

## DESIGN FOR EXTREMES: A CONTOUR METHOD FOR DEFINING REQUIREMENTS BASED ON MULTIVARIATE EXTREMES

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### ABSTRACT

The design of various products is driven by requirements that describe extremes. In marine structural design, joint extremes of environmental variables like wave height and wind speed are used to define load cases. Similarly, in ergonomic design minimum and maximum values of anthropometric variables are considered to make sure a product is suitable for a wide range of users. Here, we present a method that supports designers to define requirements using joint extreme values: the requirements contour method. The method is based on structural engineering's environmental contour method and uses a dataset and statistical methods to specify a region in the variable space that must be considered in the design process. That region's enclosure is the requirements contour and holds the joint extremes. After formally describing the method, we give an illustrative example of its usage: we use it to define requirements for the design of an ergonomic handle for a power tool. The requirements contour method is a field-independent approach to design for extremes. In the tradition of design for X, we think that a design project can benefit from applying methods that focus on different 'X's.

**Keywords:** Design for X (DfX), Requirements, Computational design methods, Probabilistic design, Environmental contour method

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# 1 INTRODUCTION

A product's design is often driven by requirements that describe extremes. A wind turbine must preserve structural integrity under all environmental conditions in which it is intended to be used. Its design will be driven by the expected maximum wind speed. Similarly, a car must drive properly under all environmental conditions that can be expected in its intended geographic region of operation. To prevent the freezing of liquids and malfunctions due to contracting materials, the design of a car should thus take into account the expected minimal temperature. The car's seat design, on the other hand, will be driven to be ergonomic for both extraordinarily short and extraordinarily tall persons.

Challenges arise when extremes of multiple variables need to be controlled for simultaneously. For example, the designer of a car seat might consider two variables, a person's total height and a person's hip height. The seat should then fit a person with joint maximum total height and hip height but also a person with joint minimum total height and hip height. However, it is questionable whether the seat should also be designed for someone with maximum total height and minimum hip height, as the existence of such a person is extremely unlikely.

The problem of how to combine extremes of multiple variables has been discussed extensively for the design of marine structures. A marine structure, for example an offshore wind turbine, must withstand the joint load from simultaneously occurring maximum wind speeds and maximum wave heights. Standards (International Electrotechnical Commission, 2009) and guidelines (DNV GL, 2017) describe how this can be achieved in practice.

In the design of marine structures arguably the most systematic approach to obtain joint extremes of environmental variables is to consider a contour that encloses all environmental states where the joint probability density function of the variables of interest is not negligible. This approach is called the environmental contour method (Figure 1). It is often used to define extreme environmental loads for structural integrity calculations and its origin can be traced back to the work of Haver (1985, 1987). Haver's environmental contour, originally called design curve, is a continuous curve parameterized by two variables of interest.

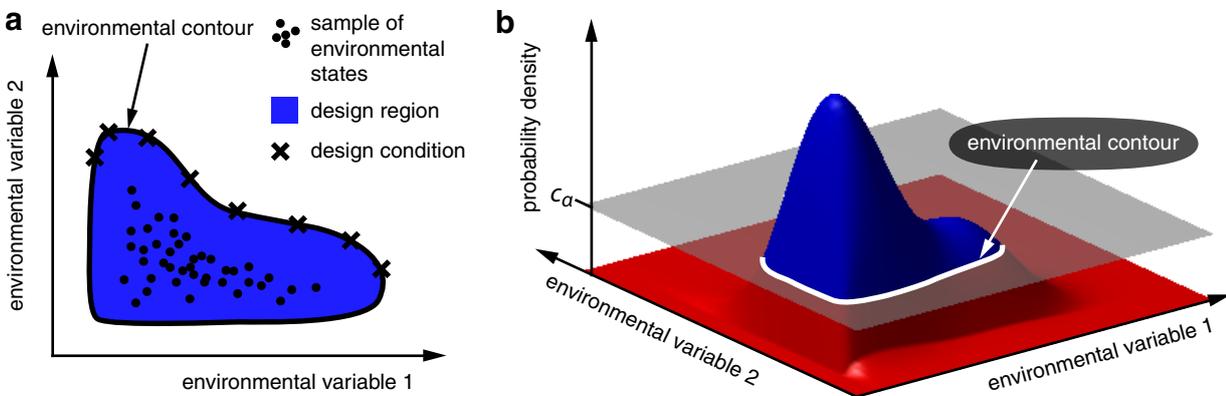


Figure 1. (a) Concept of an environmental contour. (b) Environmental contours are based on the joint probability density function of the considered environmental variables. In the illustrated case, the contour is defined via a threshold parameter  $c_\alpha$ . This parameter is chosen such that the probability of the occurrence of an environmental state outside the region enclosed by the environmental contour is  $\alpha$ . This specific construction method is called the highest density contour method.

Later, Winterstein *et al.* (1993) proposed a new way of constructing environmental contours, the so-called inverse first-order reliability method (IFORM), which was developed within the framework of structural reliability theory [see, for example, Madsen *et al.* (2006)]. To this day, numerous methods for the construction of an environmental contour have been proposed (see, for example, Huseby *et al.*, 2013; Jonathan *et al.*, 2014; Haselsteiner, Ohlendorf, Wosniok and Thoben, 2017; Chai and Leira, 2018; Manuel *et al.*, 2018; Vanem, 2018). The process of developing an environmental contour typically involves the estimation of the joint probability density function of the variables of interest and contour construction in the narrower sense. For the latter, an algorithm has to be applied that yields a contour which encloses all environmental states that can be reasonably expected to occur during the lifetime of

the considered product. Note that in multivariate statistics, there exist multiple definitions for extreme value exceedence [see, for example, [Serinaldi \(2015\)](#), or [Einmahl et al. \(2013\)](#)] and that the shape of the obtained environmental contour strongly depends on which of those definitions is used.

