usually tree rings)	d_seq { r_date 3200 40;gap 50;r_date 3160 40; gap 50; r_date 3090; };
v_seq	v_seq	v_seq *[name]* { *event*; gap *gap error*; *event*; gap *gap error*; ... };	similar to d_sequence except with errors on the gaps	v_seq { r_date 3200 40;gap 50 20;r_date 3160 40; gap 50 20; r_date 3090; };
u_seq	--	u_seq *[name]* { *event*; gap *rel_gap*; *event*; gap *rel_gap*; ... };	a series of events related to uniform growth or sedimentation rate	u_seq { r_date 3200 40;gap 42;r_date 3160 40; gap 31; r_date 3090; };
w_seq	--	w_seq *[name]* { *event*; gap *rel_gap*; *event*; gap *rel_gap*; ... };	similar to u_sequence except that the rate is allowed to vary	w_seq { r_date 3200 40;gap 42;r_date 3160 40; gap 31; r_date 3090; };
exp*[onential]*	--	exp*[onential] [name]* { *eventlist*; end *name*;t_constant *name*;}	material all earlier than an event in an exponential distribution.	exp { r_date 4000 30; r_date 3210 40; r_date 3100 40; r_date 3500 30; r_date 3150 40; end "Destruction";
		exp*[onential] [name]* { begin *name*; *eventlist*; t_const*[ant]* *name*;}	material all later than an event in an exponential distribution.	t_const "age profile";};
gauss*[ian]*	--	gauss*[ian] [name]* { *eventlist*; mean *name*; sigma *name*;}	material clustered around a mean date with an assumed gaussian distribution	gauss { r_date 4000 30; r_date 3010 30; r_date 3500 40; r_date 3200 40; mean "Mean"; sigma "Sigma";};

TABLE 1. Summary of Chronological Query Language (CQL) Elements *(Continued)*

CQL Statement	OxCal v2.18	Syntax*	Explanation	Example
QUERIES AND CALCULATIONS				
Queries				
corr*[elation]*	corr	corr *name name1 name2*;	produce a correlation plot of events with *name1* and *name2*	corr "start_end_correlation" "start" "end";
first	first	first *[name]*;	find the date of the first event in this group	phase { first; ..... };
last	last	first *[name]*;	find the date of the last event in this group	phase {..... ; last; };
span	span	first *[name]*;	find the span of the dates in this group	phase {..... ; span;};
inter*[val]*	inter	inter *[name]*;	find the interval between events	seq { r_date 990 30;inter; r_date 900 30;};
order	order	order *[name]* {*eventlist*};	find the probability of possible event orders	order {r_date "a" 3200 30; r_date "b" 3100 30; r_date "c" 3150 30; };
quest*[ion]* ?	quest ?	*event*; quest*[ion]*; *event*?	find the probability this event occurs here in the model	seq {bound "start"; phase {r_date 3005 40; r_date 3210 40? };bound "end"; };
Event calculations				
before	before	before *[name]* {*eventlist*};	probability of being before the listed events	before {r_date 3005 40; r_date 3210 40;};
after	after	after *[name]* {*eventlist*};	probability of being after the listed events	after {r_date 3005 40; r_date 3210 40;};
first	first	first *[name]* {*eventlist*};	calculate the date of the first event in a list	first {r_date 3005 40; r_date 3210 40;};
last	last	last *[name]* {*eventlist*};	calculate the date of the last event in a list	last {r_date 3005 40; r_date 3210 40;};
sum	sum	sum *[name]* {*eventlist*};	find a frequency distribution for the events	sum {r_date 3005 40; r_date 3210 40;};
diff	diff	diff *[name] name1 name2*;	calculate the difference in age	diff "length" "end" "start"
shift	shift	shift *[name] name1 name2*;	add two ages together	shift "date2" "date1" "length"
DISPLAY CONTROL				
plot	plot	plot *[name]* { *elementlist* };	group elements simply for plotting purposes	plot { r_date 3000 30; r_date 2000 20;};
axis	axis	axis *min max*;	define calendar axis limits	axis 500 1500;
comm*[ent]*	comm	comm*[ent]* "*comment*";	put a comment in the model	comm "Assuming uniform distribution";
label	label	label "*label*";	put a label on the plot	label "Bone Sample";
line	line	line	put a horizontal dividing line on the plot	line;
page	page	page	put a page break in the plot	page;

*An *eventlist* is a list of *events*, whereas an *elementlist* can be a mixed list of *groups* and *events*.

