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Modifying diets to satisfy nutritional requirements using linear programming

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A computational method for constructing individually acceptable diets by modifying a chosen diet to meet nutritional requirements is described. The effects on food quantities of imposing different nutrient requirements on a sample diet are demonstrated and techniques which can ensure the acceptability to the individual of the modified diet are described. The starting point in the calculation is the person's current dietary intake. This is modified using linear programming methods which make the smallest changes to the food quantities to meet specific targets. Sequential modification can be used to identify changes that are acceptable to the individual. The computer program has been developed in collaboration with practising dietitians and is in use in some leading UK hospitals.

Mathematically constructed diets: Nutrient requirements: Dietary modifications

The painstaking task of calculating the amounts of nutrients in a diet can now easily be done by computer (Bassham *et al.* 1984) and programs which can efficiently perform this task are widely available.

Tailoring a diet to satisfy predetermined nutrient requirements is a more difficult task which is often done by hand calculation using trial and error methods developed by the individual dietitian. For a given set of nutrient requirements there may be an infinite number of possible diets which could satisfy the requirements and automatic generation of such diets for specific applications has been described (Smith, 1959; McCann-Rugg *et al.* 1983; Suiter *et al.* 1983; Balintfy & Lancaster, 1985). However, one of the greatest difficulties lies in finding from this infinity of solutions a diet which is acceptable to the individual for whom the diet is planned. The main achievement of the work described here is mathematically to construct diets which are based on individual food preferences, and which are, therefore, more likely to be considered acceptable, but which also satisfy general healthy eating guidelines as well as clinical prescriptions.

The approach adopted is to ask the subject to list the foods eaten on an actual or typical day. Quantities of the foods in this diet are then changed by the smallest amounts necessary to satisfy exactly chosen nutrient requirements. Foods can then be exchanged or extra foods can be included as necessary while still satisfying nutrient target levels. The analysis is not specific to one disease since nutrient targets related to any disease can be specified.

Many mathematical methods are available which could perform the necessary calculations to find the changes to foods, but for the work described here linear programming (LP) (Dantzig, 1963; Ignizio, 1982; Chvátal, 1983) was used. The module which performs the LP analysis is now an extension to the dietary analysis program 'Microdiet' (Salford University 'Microdiet' System, 1990) and uses the same database of nutrient values taken from nutritional tables (Paul & Southgate, 1978; Paul *et al.* 1980). The user can select any of the foods from these tables for consideration within the LP analysis.

Table 1. *Starting diet representing a typical day's intake (g) for a 50-year-old overweight female subject*

Breakfast	
Orange juice	150
Shredded wheat	20
Milk, fresh, whole	100
White bread, toasted	40
Butter	10
Marmalade	20
Lunch	
Tuna, canned in oil	70
Lettuce	30
Cucumber	25
Raw carrot	25
Salad cream	20
Fruit yogurt	150
Wholemeal bread	30
Butter	10
Dinner	
Roast chicken	110
Potatoes, old, chips	180
Peas, fresh, boiled	80
Sponge pudding, steamed	150
Custard, made with powder	80
Snacks and drinks	
Coffee (4 cups)	600
Tea (1 cup)	150
Milk, fresh, whole (30 g per cup)	150
Sugar (5 g per cup)	25
Digestive biscuits	30

Table 2. *Nutrient totals for the starting diet representing a typical day's intake for a 50-year-old overweight female subject*

Energy	10·88 MJ (2602 kcal)
Fibre	15 g
Sodium	2070 mg
Fat	109 g (38 % of energy)

METHODS

Procedure

The procedure for using the LP software package is as follows. The dietitian and client sit together at the microcomputer and input a typical day's diet in terms of foods eaten and the quantities. Any foods from the nutritional tables can be entered at this stage. The dietitian then sets nutrient targets for daily intakes for the client, based on the dietitian's expertise or recommended daily intakes (Committee on Medical Aspects of Food Policy, 1979; National Research Council, Food and Nutrition Board, 1980; National Advisory Committee on Nutrition Education, 1983) or clinical requirements. Several types of target can be set:

(1) upper or lower restrictions, or both, can be placed on nutrients or food quantities,

Table 3. *Nutrient targets set for daily intake for a 50-year-old overweight female subject*

Energy	⩽ 6·27 MJ (1500 kcal)
Fibre	⩾ 20 g
Sodium	⩽ 1600 mg
Fat	⩽ 35% of energy

(2) a selected subset of foods can be required to provide a chosen amount of a nutrient,
(3) carbohydrate, protein or fat can be required to be greater than, less than or equal to a chosen percentage of energy,
(4) foods can be required to satisfy a chosen polyunsaturated:saturated fat (P:S) ratio,
(5) two foods can be linked so that final quantities are in proportion.