In summary, a wide range of products should be designed to properly function under the influence of joint extremes. Consequently, product designers need methods that provide realistic design conditions that ensure a high reliability. At the same time, the methods should not provide unrealistic design conditions that put unnecessary burden on the design task. Marine structures, like offshore wind turbines or ships, are a category of products for which this problem has already been explored extensively. Possibly, the concepts of methods that have been developed for marine structural design, can be transferred to other fields that deal with designing for extremes. In this paper, we show how the environmental contour method can be changed to suit non-environmental requirements such that it can be applied to other fields of engineering design.

The paper is organized in four sections: First, we describe the environmental contour method (Section 2). Then, we formulate a field-independent and time-undefined version of the method: the requirements contour method. After describing this method formally (Section 3), we give an illustrative example that deals with the design of an ergonomic handle for power tools (Section 4). The requirements in this example will be the user's hand width and hand height.

## 2 ENVIRONMENTAL CONTOUR METHOD

The environmental contour method is a rational way of defining extremes of environmental variables for engineering design. In practice, designers or analysts use it for obtaining a set of design conditions from an observed or artificially generated sample of environmental states. The overall approach usually comprises three steps: (i) statistical modelling, (ii) contour construction and (iii) design condition selection. Sometimes a statistical model can be defined beforehand such that only the last two steps are required.

### 2.1 Statistical modelling

In engineering, environmental conditions such as waves and wind are often modeled as random processes. It is common practice to assume that the long-term evolution of environmental conditions can be considered as a sequence of stationary processes ([Naess and Moan, 2013](#)). That is, the random process associated with a certain environmental condition is assumed to be stationary for fixed time intervals of equal length. Common choices for the state duration  $T_S$  are, for example,  $T_S = 1$  hour,  $T_S = 3$  hours, or  $T_S = 6$  hours. The statistical description of the environment during the state duration  $T_S$  is then referred to as 'short-term statistics' while so-called 'long-term statistics' are used to describe the environment on a time-scale of years. Long-term statistics deal with time-integrated parameters like the 'significant wave height', the 'spectral peak period' [these two parameters would describe a 'sea state', see, for example, [Ochi \(1998\)](#)] or the '1-hour mean wind speed' (together, these three parameter would describe an 'environmental state'). These time-integrated parameters are then used to parametrize spectra that describe the short-term statistics. The basis of the spectra could be the water surface elevation or the wind's instantaneous velocity.

The environmental contour method deals with the environment's long-term statistics. The environment is modeled with  $d \in \mathbb{N}$  random variables, where each random variable represents one environmental variable. We use  $\mathbf{X} = (X_1, \dots, X_d)^T$  to denote the respective random vector, and  $f(\mathbf{x})$  to denote the associated joint density function, where  $\mathbf{x} \in \mathbb{R}^d$  is a realization of  $\mathbf{X}$ .

Most of the time, the exact probability distribution of  $\mathbf{X}$  is not known *a priori*. In such a setting, the density function  $f(\mathbf{x})$  can be estimated using a set of measured (or simulated) observations  $O$  that are representative for the environmental variables during the respective time period. For simplicity, it is assumed that these observations are independent and identically distributed.

In the context of the environmental contour method, density estimation is often done by fitting the parameters of a previously selected distribution to the obtained sample of environmental states via maximum likelihood estimation (MLE) [see, for example, [Li et al. \(2015\)](#)]. However, some authors have also considered non-parametric methods, specifically kernel density estimation ([Eckert-Gallup and Martin, 2016](#); [Haselsteiner, Ohlendorf and Thoben, 2017](#)).

## 2.2 Contour construction

The purpose of contour construction in the narrower sense is to define an environmental contour based on the random vector  $\mathbf{X}$  that encloses all environmental states in  $\mathbb{R}^d$  that can be reasonably expected to occur.

As described in the introduction, various methods for the construction of an environmental contour based on a given random vector have been proposed. Here, we consider the so-called highest density contour method described in the work of [Haselsteiner, Ohlendorf, Wosniok and Thoben \(2017\)](#), which is based on the statistical concept of the highest density region. Based on the work of [Hyndman \(1996\)](#), a highest density region  $R(c_\alpha) \subset \mathbb{R}^d$  for a probability density function  $f(\mathbf{x})$  and a parameter  $\alpha \in [0, 1]$  can be defined in the following manner:

$$R(c_\alpha) = \{\mathbf{x} \in \mathbb{R}^d : f(\mathbf{x}) \geq c_\alpha\}, \quad (1)$$

where  $c_\alpha$  is chosen as the largest threshold that yields a region, which covers at least  $1 - \alpha$  of  $f$ , that is

$$c_\alpha = \operatorname{argmax}_{c \in [0, \infty)} \Pr(\mathbf{X} \in R(c)) \geq 1 - \alpha. \quad (2)$$

Then, the  $100(1 - \alpha)\%$  contour is defined as the set  $C(\alpha) \subset R(c_\alpha)$  that contains exactly the environmental states at which the probability density equals  $c_\alpha$ :

$$C(\alpha) = \{\mathbf{x} \in \mathbb{R}^d : f(\mathbf{x}) = c_\alpha\}. \quad (3)$$

Note that in practice, one typically uses numerical approximations to obtain  $R(c_\alpha)$  and the associated contour  $C(\alpha)$  [see, for example, [Haselsteiner, Ohlendorf, Wosniok and Thoben \(2017\)](#)].