Future versions of OxCal will be able to read existing models. Note that all commands can be given as upper or lower case. Changes to the model definition are:

- replacement of date, rand and cal with r_date, r_sim[ulate] and c_date
- replacement of cal for luminescence dates with l_date
- replacement of cal for uranium series dates with th_u_date and pa_u_date
- replacement of file with event (where there is no prior information) and prior where there is
- addition of specific combination method l_comb
- addition of dating simulation commands l_sim[ulate], th_u_ sim[ulate], pa_u_simulate and c_ sim[ulate
- addition of new models for specific circumstances: u_seq, w_seq, expon[ential], gauss[ian]

```
sequence { boundary "colonisation";
  phase "I" { r_date 3310 40; .... }; boundary "destruction event";
  phase "II" { r_date 3200 50; sequence "site x"{ r_date 3220 40; r_date ... }; ...};
  boundary "invasion";
  phase "III" {r_date 3110 30; .... };boundary "volcanic eruption";};
```

Subsequent analysis of the model will then not only constrain the elements of the three phases to be in uniform distributions but also provide estimates of the "boundary" events (in this case colonization, a destruction event and a volcanic eruption).

Wiggle matching of tree ring sequences can be achieved in CQL by using the **d_sequence** statement or the **v_sequence** statement if the gaps between the elements of the sequence are only approximately known.

As indicated above, several new models (not present in current versions of OxCal) would also be useful in a variety of situations. One such is the exponential distribution that might be applied, *e.g.*, to the material in a destruction layer. Such a model is defined as:

```
exponential "pre-destruction"
  { r_date 3110 40; r_date 2930 40; ... r_date; 3100 40;
  end "destruction"; t_constant "average age";};
```

Analysis using this model would yield estimates of both the date of the destruction and the exponential time constant **(t_constant)** relating to the average age of objects at the time of destruction. Finally, a **gaussian** model would allow the treatment of events that cluster together but not in the uniform phases described above.

Considerations in Implementation

Planning

The formalism described above allows complex models to be defined, but analysis of this sort usually requires a large number of scientific dating measurements to be effective in answering archaeological or environmental questions. It is clearly important to assess whether this investment is going to be worthwhile and which samples should be chosen for dating. This can be achieved by performing analysis before the dating is undertaken, although this obviously relies on guesses about the chronology of the site in question; it can therefore never be definitive or watertight. To allow such analysis, the OxCal program incorporates the **rand** function that generates simulated ^{14}C measurements of the sort you would expect to get for objects of a certain age. The scatter associated with a measurement of this kind is generated randomly. In general, this method would be useful for all kinds of dating, so four statements are included in the new definition of CQL (**r_simulate, l_simulate, th_u_simulate, pa_u_simulate** and **c_simulate**) to allow the simulation of ^{14}C, TL, U-series and general dating techniques, respectively.

Reliability Testing

With so many possible models, it is very important that the reliability of all aspects of the analysis be tested, especially because, except for Gaussian probability distributions, a simple χ^2 test cannot be used. There are three concerns here: the results of the scientific dating measurements, the choice of model (using stratigraphic and other evidence) and the statistical analysis itself. The last of these is dealt with below in the section on the limitations of numerical methods.

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Both the dating measurements themselves and the stratigraphic evidence should be subject to very careful scrutiny and quality control before any analysis takes place. In particular, the standard uncertainty terms must be realistic since under- or overestimates of these will cause problems. This is the responsibility of dating laboratories and is largely achieved by checking that measurements made on known-age material have a Gaussian distribution about the expected mean (see Fig. 1). Stratigraphic interpretation is primarily the responsibility of the archaeologist or environmental scientist. Analysis of the chronological model and the dating evidence together can, however, be used to test whether these two agree, and if not, where the problems seem to lie. Such problems of association, contamination, residuality, *etc.*, are inevitable in even the most thoroughly excavated sites. All of the scientific methods give some sort of a prior probability distribution that can be compared to the posterior distribution by means of an overlap integral to give an agreement index (see Bronk Ramsey 1995a for the exact method employed by OxCal).