The LP program is then run and the amounts of the chosen foods modified by the program so that, in total, they meet the targets. If the modifications are acceptable to the client then the new daily diet is printed out. On the other hand if the modifications are not acceptable, if, for example, too large quantities of some food items are suggested by the program, then other options within the software can be utilized. Further constraints can be set to limit increases in food quantities to practical levels, or the introduction of new foods into the diet can be examined. Subsidiary results from the LP analysis enable all foods in the database to be ranked in terms of their value to the particular diet and nutrient targets. This ensures that foods need only be considered for inclusion if they both assist in meeting the targets and are acceptable to the client. The LP analysis can be re-run at any stage and further modified food quantities produced. This exploration of the diet and food preferences can be continued until a daily diet acceptable to the client and meeting original nutrient targets is produced.

The whole process can be repeated for another day's food intake and a new daily diet meeting the nutrient targets produced. Because the starting point of the analysis is actual food preferences, the resulting daily diet suggestions are as varied as the client's present diet. Moreover, they see modifications to their own diets, not some hypothetical diets remote from their own habits.

Example diet

A sample diet has been chosen to demonstrate results which can be obtained using the diet prescription program. The starting diet, shown in Table 1, represents a typical day's eating pattern for a particular subject (50-year-old overweight female). The nutrient totals for this diet are given in Table 2.

The nutrient targets selected for this subject are given in Table 3. The final diet, balanced for nutrient requirements, is shown in Table 4 and corresponding nutrient totals are shown in Table 5. Several steps were followed in the analysis in order to achieve the balanced diet and the food quantities for each of these steps are shown in Table 6. Stage 1 was the starting quantities selected by the subject. Stage 2 set a daily requirement for energy of 6·27 MJ (1500 kcal) and produced food quantities representing what is, mathematically, the smallest overall change to those in stage 1 necessary to reduce the total energy content to this. The LP procedure achieved this by finding the most concentrated sources of energy in the diet and removing them successively until the overall energy content of the remaining foods is reduced to 6·27 MJ (1500 kcal). The constraints shown in Table 3 were set for all the following stages and produced the quantities in stage 3. The food quantities listed in stage

Table 4. *Balanced diet obtained by linear programming representing a typical day's intake (g) for a 50-year-old overweight female subject*

Breakfast	
Orange juice	150
Shredded wheat	20
Milk, fresh, whole	100
White bread, toasted	40
Butter	5
Marmalade	20
Lunch	
Mackerel, fried	70
Lettuce	30
Cucumber	25
Raw carrot	25
Salad cream	20
Fruit yogurt	150
Wholemeal bread	30
Butter	5
Dinner	
Roast chicken	110
Potatoes, new, boiled	180
Peas, fresh, boiled	80
Dessert apple, raw	150
Snacks and drinks	
Coffee (4 cups)	600
Tea (1 cup)	150
Milk, fresh whole (30 g per cup)	150
Digestive biscuits	30

Table 5. *Nutrient totals for the balanced diet representing a typical day's intake for a 50-year-old overweight female subject*

Energy	6·27 MJ (1500 kcal)
Fibre	20 g
Sodium	1324 mg
Fat	46 g (28% of energy)

1 formed the starting point of each of the LP analyses. The LP procedure found new quantities so that the sum of the differences in food quantities between stage 1 and 3 was the smallest it could be for the nutritional targets in Table 3 to be met. An extra constraint linking sponge pudding and custard resulted in the quantities at stage 4. Additional numerical results produced as an integral part of the LP analysis showed that restricting the fat content of the diet was the most important cause of reductions in food quantities. To reduce the fat content, tuna fish was replaced by mackerel and chips were replaced by boiled potatoes, and stage 5 is the resultant quantities after a further LP run. Finally, quantities of sponge pudding and custard were now too low to be considered to be a reasonable portion and they were removed from the list and replaced by a dessert apple. This allowed the salad cream and half the butter back into the diet at stage 6. This diet was satisfactory to the subject and is the one shown in Table 4. The nutrient totals at each of the stages are shown in Table 7. Food quantities are rounded by the program to the nearest 1 g.

