PROJECTIVE CHARACTERS OF DEGREE ONE
AND THE INFLATION-RESTRICTION SEQUENCE

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Abstract

Let $G$ be a finite group, $\alpha$ be a fixed cocycle of $G$ and $\text{Proj}(G, \alpha)$ denote the set of irreducible projective characters of $G$ lying over the cocycle $\alpha$.

Suppose $N$ is a normal subgroup of $G$. Then the author shows that there exists a $G$-invariant element of $\text{Proj}(N, \alpha_N)$ of degree 1 if and only if $[\alpha]$ is an element of the image of the inflation homomorphism from $M(G/N)$ into $M(G)$, where $M(G)$ denotes the Schur multiplier of $G$. However in many situations one can produce such $G$-invariant characters where it is not intrinsically obvious that the cocycle could be inflated. Because of this the author obtains a restatement of his original result using the Lyndon-Hochschild-Serre exact sequence of cohomology. This restatement not only resolves the apparent anomalies, but also yields as a corollary the well-known fact that the inflation-restriction sequence

$$\{1\} \rightarrow M(G/N) \rightarrow M(G) \rightarrow M(N)$$

is exact when $N$ is perfect.


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All groups, $G$, considered in this paper are finite and all representations of $G$ are defined over the complex numbers. The reader unfamiliar with projective representations is referred to [3] for basic definitions and elementary results.

The purpose of this paper is to investigate under which circumstances the following well-known corollary to Clifford’s theorem can be generalized to projective characters.

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Let $N \trianglelefteq G$, and $\chi \in \text{Irr}(G)$ such that $[\chi_N, 1_N] \neq 0$, where $1_N$ denotes the trivial character of $N$. Then $N \leq \text{Ker}(\chi)$.

Our generalization will take the form of answering the following question.

Let $\alpha$ be a cocycle of $G$ and $N \trianglelefteq G$. Under which necessary and sufficient conditions does there exist a $G$-invariant projective character of $N$ with degree 1 and cocycle $\alpha_N$?

Our motivation for investigating this problem is provided by Haggarty and Humphreys [1] who said “Given a subgroup $L$ of $G$, a cocycle $\alpha$ of $G$ determines a cocycle $\alpha_L$ of $L$ by restriction. However elements which are $\alpha_L$-regular in $L$ need not be $\alpha$-regular in $G$. This fact complicates the theory of projective characters.”

The implication here can be taken to be that given that every element of $L$ is $\alpha$-regular (see 1.1 for this definition), then the theory of projective characters is similar to ordinary character theory. Indeed as the theory applies to induction from $L$ to $G$ this is the case, however we shall show in Section 1 that this is certainly not the case when looking at the restriction from $G$ to $L$, where $L \trianglelefteq G$.

1. Projective characters of degree one

To avoid repetition we fix the following notation for the rest of this paper. Let $G$ be a group, $\alpha$ be a cocycle of $G$, and $\text{Proj}(G, \alpha)$ denote the set of irreducible projective characters of $G$ with cocycle $\alpha$. We shall also use without further reference the fact due to Schur that $o([\alpha])$ in $M(G)$ divides $\xi(1)$ for all $\xi \in \text{Proj}(G, \alpha)$, where $M(G)$ denotes the Schur multiplier of $G$. We thus have that there is an element of $\text{Proj}(G, \alpha)$ of degree 1 if and only if $[\alpha] = [1]$. We now recall some more well-known facts about projective characters in a series of lemmas. We start however with a basic definition.

**DEFINITION 1.1.** An element, $x$, of $G$ is said to be $\alpha$-regular if $\alpha(x, g) = \alpha(g, x)$ for all $g \in C_G(x)$.

It is easy to check that if $[\alpha] = [\beta]$, then $x$ is $\alpha$-regular if and only if $x$ is $\beta$-regular. Also any conjugate of an $\alpha$-regular element is $\alpha$-regular, so that we may speak of the $\alpha$-regular conjugacy classes of $G$.

**LEMMA 1.2.** (i) There exists $\beta \in [\alpha]$ such that

$$\frac{\beta(g, x)\beta(gx, g^{-1})}{\beta(g, g^{-1})} = 1$$

for all $\beta$-regular $x \in G$, and all $g \in G$. 
(ii) An element, \( x \), of \( G \) is \( \alpha \)-regular if and only if there exists \( \xi \in \text{Proj}(G, \alpha) \) such that \( \xi(x) \neq 0 \).

**PROOF.** See (7.2.4) and (7.2.5) of [3].

We call a cocycle satisfying the condition of 1.2(i) a class-function cocycle, in the sense that by (ii) the elements of \( \text{Proj}(G, \beta) \) are then class functions.

Let \( N \leq G \), for \( \zeta \in \text{Proj}(N, \alpha_N) \) we define the \( g \)-conjugate, \( \zeta^g \), of \( \zeta \) by

\[
\zeta^g(x) = f_\alpha(g, x)\zeta(g x g^{-1})
\]

for \( g \in G \), and all \( x \in N \); where \( f_\alpha(g, x) = \alpha(g, x)\alpha(g x, g^{-1})/\alpha(g, g^{-1}) \). This defines an action of \( G \) on \( \text{Proj}(N, \alpha_N) \) for which Clifford's theorem holds. Having defined our action we can now begin to look at the relationship between \( \alpha_N \)-regular elements of \( N \) and \( \text{Proj}(N, \alpha_N) \).