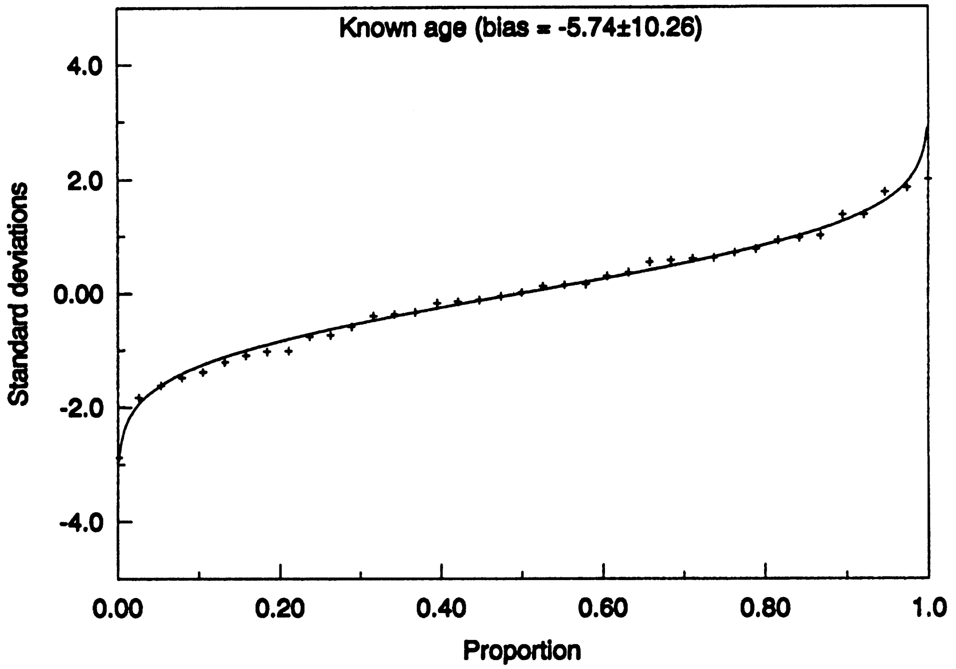

Fig. 1. Known-age samples measured at Oxford in 1996 (39 in total). These have been plotted in terms of the number of standard deviations they are from the expected value and are sorted by the same value. The curve is the expected Gaussian distribution. This method allows the validity of standard uncertainty terms to be checked.

Limitations of Numerical Methods

Although the overall definitions of chronological models can be very well specified and watertight, this is often not the case with the methods of numeric analysis used. Because of the complex nature of the models and the flexibility required in their definition, purely analytical methods are rarely employed. In particular, Gibbs sampling is a very flexible method well suited to this type of problem, but it does have some limitations, working well only with continuous distributions. (For details on use of the method, see Bronk Ramsey 1995a,b; Buck, Litton and Smith 1992; Gelfand and Smith 1990.) If there are a number of discrete possibilities, the method can become "stuck in a rut". To some extent, this problem can be monitored by starting the analysis from many different points. If all of these give similar results we can be fairly confident that the method is converging on a single solution. Such a test is built into current versions of OxCal, although in some cases it has been found to be necessary to analyze a model several times to look for possible problems. If there are problems with convergence, any results should be treated with the utmost caution. In principle, if the analysis were continued indefinitely, a truly representative picture would be built up, but this is not practical.

There are also other, unrelated, limits to the sorts of analysis that are possible. It is clearly unwise to try to wiggle-match tree ring sets to a calibration curve when the density and precision of the measurements to be fitted are greater than those of the calibration curve itself: we should always bear in mind the inherent limitations of the curve used (*e.g.,* a bidecadal curve cannot in general yield results accurate to better than 20 yr). It is also unwise to build a very elaborate model around a small number of measurements. In all cases, common sense is needed in assessing how far analysis should be pushed and how strong the underlying assumptions are.

Inherent Biases in Dating Methods

A number of biases are associated with different dating methods. ***Bias*** is here used to mean that the uncertainties quoted give a systematically misleading impression of the true range of probabilities. In ^{14}C dating these have been largely overcome by the process of calibration using the probability-based methods. ^{14}C measurement beyond the range of calibration undoubtedly has some unknown offset that varies with time but it is also subject to an inherent bias to younger ages—a feature common to many dating methods, including (TL) and U-series.