Equations 1 – 3 indicate that constructing an environmental contour for a probability density function  $f(\mathbf{x})$  is done by first selecting  $\alpha$ , second computing  $R(c_\alpha)$  and eventually computing  $C(\alpha)$ . In the environmental contour method,  $\alpha$  is usually chosen based on a desired value for the so-called return period  $T_R$  in the sense that

$$\alpha = \frac{T_S}{T_R}, \quad (4)$$

where  $T_S$  is the state duration associated with the environmental variables and  $T_S$  and  $T_R$  are assumed to be given in the same unit of time. A typical value of  $T_R$  for the design of marine structures is 50 years. Due to the definition of the highest density region, the return period  $T_R$  is equal to the expected period of time between two events of exceedance. An event of exceedance  $\mathbf{x}_e$  is defined as the occurrence of an event anywhere outside the design region, that is,  $\mathbf{x}_e \in \mathbb{R}^d \setminus R(c_\alpha)$ .

Note that the definition of an environmental contour in Equation (3) does not necessarily yield a contour in the sense of a single closed curve or surface. In particular, this might not be the case if the considered probability density function  $f(\mathbf{x})$  is not continuous, constant in one or more neighborhoods of  $\mathbb{R}^d$ , or if  $f(\mathbf{x})$  is multimodal. In practice, we strongly advice to only use the environmental contour method if  $f(\mathbf{x})$  can be guaranteed to be at least continuous.

## 2.3 Design condition selection

The traditional environmental contour method is often used for structural analysis simulations where points along the contour serve as input parameters of the simulation. Note that an environmental contour typically contains infinitely many points. However, in practice, only finitely many simulations can be performed. Selecting a set  $P = \{\mathbf{p}_1, \dots, \mathbf{p}_m\} \subset C$  of  $m \in \mathbb{N}$  so-called design conditions is thus a critical task in the preparation of a structural analysis simulation. Currently, however, to the authors' knowledge there exist no formal methods or engineering guidelines that cover this step.

## 3 REQUIREMENTS CONTOUR METHOD

Because the environmental contour method is time-defined in the sense that it is used to find a contour with return period  $T_R$ , the method is only suited for variables that are time-defined. Thus, the environmental contour method requires the vector  $\mathbf{X}$  to represent environmental states that have an associated state duration  $T_S$  (see Section 2.1). However, the method's rational of giving designers a set of well-defined joint extremes, which serve as design requirements, could also be useful for requirement

variables that have no duration associated to them. Then the method could also be applied to requirements that are not based on the environment, but on other entities like humans, other biological systems or technical systems.

### 3.1 Statistical modelling and contour construction

Let us formulate a contour method that can be applied to generic, time-undefined properties of an entity, which can be described using random variables. Similar to the environmental contour method the method comprises the three steps statistical modelling, contour construction and design condition selection (Figure 2 a). Statistical modelling deals with estimating the random vector  $\mathbf{X}$ . In the requirement contour method,  $\mathbf{X}$  holds  $d$  'requirement variables' that represent  $d$  properties of a common entity (Figure 2 b). In ergonomic design the entity could, for example, be the human hand and its properties hand width and hand length. However, the requirement variables do not need to describe anthropometric properties: consider a robot that shall be designed to safely pick up soft objects, for example fruits. Then, the entity would be the fruit of interest, say a kiwi, and its properties, for example, the kiwi's stiffness, the kiwi's horizontal perimeter and the kiwi's vertical perimeter.

In comparison to the environmental contour method's 'environmental variables', these 'requirement variables' have no state duration  $T_S$ . This difference does not greatly influence the available methods that are suitable for statistical modelling.  $\mathbf{X}$  can be estimated using similar methods as those being applied in the context of the environmental contour method. Consequently, parametric statistical models that model the properties with simple distributions and dependence functions might be applied. Alternatively, non-parametric methods like multivariate kernel density estimation can be used.

Contour construction, however, has to be adapted because a generic requirement variable has no associated state duration and consequently we cannot define  $\alpha$  based on the state duration and return period (see Equation 4). Instead, we calculate  $\alpha$  based on the expected number of events between two consecutive events of exceedance,  $\bar{n}$ :

$$\alpha = \frac{1}{\bar{n}}. \quad (5)$$

For example, consider that we would like to design a car seat such that on average, all but every tenth randomly selected adult person could comfortably sit in the seat. Then  $\bar{n} = 10$  and  $\alpha = 0.1$ .

Based on a given value for  $\alpha$  the contour construction technique described in Section 2.2 can be applied. Other contour construction methods are suitable too and based on the specific design project, one might apply another technique (see, for example, Winterstein *et al.*, 1993; Huseby *et al.*, 2013; Jonathan *et al.*, 2014; Chai and Leira, 2018). Constructing a contour based on a highest density region, however, has a potential advantage for the design process: for a given probability content this region occupies the smallest possible volume in the sample space. That means that a smaller range of variable values must be taken into account compared to other definitions for a design region that contains  $1 - \alpha$  probability.

