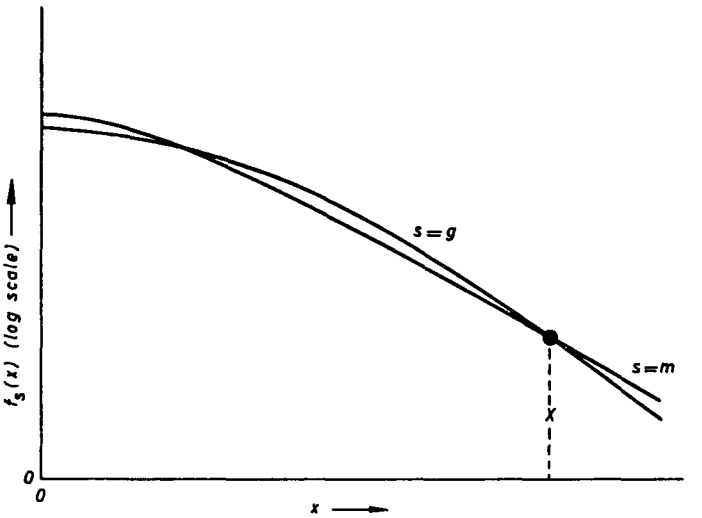

FIG. 2. Schematic comparison between any general $f_m(x)$ and its corresponding $f_g(x)$

is as follows. Since all the w_i and σ_i are positive it is sufficient to prove that:

$$\sum_{i=1}^{n} w_i\sigma_i^2 > \left(\sum_{i=1}^{n} \frac{w_i}{\sigma_i}\right)^{-2}$$

Consider all the w_i replaced by rational numbers m_i/M, where $\sum_{i=1}^{n} m_i = M$. (Even any irrationals can be approximated as closely as we like, for sufficiently large M).

Then, remembering that the σ_i are not all equal, from Tchebychef's inequality:

$$\sum_{i=1}^{n} w_i\sigma_i^2 = \frac{1}{M}\sum_{i=1}^{n} m_i\sigma_i^2 > \frac{1}{M^2}\left(\sum_{i=1}^{n} m_i\sigma_i\right)^2.$$

So, since the arithmetic mean for unequal numbers is greater than the harmonic mean, as required:

$$\sum_{i=1}^{n} w_i\sigma_i^2 > \left[\frac{M}{\sum_{i=1}^{n}(m_i/\sigma_i)}\right]^2 = \left(\sum_{i=1}^{n} \frac{w_i}{\sigma_i}\right)^{-2}.$$

REFERENCE

1 Anderson, O. D. (1976). On error distributions in navigation. This *Journal*, **29**, 69.

The Concept of *Jocobrad*

JOCOBRAD (devised by K. D. Jones, G. R. Cowap and M. R. Bradshaw) has been devised in the light of research into the reactions and responses of mariners to various computer-based radar plotting and forecasting systems. It is a new concept in computer-based radar plotting systems which allows the

operator to ascertain very quickly a safe manœuvre in any encounter, having regard to every other target in the vicinity and to the time period associated with a contemplated manœuvre. The system assumes that some means of echo filtering, acquisition and tracking is available and that a simple program calculates the true course and speed of every target which approaches within radar range. Automatic acquisition and plotting of every such target is essential as manual acquisition would detract from the more important features of JOCOBRAD. JOCOBRAD can be operated in one of three separate modes; useful information being conveyed by interswitching between true and relative motion while operating in two of the three.

1. *The detection of a dangerous encounter.* This is obtained by a vector attached to every acquired target and is projected ahead, thereby conveying the true or relative track of the target; interswitching between true and relative motion indicates stationary targets and targets on a parallel course very quickly and easily.

The vector is of fixed time length and its terminal point represents the target's forecast position at the end of thirty minutes. This is an existing method of presenting data; however the JOCOBRAD vectors differ in that the 30-minute vector can be divided into three separate sections by the use of two controls. When both these controls are at zero the vector is displayed as a solid line. The controls allow the line to be sub-divided into an initial solid line, a central section of dotted line and a final section of solid line, the length of each section being determined by the time settings on the controls. This enables the operator to estimate intermediate time intervals to various crucial situations such as collision or nearest approach.