To see why this is the case, it is useful to consider a hypothetical ^{14}C measurement of, *e.g.*, 1.0 ± 0.4 pMC. The standard uncertainty quoted applies to the ^{14}C measurement made, but it is usually assumed that confidence intervals in the ^{14}C age of the sample can be directly calculated. This problem can be tackled by either classical or Bayesian statistical methods, but only the latter will be considered here.

The 95.4% confidence interval for the ^{14}C age of our postulated sample would normally be taken as that corresponding to 0.2–1.8 pMC. We come to an identical conclusion using Bayesian statistics, with a 95.4% probability interval, if we assume that this measurement implies that the most likely value is 1.0% with a Gaussian probability distribution having a standard deviation of 0.4. However, we can see that this is not realistic, as we have more information:

- We know the value cannot be below zero;
- the range 0.2–1.0 covers a much larger time range than 1.0–1.8 (by a factor of almost three);
- there must be a significant probability that the real value is very close to zero since such a value corresponds to a huge time range.

In using this Gaussian model, we are in practice biasing the measurements with a prior probability distribution that is simply the differential of the age equation

$$p_{RC} \propto dA/dt = -\left(\frac{100}{8033}\right)\exp\frac{-t}{8033} \quad . \tag{1}$$

The nature of this "^{14}C prior" can be seen in Figure 2 and the corresponding posterior distribution for an example measurement in Figure 3.

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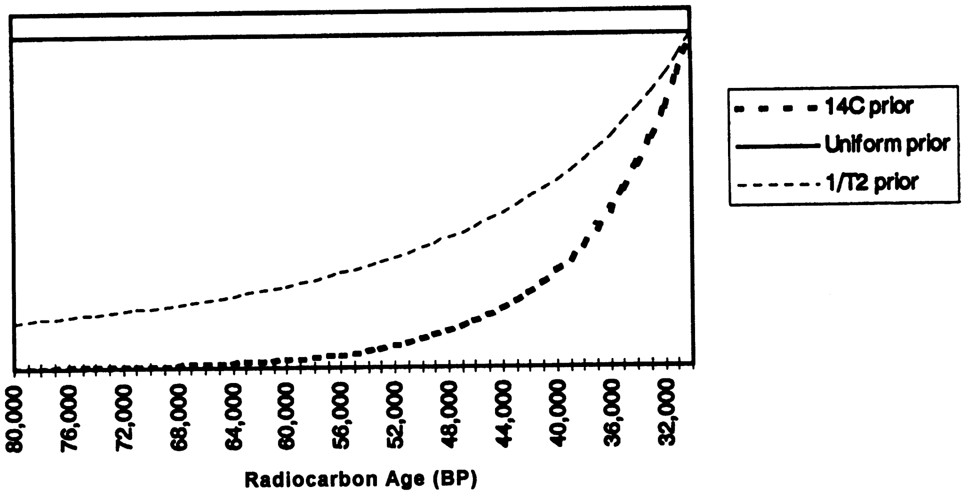

Fig. 2. Comparison of weighting inherent in using the ^{14}C ratios as the basis for age determination (^{14}C prior) compared to an inverse square model and the uniform prior normally used for Bayesian analysis. The scale here is arbitrary. These biases are all only significant close to the limit of the technique.