### 3.2 Selecting and using design conditions

Which design conditions shall be selected, depends on the particular context and we do not try to lay out an algorithm for that task. In abstract terms, the design conditions  $P$  are used for engineering design synthesis, analysis and evaluation (Figure 3 a).

In the environmental contour method, the typical concrete use case for working with design conditions is performing a structural integrity analysis (analysis and evaluation activities) and an iterative optimization of the structural design (an engineering design synthesis activity). Each design condition represents an environmental state and the designers expect that their structural design withstands the extreme environmental loads that correspond to the given environmental states. To evaluate whether this is true, the designers analyze if their design actually withstands the environmental loads by performing a limit state analysis.

For that purpose, often a failure function,  $g(\mathbf{x})$ , is defined. The failure function separates the sample space into three sets, a survival region  $S$ , a limit surface  $L_Z$  (that contains the limit states) and a failure region  $\mathcal{F}$  (Madsen *et al.*, 2006). Although failure function and limit state analysis are concepts from structural reliability theory, its rational and mathematics can be applied to other fields of design as well. Consequently, let the failure function describe whether a design  $D$  fulfills the requirements for entity  $\mathbf{x}$ . Then  $g(\mathbf{x}, D) > 0$  describes that the design  $D$  fulfills the requirement for entity  $\mathbf{x}$  and  $g(\mathbf{x}, D) < 0$

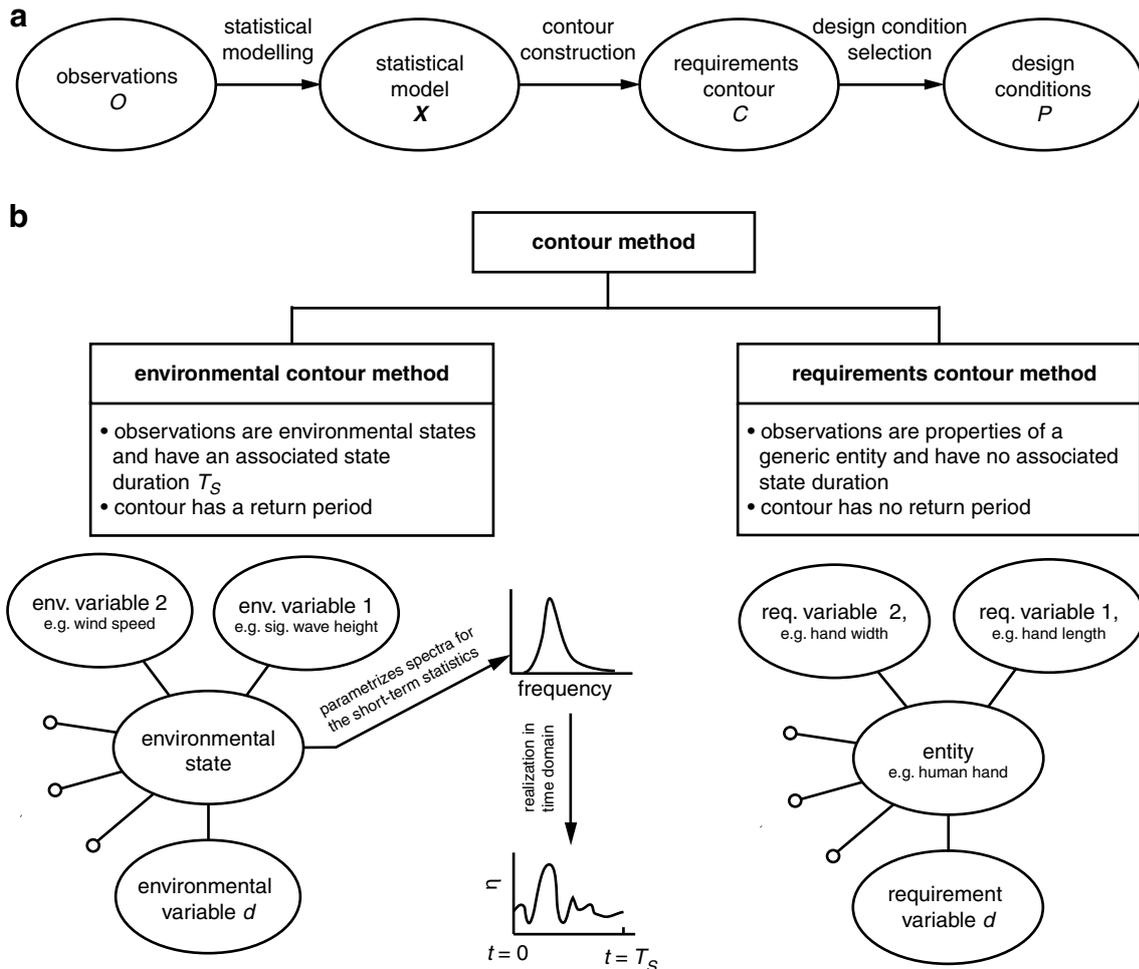


Figure 2. Requirements contour method and its difference to the environmental contour method. (a) In three steps, the requirements contour method maps a sample of properties onto a set of ‘design conditions’. (b) While the environmental contour method is based on time-defined ‘environmental variables’, the requirements contour method deals with ‘requirement variables’, which are time-undefined properties of a common entity.