Table 6. *Food quantities obtained by linear programming for intermediate stages in balancing the diet representing a typical day's intake (g) for a 50-year-old overweight female subject*

	Food quantities at stage:*					
Foods	1	2	3	4	5	6
Orange juice	150	150	150	150	150	150
Shredded wheat	20	0	74	66	41	20
Bread, white, toasted	40	16	40	40	40	40
Butter, salted	20	0	0	0	0	10
Marmalade	20	20	20	20	20	20
Coffee, infusion	600	600	600	600	600	600
Milk, fresh, whole	250	250	250	250	250	250
Sugar, white	25	0	0	0	0	0
Tuna, canned in oil	70	70	0	0		
Mackerel, fried					70	70
Lettuce, raw	30	30	30	30	30	30
Cucumber, raw	25	25	25	25	25	25
Carrots, old, raw	25	25	25	25	25	25
Salad cream	20	0	0	0	0	20
Yogurt, fruit	150	150	150	150	150	150
Bread, wholemeal	30	30	30	30	30	30
Tea	150	150	150	150	150	150
Digestive biscuits	30	0	30	30	30	30
Roast chicken	110	110	110	110	110	110
Potatoes, old, chips	180	180	81	0		
Potatoes, new, boiled					180	180
Peas, fresh, boiled	80	80	80	80	80	80
Sponge pudding, steamed	150	0	0	80	34	
Custard, made with powder	80	80	80	43	18	
Apple, raw						150

* 1, Starting quantities selected by subject; 2, setting a requirement for 6·27 MJ (1500 kcal) the quantities represent, mathematically, the smallest overall change to those in stage 1 necessary to reduce total energy to 6·27 MJ (1500 kcal); 3, quantities produced by the constraints of Table 3; 4, quantities produced by the extra constraint linking sponge pudding and custard; 5, quantities produced by replacing tuna by mackerel and chips by boiled potatoes to reduce fat; 6, sponge pudding and custard replaced by eating apple, allowing salad cream and half the butter back into diet.

Description of the mathematical method

The unknown variables for the problem are the food quantities which together meet the nutrient constraints. The variety of possible solutions (the solution space) is decided by the constraints. The starting vector of food quantities selected by the individual, although palatable, is not generally in the solution space. The problem is that of finding a vector of food quantities from this solution space which varies 'as little as possible' from the starting vector of food quantities. The LP formulation used provides a vector of food quantities where the distance between the two vectors is minimized. The requirement 'as little as possible' is formulated by minimizing the sum of the deviations in food quantities from those chosen. This model is shown in algebraic form in Table 8.

Mathematically any combination of nutrient constraints can be accommodated. In practice they will depend on the individual subject together with any diet-related clinical requirements. For example, if the subject is a renal patient then there might be constraints set for energy, protein, sodium, potassium.

Table 7. *Nutrient totals obtained by linear programming for intermediate stages in balancing the diet representing a typical day's intake (g) for a 50-year-old overweight female subject*

Nutrients	Nutrient totals at stage:*					
	1	2	3	4	5	6
Energy (MJ (kcal))	18·88 (2602)	6·27 (1500)	6·27 (1500)	6·27 (1500)	6·27 (1500)	6·27 (1500)
Fibre (g)	15	9	20	20	20	20
Sodium (mg)	2070	976	962	1171	1187	1324
Fat (g)	109	55	38	41	39	46

* For explanation see footnote to Table 6.

Table 8. *Simplified mathematical model for food quantities which meet nutrient constraints*

Find $x_1, x_2, \ldots, x_n$ so as to
minimize $e_1 + e_2 \ldots + e_n$
such that

$$\sum_{j=1}^{n} a_{ij}[f_j - e_j] \lesseqgtr t_i \quad i = 1, 2, \ldots, m$$

and $f_j + e_j \geqslant 0$

where: x_j $(f_j + e_j)$ is the quantity of food j in the solution, f_j is the original quantity of food j, e_j is the deviation in quantity of food j from the original, m is the number of nutrients considered, n is the number of foods, a_{ij} is the number of units of nutrient i in 100 g of food j, t_i is the number of units of nutrient i required.