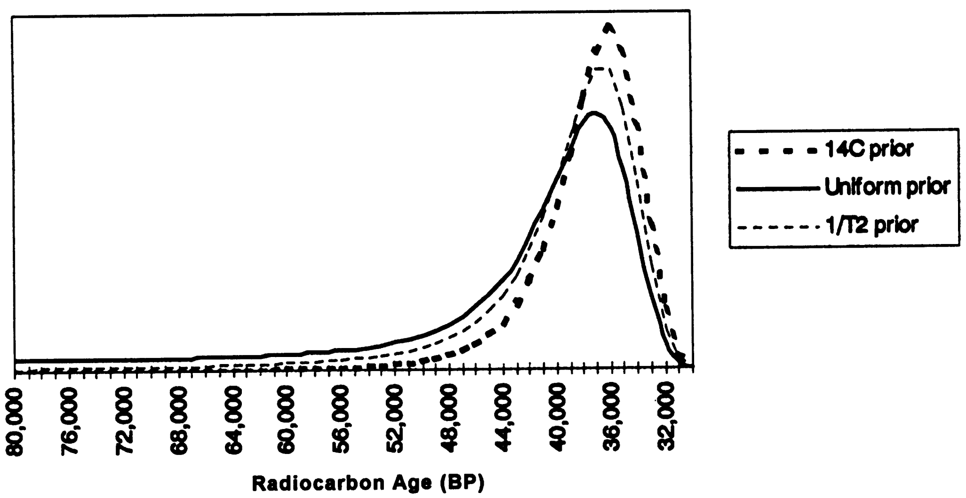

Fig. 3. Probability distributions derived from a ^{14}C measurement of 1.0 ± 0.4% using different prior assumptions; all have been calculated using Bayesian statistics, but the ^{14}C prior method is analogous to using confidence limits derived directly from the ratio and standard uncertainty. The scale here is arbitrary, as the Uniform prior distribution cannot be normalized.

A New Probability Model for Large Time Scales

In most Bayesian or probabilistic modeling, a uniform prior distribution in time (rather than ^{14}C concentration) is used. This means that we assume any event is equally likely to have occurred in any individual year. When the dating methods we are using are fairly precise, this is a very reasonable assumption. For longer time scales, however, normalization is almost impossible even if some arbitrary cutoff point is defined. For ^{14}C measurements with an activity of A and an uncertainty of σ the choice of this cutoff point makes a real difference to any deductions made if $A < \sim 6\,\sigma$ and has a dom-

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inant effect where $A < 3\,\sigma$. In addition, such a model is clearly unrealistic: any traces of living matter are much more likely to be recent than they are to be extremely ancient, all other things being equal.

The first criterion when choosing a new model for this time scale is that normalization of the probabilities should be possible. Ideally, if $p(t)$ were our prior probability distribution we would be able to calculate

$$\int_{-\infty}^{0} p(t)dt \qquad (2)$$

or at least (since we can always define a definite latest point in time)

$$\int_{-\infty}^{T} p(t)dt \quad . \qquad (3)$$

Ideally, the function would not depend on the units of time involved and would vary only gently over the time scale in which we are interested. The obvious choice mathematically is $1/t^2$ (since this is the lowest negative power of t that can be integrated from $-\infty$). It is, however, only one possible model, which we will look at here in order to see how it might be used.

When using a nonuniform model of this sort, care has to be exercised in the choice of algorithms for estimating chronological ranges with defined probabilities. For example, if we are interested in a 95.4% probability, the normal procedure (with uniform prior distributions) is to select the 95.4% of the probability distribution that has the highest probability density. Exactly the same procedure can be employed for the inverse square model by plotting the distributions on a t^{-1} scale. In this "gauge" the prior distribution is once more uniform. This method provides both a useful way of visualizing the distributions over a long time range and assurance that the ranges generated are not themselves biased by the model.

To see how this works in practice, we will consider six hypothetical ^{14}C dates with activities of 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0 pMC, with uncertainties of 0.4 pMC. These uncertainties are perhaps somewhat larger than usually obtained for well-preserved samples of reasonable size, but they will illustrate the effects we are looking at. Figure 4 shows the plotted distributions with their 95.4% ranges calculated by the probability method. Table 2 also gives the ranges as they would be quoted conventionally, using a uniform prior distribution, and for higher activities.

TABLE 2. Interpretation of ^{14}C Measurements as Ages Using Different Models

^{14}C activity ($A \pm \sigma$)	"Conventional" quoted date (yr BP)	2 σ asymmetric range (yr BP)	Uniform prior (95.4%) (yr BP)	Inverse square prior (95.4%) (yr BP)
0.0 ± 0.4%	>38,780	∞–38,780	>~100k*	>43,860
0.2 ± 0.4%	>36,990	∞–36,990	>44,250	>40,650
0.4 ± 0.4%	>35,530	∞–35,530	>38,760	>38,460
0.6 ± 0.4%	>34,290	∞–34,290	>35,460	>35,460
0.8 ± 0.4%	>33,200	∞–33,200	>33,110	>33,110
1.0 ± 0.4%	36,990 ± 3,220	49,920–32,270	>31,440	138k–31,440
2.0 ± 0.4%	31,425 ± 1,610	35,530–28,720	>~26k*	35,740–28,570
4.0 ± 0.4%	25,860 ± 803	27,650–24,390	27,600–24,340	27,600–24,350