describes that the design fails to fulfill the requirements for entity  $\mathbf{x}$ . For simplicity, without  $D$ :

$$\begin{aligned}
 g(\mathbf{x}) &> 0, & \mathbf{x} &\in S \\
 g(\mathbf{x}) &= 0, & \mathbf{x} &\in L_Z \\
 g(\mathbf{x}) &< 0, & \mathbf{x} &\in \mathcal{F}.
 \end{aligned} \tag{6}$$

To analyze if the design performs satisfactory at the required  $\alpha$ -value (meaning that it fulfills the requirements for all entities that must be considered), one evaluates if the failure function is positive for all selected design conditions. If this criteria is fulfilled, one can reasonably assume that the failure region does not overlap with the design region (Figure 3 b) and consequently that the design has a failure probability  $P_f < \alpha$  where  $P_f$  is defined as the likelihood that an event within the failure region occurs,  $P_f = \Pr(\mathbf{X} \in \mathcal{F})$ .

This assumption, however, is only reasonable if the worst outcome among all entities within the design region, occurs at an entity along the contour. In other words, if the failure function were evaluated for all  $\mathbf{x} \in R$ , its minimum must occur at  $\mathbf{x} \in C$ :

$$\underset{\mathbf{x} \in R}{\operatorname{argmin}} g(\mathbf{x}) \in C. \tag{7}$$

This condition justifies the requirements contour method’s (and the environmental contour method’s) efficient approach of considering only design conditions along the contour, compared to an analysis where the complete design region is evaluated.

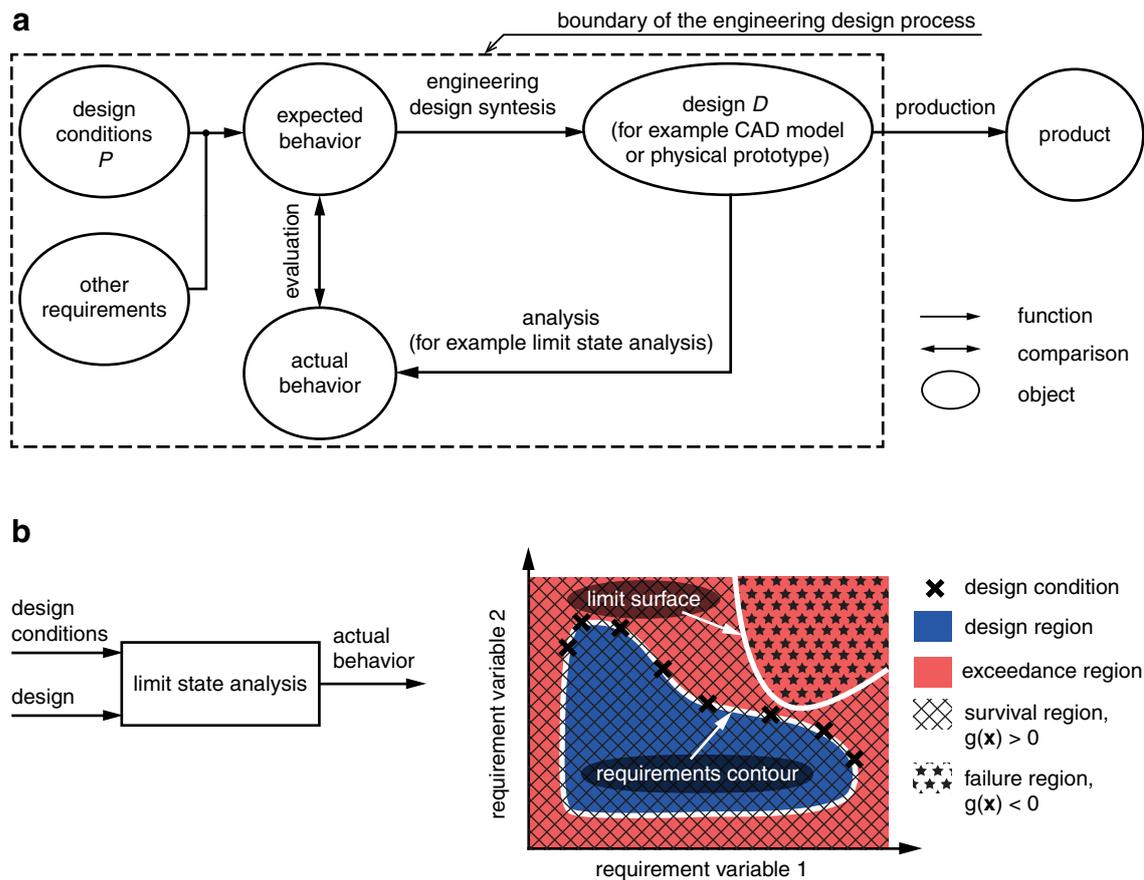


Figure 3. Working with 'design conditions'. (a) Model of the engineering design process and the role that design conditions play in it. Parts of the model's structure are based on Gero (1990). (b) As in structural design, design conditions from a requirements contour can be used in a subsequent limit state analysis. If the failure region does not overlap with the design region the design fulfills the requirement at the specified  $\alpha$ -value.

#### 4 ILLUSTRATIVE EXAMPLE: HANDLE DESIGN OF A POWER TOOL

To illustrate the usage of the requirements contour method, let us consider an example that deals with ergonomics. For many tasks, humans operate tools and often, the preferred way of interacting with the tool is holding it with their hands. The tool's handle is the interface between the tool and the hand. It should enable a secure and comfortable grip. Designing a handle with these properties is a typical activity of ergonomics and is also a subject of academic research.