This mode, when used in conjunction with relative motion, will immediately indicate targets which threaten the observer's domain, and therefore represent a danger. A variable range ring can be placed around own ship at a radius which represents a minimum clearing distance. The area within this circle becomes own ship's domain and any vector that protrudes into that circle is an imminent danger. Targets on collision or near collision courses indicating that a collision will occur sometime in the future, but more than 30 minutes hence, will be associated with vectors that point towards own ship's domain but do not enter it.

2. *Selection of a safe manœuvre and detection of a potentially dangerous manœuvre.* In other systems this type of information is available in a manner which is time consuming and requires the operator to select a manœuvre he considers suitable, then setting the controls to the numerical values of the proposed manœuvre and observing the information on the display; which he can then readily accept or dismiss. The JOCOBRAD system on the other hand tackles the decision making problem in a different manner. The three separate sections of the JOCOBRAD vector, as described in the Mode 1 operation, define three separate legs of any deviation.

(i) The first leg represents the track of every target, whilst own ship maintains her present course and speed, during a short decision period prior to the execution of any manœuvre.

(ii) The second leg represents the forecast track of every target during the time interval represented by the length of this second vector, for which a proposed manœuvre will be maintained.

(iii) The third leg represents the forecast track of every target during the remaining time of the 30-minute vector, for which the original course and

speed is resumed. It is parallel to first leg vector. If the total manœuvre exceeds 30 min. this vector will not appear, thus indicating an unacceptably lengthy manœuvre.

The length of the second vector can be altered by adjusting the 'hold' control and the length of the third vector is then the remaining length, such that the combined time period of the three vectors represents 30 min. A 'trial course' and 'trial speed' control adjusts the direction of the centre vector on each target. The final vector is tied to the terminating point of the second vector and the second vector commences at the terminal point of the first vector. These three vectors define the motion on the PPI of all the targets for a proposed alteration of course and/or speed which will take place after a time defined by the 'delay' control and maintained for a period defined by the 'hold' control, after which it is supposed that the original course and speed will be resumed. When in relative motion each target will have a 'dog-leg' vector. By adjusting the four controls mentioned above, the vectors attached to each target can be adjusted such that they all clear the predetermined own ship's domain.

The essential concept of JOCOBRAD is that an acceptable manœuvre is determined by observing the display while controls are adjusted to produce a 'picture' representative of a safe and acceptable manœuvre. When the 'picture' has been properly adjusted the settings on the controls can be read to give

(i) Delay time before any action taken.
(ii) Course and speed necessary for proposed action.
(iii) Time to hold the proposed action before returning to the original course and speed.

When in true motion each target will have a straight vector, but made up of the three separate sections described above. In addition, own ship will have the 'dog-leg' vector. This mode is not as useful in selecting an anti-collision manœuvre, but is useful when manœuvring in a buoyed channel or in restricted waters, as the vectors associated with own ship represent the true track throughout the manœuvre and can be more readily associated with the chart.

3. *Target tracking during the execution of a manœuvre.* Any decision made, concerning a collision avoidance action, is made on the assumption that all targets maintain their course and speed. In any multiple encounter such an assumption is false. Other targets will also take evasive action which would hopefully improve the situation and increase the clearance of own ship. It is essential that every target is watched to ensure that the situation does not become dangerous due to some subsequent action of any target.

To do this JOCOBRAD incorporates a mode of operation which allows the operator to freeze the vectors on the PPI and at the same time initiates a clock. At this stage the echoes start to move along the vectors. Providing that the operator (by using the clock) carries out the proposed manœuvre at times indicated by the controls, then the echoes should track along the vector providing no action is taken by any target. Any subsequent action by a target will cause the echo to deviate from the fixed vector and the operator can immediately detect whether any such manœuvre will improve or endanger the situation, by observing the way in which the echo deviates from the fixed vector. The operator will immediately observe any such development at a glance and need not maintain constant radar watch.