*This is strongly dependent on the cutoff used (here 10^7 yr)

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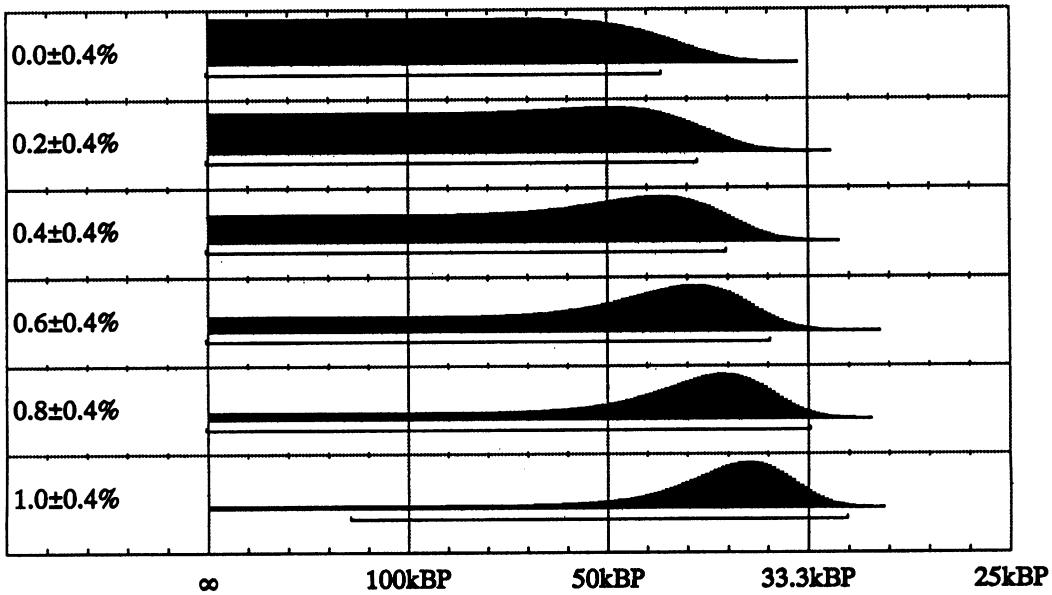

Fig. 4. Probability distributions for a series of ^{14}C dates using an inverse square model and plotted on a 1/t axis. 95.4% probability ranges are also shown.

From this table it can be seen that using a probabilistic approach of this sort does give significantly different ranges from the asymmetric ranges based purely on the ^{14}C measurements where the measurements are close to background (particularly when σ/A is between 2 and 4). As stated above, the uniform prior model has problems and moreover is unrealistic in this region. The inverse square model and the standard interval limits give very similar values as A rises above ~5 σ. When $A > 6\ \sigma$ all of the methods give indistinguishable results within the resolution of the calculations performed here. In the region close to background, however, both probabilistic models show the extent to which the conventional confidence limit ranges underestimate both the probable antiquity of samples and the overall uncertainty in the age. The inverse square model is well behaved over this entire region. It also imposes a much smaller bias than the raw ^{14}C calculation, and in most circumstances it will be relatively realistic. Similar calculations could be performed for U-series and TL dates using exactly the same prior probability, allowing the measurements to be meaningfully compared.

The rationale for an approach like this is not only that we can arrive at more realistic age ranges for individual measurements but also that it provides a possible statistical framework within which further analysis can take place. As an example, we will consider the six hypothetical measurements shown in Figure 4. Supposing we also have stratigraphic evidence demonstrating that these samples should be in chronological order. Using the inverse square model it is then possible to perform a Gibbs sample analysis of the sequence; the results are shown in Figure 5 and Table 3.

Clearly, any deductions made beyond *ca.* 60 ka BP are going to depend very little on the measurements made and almost entirely on the model, and they should therefore be treated with caution. However, the introduction of other dating information into this picture is now possible given the overall framework.