Wang and Cai (2016) developed a guideline for hand tool handle design based on anthropometric measurements of hands. They proposed that the most comfortable design for the handle is an elliptic cylinder with a ratio of 1:1.25 between the minor and major diameter of the ellipse. Recently, Matthiesen and Germann (2018) studied power tool handle design. A power tool's handle usually comprises one or multiple buttons to control the tool (Figure 4 a). Consequently, the handle cannot be a simple elliptic cylinder. Matthiesen and Germann (2018) investigated predictors for the suitability of usage of power tools and analyzed the handle's geometry as well as anthropometric variables. They found that the anthropometric variables thumb length, hand length and the hand width are meaningful predictors for the suitability of usage. Although such results have not been presented by Matthiesen and Germann (2018), to motivate our example, let us assume that the suitability of usage was worst for users with hands whose anthropometric variables were extreme. Under this assumption the requirements contour method is an appropriate method to support ergonomic design.

Thus, we will lay out how the requirements contour method can be applied to design the handle of a power tool. We will derive a requirements contour based on hand measurements. This contour represents 'extreme users' and we expect that some of them experience more difficulties in holding the tool comfortably compared to 'average users'. Then, we will select a set of design conditions along the contour. The design and evaluation of the handle itself, however, are only considered theoretically.

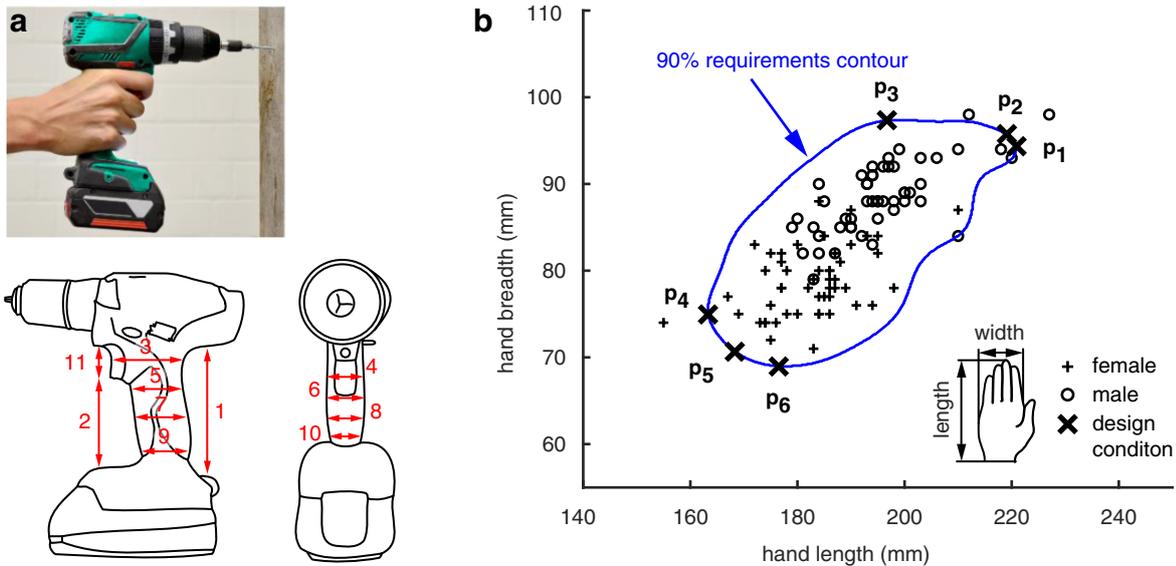


Figure 4. (a) Power tool handle design [adapted from Matthiesen and Germann (2018) with permission from the authors]. The numbers refer to the geometric parameters that were analyzed by Matthiesen and Germann (2018). (b) Requirements contour for the handle design of a power tool.

#### 4.1 Data and methods

We used anthropometric data covering 50 female and 50 male individuals from the ANSUR dataset (Gordon *et al.*, 2014) as the sample  $O$ . Then, we estimated the probability density using bivariate kernel density estimation with Gaussian kernels whose bandwidth in dimension  $i$ ,  $b_i$ , was selected using Silverman's rule of thumb (Silverman, 1998):

$$b_i = \sigma_i \left( \frac{4}{(d+2)n} \right)^{1/(d+4)}, \quad (8)$$

with  $\sigma_i$  being the standard deviation in dimension  $i$ ,  $d = 2$  being the number of dimensions and  $n = 100$  being the number of observations. Based on the estimated density function, we constructed a 90% highest density contour using the open-source implementation `compute-hdc` (Haselsteiner, 2018). The contour was defined by  $\bar{n} = 10$ , which lead to  $\alpha = 0.1$ . Lastly, we selected six design conditions,  $P = \{\mathbf{p}_1, \dots, \mathbf{p}_6\}$ , along the contour. Four design conditions were the vectors  $(\max_{x_1, x_2} | \max_{x_1})$ ,  $(\min_{x_1, x_2} | \min_{x_1})$ ,  $(\max_{x_2, x_1} | \max_{x_2})$  and  $(\min_{x_2, x_1} | \min_{x_2})$ . The remaining two design conditions were the point along the contour where the product of the vector's elements is maximized and the point along the contour where the product of the vector's elements is minimized.