In the actual model used, deviational variables are included to represent both deviations from chosen food quantities and deviations from chosen nutrient requirements, all of which are minimized in the objective function. This results in a robust system where problems can always be handled since food quantities are changed 'as little as possible' to satisfy nutrient targets 'as closely as possible'. This is guided by the philosophy of goal programming (Anderson & Earle, 1983; Ignizio, 1985).

The module which performs the LP analysis to calculate the changes to food quantities uses the nutrient arrays produced by 'Microdiet'. There are no mathematical limits on the number of foods or nutrient targets selected, although there are some computational limitations on size of problem. However, problems containing up to 100 foods and thirty targets have been handled with ease.

DISCUSSION

The example included in the present paper has demonstrated the power of the method used to satisfy exactly nutritional targets. Clinicians and dietitians have been involved at all stages of development of the software described here and a lively debate has ensued on the benefits of such an exact method. Some feel that dietetic advice of a more general nature is preferable to provision of exactly specified foods and quantities. Others feel that exact specification of diets may improve patient compliance. This aspect of the program's potential is currently being tested in clinical trials.

The need for large changes to food quantities to meet the nutrient targets can sometimes be a problem but can usually be overcome by exchanging foods in the diet or by bringing new foods in from the database. In the example, the overall fat content was reduced by replacing two particular foods by lower-fat alternatives. Instead, the whole milk could have been replaced by skimmed milk and the butter by low-fat spread if this was more to the subject's taste. Continuing with the LP analysis would produce another diet satisfying the nutrient targets but addressing different food preferences.

The normal mode of usage of the software is to introduce various nutritional and palatability constraints at successive stages of the analysis. This allows such constraints to be added as they are needed in producing a satisfactory diet. Editing the food list, by removing, introducing or exchanging foods, can also be done at any stage and in response to the changes in quantities suggested by the LP analysis. Any such editing is applied to the original food list and provides the starting point for subsequent LP analyses of food quantities. On a purely theoretical point, if all the editing and all the targets were applied in a single step and the LP analysis performed, then the resulting list of food quantities would be the same as that obtained by the iterative approach.

Quantities of foods in the example were rounded to the nearest 1 g at each step. However, for some foods (e.g. eggs, slices of bread) rounding would be needed to the nearest portion size and the deviation from nutrient targets caused by this might be unacceptable. One way of dealing with this problem is to use an extra equation to fix the food quantity at the nearest acceptable portion size and, if necessary, re-analyse the problem so that targets can be satisfied exactly by changing other foods in the diet. Alternatively, the food can be replaced by a more suitable one. This was demonstrated in the example when sponge pudding and custard had been reduced to silly quantities by the program. The subject replaced these foods by an apple and in the following step of the analysis all the foods, including the dessert apple, remained in sensible portion sizes.

The educational benefits of using the program are clear. For example, if a low target for fat has been set and the original diet included items such as butter, full-cream milk, high-fat cheese, etc. then these will be reduced by the analysis. It can be demonstrated to users that if some of these foods are replaced by low-fat alternatives such as low-fat spread, skimmed milk and cottage cheese then some of the others are allowed back into the diet while maintaining nutrients at target levels.

Conclusions

In finding a suitable diet for a particular individual, there are many combinations of foods which could satisfy given nutrient requirements. Any given individual will have their own preferences. The present work emphasizes the preferences of the individual by using a diet of foods and quantities chosen by the individual and making as little change as possible to it while meeting the nutritional requirements.

The starting vector of food quantities is already perhaps the most acceptable result for the individual but does not generally meet the requirements of good nutrition. The LP formulation described in the present paper provides the vector of food quantities which is closest to the chosen starting vector. The distance between these two vectors is the minimized change in food quantity.

Sometimes, changes made to food quantities by the program would be considered to be unacceptably large. When this occurs in practice it is necessary to introduce new foods or change some of the foods in the preliminary diet. In this context the program has proved useful as a teaching aid. When dealing with the problem of unacceptably large increases to foods, it is easy using LP to find a new food to bring in to the diet to satisfy the nutrient targets more easily.

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