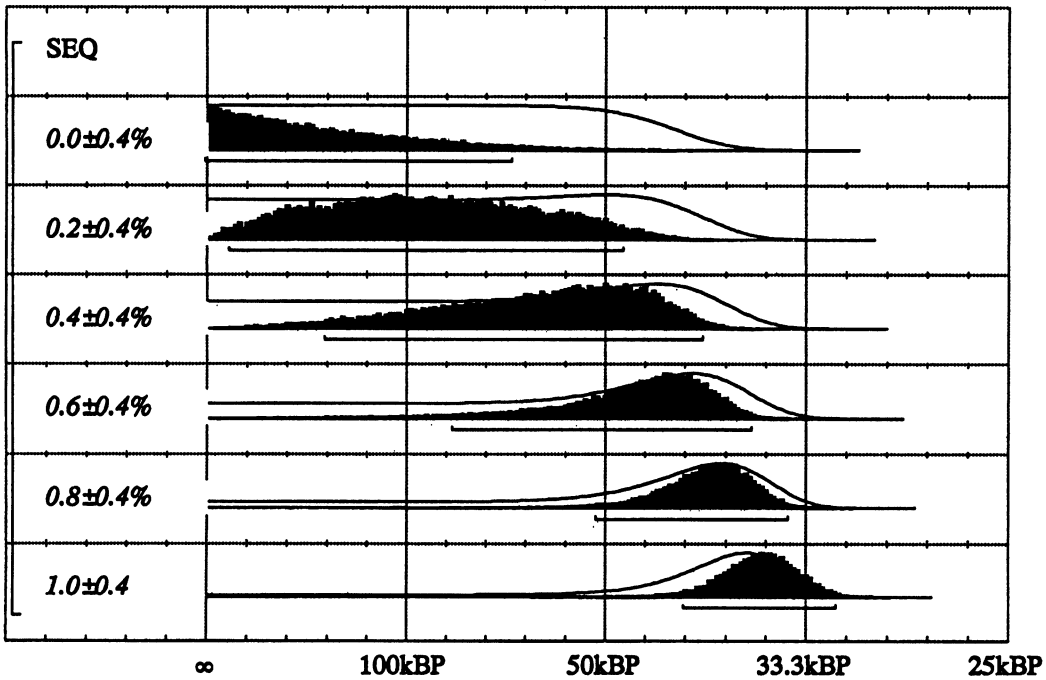

Fig. 5. Similar to Fig. 4, but incorporating the assumption that these come from a chronological series. These distributions were calculated using the Gibbs sampling routine of OxCal with modified scales.

TABLE 3. ^{14}C Measurements Interpreted Using an Inverse Square Prior With and Without the Use of Sequence Information

^{14}C activity ($A \pm \sigma$)	Inverse square prior (95.4%) (yr BP)	Sequence number	Using sequence information (95.4%) (yr BP)
0.0 ± 0.4%	>43,860	1	>69,450
0.2 ± 0.4%	>40,650	2	625,000–48,550
0.4 ± 0.4%	>38,460	3	166,670–40,320
0.6 ± 0.4%	>35,460	4	74,630–36,500
0.8 ± 0.4%	>33,110	5	50,500–34,483
1.0 ± 0.4%	138k–31,440	6	41,670–31,850

Classical statistical techniques could also be applied to this sort of problem although this becomes increasingly impractical as the constraints become more complex. A comparison of the use of such methods with the Bayesian approach for simple cases would be valuable, but is beyond the scope of this paper.

CONCLUSION

The probabilistic approach to chronology allows all kinds of evidence to be brought together in a quantitative way. It is hoped that the Chronological Query Language (CQL) outlined here will enable archaeologists and earth scientists to specify such evidence in an unambiguous way. It will also form the basis for future developments of the computer program OxCal, which can be used for analysis of this sort.

The question of how to deal with ^{14}C dates when they are close to background has been addressed. The conventional method of quoting date ranges directly from ^{14}C concentrations is shown to exhibit a bias to younger ages. Similar effects would also be seen with other scientific dating methods. By using a method-independent (and less extreme) inverse square prior probability, this problem can be largely overcome. Probabilistic analysis then becomes possible, so that the full wealth of evidence available in any context can be used.

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