#### 4.2 Results and possible next steps

The 90% requirements contour encloses all but four measurements (Figure 4 b) and the selected design conditions range from 177 to 221 mm hand length and from 69 to 97 mm hand width.

As a next step, the six design conditions could be used to analyze a potential handle design. The analysis could be based on a limit state analysis approach as outlined in Section 3.2. Thus, one would define a failure function,  $g(\mathbf{x})$ . As a simple case, the failure function could be based on the comfort of the handle and could be approximated by asking participants to rate the comfort of the handle on a scale from 1 to 5 (with 5 being the best): let  $h(\mathbf{x}) \in \{0, \dots, 5\}$  be that rating. Then a minimum value for the handle's comfort, for example  $h_{min} = 3$ , could be defined. In that case the failure function would be  $g(\mathbf{x}) = h(\mathbf{x}) - h_{min}$ .

Next, one would need to recruit participants whose hands resemble the set of design conditions and let these participants test and rate the potential handle design of the power tool. If the failure function based on the ratings of these persons is positive, that means  $g(\mathbf{x} = \mathbf{p}) > 0$  for all  $\mathbf{p} \in P$ , the design fulfills the requirements. On the other hand, if  $g(\mathbf{x} = \mathbf{p}) \leq 0$  for any  $\mathbf{p} \in P$ , either the design has to be changed or the requirements have to be adapted.

## 5 DISCUSSION AND CONCLUSIONS

Here, we showed how the concept of structural engineering's environmental contour method can be used to formulate a method that deals with generic, time-undefined requirement variables. The presented requirements contour method is a rational way of defining quantitative requirements for products that need to properly function in the presence of joint extremes. Compared to some other approaches that consider the complete 'design region', in other words, approaches that consider all statistically relevant values of a requirement variable, the contour method leads to fewer design conditions that need to be considered. Consequently, the design process could be accelerated. This acceleration, however, is only justified if the worst situation for the design occurs at an extreme.

In this paper, we gave an illustrative example from the field of ergonomics. We note that researchers in ergonomics have developed their own set of methods to statistically define regions of anthropometric variables that designers should consider. Often designers decide to exclude about 1.5 to 10% of the target population (Molenbroek *et al.*, 2003). Possibly, the most popular approach among practitioners is to use percentiles of individual variables. Often the 5th and 95th percentile are used to define the minimum and maximum values that are considered [see, for example, Jürgens *et al.* (1998)]. However, researchers have also developed multivariate approaches for ergonomics, one example is the tool 'Ellipse' that is based on bivariate normal distributions that are fitted to anthropometric data (Molenbroek, 2000). This tool gives designers insights, how much percentage of the target population they exclude, if they use arbitrary univariate percentiles for the two anthropometric variables of interest. To the authors' knowledge, however, no ergonomics method applies the concept of a highest density region to an arbitrary joint distribution or uses mathematical reasoning to justify why only joint extremes, instead of the whole body of a distribution, are considered.

The method we presented here is field-independent and was not developed specifically for ergonomic design. It could potentially be useful in other fields of engineering design, for example for tolerance management. Consider that the entity of interest is a mass-produced mechanical part. Usually, tolerances are given individually for each dimension of the part. The requirements contour method could be used to define joint tolerances of multiple dimensions based on a joint probability distribution. Potential applications are whenever joint extremes are a good description for requirements. In principle, the entity whose properties serve as requirements could be any biological or a technical system.

In the product development process, designers need to fulfill different kinds of requirements: some are qualitative, some are quantitative, and some of the quantitative requirements describe extremes. In the tradition of design for X [DFX; Kuo *et al.* (2001)], here, we introduced a method to design for one specific aspect, a method to design for extremes. The idea is that, in practice, during a holistic product development project, designers use 'design for extremes methods' when the situation asks for them.

We believe that the increasing amount of available data and increasing advancement and usage of computer methods in engineering design supports the use of formal probability-based methods. The proposed requirements contour method is a general purpose formal method to assist designers when they design for extremes.

## DATA AVAILABILITY AND ACKNOWLEDGEMENTS

The datasets analyzed during this study and the used software are available in the GitHub repository 'iced2019', <https://github.com/ahaselsteiner/iced2019>. The requirements contour shown in Figure 4 can be reproduced by running the contained Matlab file `computeContourForHandleDesign.m`.

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## REFERENCES

- Chai, W. and Leira, B. J. (2018), "Environmental contours based on inverse SORM", *Marine Structures*, Vol. 60, pp. 34–51.
- DNV GL (2017), Recommended practice DNVGL-RP-C205: Environmental conditions and environmental loads, Technical report.
- Eckert-Gallup, A. and Martin, N. (2016), "Kernel density estimation (KDE) with adaptive bandwidth selection for environmental contours of extreme sea states", in *OCEANS 2016 MTS/IEEE Monterey*, IEEE, Monterey, CA, USA, pp. 1–5.

- Einmahl, J. H. J., Haan, L. D. and Krajina, A. (2013), “Estimating extreme bivariate quantile regions”, *Extremes*, Vol. 16, pp. 121–145.
- Gero, J. S. (1990), “Design prototypes : A knowledge representation schema for design”, *AI Magazine*, Vol. 11 No. 4.
- Gordon, C. C., Blackwell, C. L., Bradtmiller, B., Parham, J. L., Barrientos, P., Paquette, S. P., Corner, B. D., Carson, J. M., Venezia, J. C., Rockwell, B. M., Mucher, M. and Kristensen, S. (2014), 2012 Anthropometric survey of U.S. army personnel : methods and summary statistics, Technical report, U.S. Army Natick Soldier Research, Development and Engineering Center.
- Haselsteiner, A. F. (2018), “compute-hdc: An open-source implementation of the highest density contour method in Matlab (version 1.1.0)”. <https://github.com/ahaselsteiner/compute-hdc/releases/tag/1.1.0>
- Haselsteiner, A. F., Ohlendorf, J.-H. and Thoben, K.-D. (2017), “Environmental contours based on kernel density estimation”, in *Proc. 13th German Wind Energy Conference (DEWEK 2017)*, Bremen, Germany.
- Haselsteiner, A. F., Ohlendorf, J.-H., Wosniok, W. and Thoben, K.-D. (2017), “Deriving environmental contours from highest density regions”, *Coastal Engineering*, Vol. 123, pp. 42–51.
- Haver, S. (1985), “Wave climate off northern Norway”, *Applied Ocean Research*, Vol. 7 No. 2, pp. 85–92.
- Haver, S. (1987), “On the joint distribution of heights and periods of sea waves”, *Ocean Engineering*, Vol. 14 No. 5, pp. 359–376.
- Huseby, A. B., Vanem, E. and Natvig, B. (2013), “A new approach to environmental contours for ocean engineering applications based on direct Monte Carlo simulations”, *Ocean Engineering*, Vol. 60, pp. 124–135.
- Hyndman, R. J. (1996), “Computing and graphing highest density regions”, *The American Statistician*, Vol. 50 No. 2, pp. 120–126.
- International Electrotechnical Commission (2009), Wind turbines - part 3: Design requirements for offshore wind turbines, Technical Report IEC 61400-3:2009-02.
- Jonathan, P., Ewans, K. and Flynn, J. (2014), “On the estimation of ocean engineering design contours”, *Journal of Offshore Mechanics and Arctic Engineering*, Vol. 136 No. 4, pp. 41101–1 to 041101–8.
- Jürgens, H., Matzdorf, I. and Windberg, J. (1998), “Internationale antropometrische Daten als Voraussetzung für die Gestaltung von Arbeitsplätzen und Maschinen”, in *Arbeitswissenschaftliche Erkenntnisse: Forschungsergebnisse für die Praxis*, Bundesanstalt für Arbeitsschutz und Arbeitsmedizin, Dortmund, Germany.
- Kuo, T. C., Huang, S. H. and Zhang, H. C. (2001), “Design for manufacture and design for ‘X’: Concepts, applications, and perspectives”, *Computers and Industrial Engineering*, Vol. 41 No. 3, pp. 241–260.
- Li, L., Gao, Z. and Moan, T. (2015), “Joint environmental data at five European offshore sites for design of combined wind and wave energy devices”, *Journal of Offshore Mechanics and Arctic Engineering*, Vol. 137, p. 031901.
- Madsen, H. O., Krenk, S. and Lind, N. C. (2006), *Methods of structural safety*, Dover Publications, Mineola, New York, USA.
- Manuel, L., Nguyen, P. T. T., Canning, J., Coe, R. G., Eckert-Gallup, A. C. and Martin, N. (2018), “Alternative approaches to develop environmental contours from metocean data”, *Journal of Ocean Engineering and Marine Energy*, Vol. 4 No. 4, pp. 293–310.
- Matthiesen, S. and Germann, R. (2018), “Meaningful prediction parameters for evaluating the suitability of power tools for usage”, in *Procedia CIRP*, Vol. 70, pp. 241–246.
- Molenbroek, J. F. (2000), “Making an anthropometric size system interactively”, in *Proc. of the Human Factors and Ergonomics Society Annual Meeting*, Vol. 44, pp. 766–769.
- Molenbroek, J. F., Kroon-Ramaekers, Y. M. and Snijders, C. J. (2003), “Revision of the design of a standard for the dimensions of school furniture”, *Ergonomics*, Vol. 46 No. 7, pp. 681–694.
- Naess, A. and Moan, T. (2013), “Random environmental process”, in *Stochastic Dynamics of Marine Structures*, Cambridge University Press, Cambridge, United Kingdom, pp. 191–208.
- Ochi, M. (1998), *Ocean waves: The stochastic approach*, Cambridge University Press, Cambridge, United Kingdom.
- Serinaldi, F. (2015), “Dismissing return periods!”, *Stochastic Environmental Research and Risk Assessment*, Vol. 29 No. 4, pp. 1179–1189.
- Silverman, B. W. (1998), *Density estimation for statistics and data analysis*, CRC press, London, UK.
- Vanem, E. (2018), “A simple approach to account for seasonality in the description of extreme ocean environments”, *Marine Systems & Ocean Technology*, Vol. 13 No. 2-4, pp. 63–73.
- Wang, C.-Y. and Cai, D.-C. (2016), “Hand tool handle design based on hand measurements”, in *MATEC Web of Conferences (IMETI 2016)*, Vol. 119, pp. 01044–1 to 01044–5.
- Winterstein, S. R., Ude, T. C., Cornell, C. A., Bjerager, P. and Haver, S. (1993), “Environmental parameters for extreme response: Inverse FORM with omission factors”, in *Proc. 6th International Conference on Structural Safety and Reliability (ICOSSAR 93)*, Innsbruck, Austria.