**LEMMA 1.3.** Let \( N \leq G \) and \( x \) be an \( \alpha_N \)-regular element of \( N \). Then for all \( g \in G \), \( x^g \) is \( \alpha_N \)-regular.

**PROOF.** Let \( \zeta \in \text{Proj}(N, \alpha_N) \) such that \( \zeta(x) \neq 0 \), and \( g \in G \). Then \( \zeta^g \in \text{Proj}(N, \alpha_N) \) and \( \zeta^g(x^g) = c\zeta(x) \) for some \( c \neq 0 \). Thus \( x^g \) is \( \alpha_N \)-regular.

Our next result shows that \( G \)-invariance, not surprisingly, does not depend upon the choice of cocycle from \( [\alpha] \).

**LEMMA 1.4.** Let \( N \leq G \), \( \mu : G \to \mathbb{C}^* \) be a mapping with \( \mu(1) = 1 \), and

\[
\beta(g, h) = \frac{\mu(g)\mu(h)}{\mu(gh)} \alpha(g, h) \quad \text{for all} \; g, h \in G.
\]

Suppose \( \text{Proj}(N, \alpha_N) = \{\zeta_1, \ldots, \zeta_t\} \). Then

(i) \( \text{Proj}(N, \beta_N) = \{\mu_N \zeta_1, \ldots, \mu_N \zeta_t\} \); 

(ii) for \( g \in G \), \( \zeta_i^g = \zeta_j \) if and only if \( (\mu_N \zeta_i)^g = \mu_N \zeta_j \).

**PROOF.** (i) See pages 72–73 of [3].

(ii) Let \( g \in G \). Then for all \( x \in N \),

\[
(\mu_\zeta)^g(x) = f_\beta(g, x)(\mu_\zeta)(gxg^{-1})
= \frac{\beta(g, x)\beta(g x, g^{-1})}{\beta(g, g^{-1})} \mu(gxg^{-1})\zeta_i(gxg^{-1})
= f_\alpha(g, x)\mu(x)\zeta_i(gxg^{-1}) = \mu(x)\zeta_i^g(x).
\]

**PROPOSITION 1.5.** Let \( N \leq G \), and let \( \inf \) denote the inflation homomorphism from \( M(G/N) \) into \( M(G) \). Then there exists a \( G \)-invariant \( \delta \in \text{Proj}(N, \alpha_N) \) with \( \delta(1) = 1 \) if and only if \( [\alpha] \in \text{Im}(\inf) \).
PROOF. Suppose there exists a $G$-invariant $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$. Let $P$ be an irreducible projective representation of $G$ with cocycle $\alpha$ which has $\delta$ as a constituent of $P_N$. Then, if $P$ has degree $n$, we have that $P$ induces a homomorphism $\overline{P} : G/N \to \text{PGL}(n, \mathbb{C})$ defined by $\overline{P}(gN) = \Pi(P(g))$ for all $g \in G$, where $\Pi$ denotes the natural homomorphism from $GL(n, \mathbb{C})$ onto $\text{PGL}(n, \mathbb{C})$. Now any section of $\overline{P}$ will be a projective representation of $G/N$ with some cocycle $\beta$ of $G/N$, moreover $[\beta]$ does not depend on the choice of section, and it is clear that $\text{inf}([\beta]) = [\alpha]$.

Conversely, suppose $[\alpha] = \text{inf}([\beta])$ for some $[\beta] \in M(G/N)$. Then regarding $\beta$ as a cocycle of $G$ we have that the trivial character, $1_N$, of $N$ is an element of $\text{Proj}(N, \beta_N)$. As such $1_N$ is $G$-invariant since $\beta$ is constant on the cosets of $N$ in $G$, and hence there exists a $G$-invariant $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$ by 1.4.

This result would appear to have easily answered our original question. However we shall now demonstrate that in certain circumstances it is possible to produce $G$-invariant projective characters of degree 1 in situations where it is not intrinsically obvious that the cocycle could be inflated.

**Lemma 1.6.** Let $N \trianglelefteq G$ such that $[\alpha_N] = [1]$. Then there exists $\delta \in \text{Proj}(N', \alpha_{N'})$ with $\delta(1) = 1$ such that $\delta$ is $G$-invariant.

**Proof.** Since $[\alpha_N] = [1]$, $\mathcal{A} = \{\delta \in \text{Proj}(N, \alpha_N) : \delta(1) = 1\}$ is non-empty and $G$ acts upon it. Now for $\delta \in \mathcal{A}$ we have that $\delta_{N'}$ is irreducible, and so by Clifford's theorem there exists a bijection from $\text{Irr}(N/N')$ onto $\mathcal{A}$ defined by $\lambda \mapsto \lambda \delta$. Now if $\xi \in \text{Proj}(G, \alpha)$ such that $\delta$ is a constituent of $\xi_N$, we have that $\xi_N = e(\delta_1 + \cdots + \delta_t)$, where $\delta = \delta_1, \ldots, \delta_t$ are the distinct $G$-conjugates of $\delta$. Thus $\xi_{N'} = et\delta_{N'}$, and so $\delta_{N'}$ is $G$-invariant.

It is obvious that if there is a $G$-invariant element of $\text{Proj}(N, \alpha_N)$ of degree 1, then necessarily every element of $N$ is $\alpha$-regular. One could conjecture, falsely as it happens, that this was also a sufficient condition, but our next major result shows that to some extent this conjecture would be justified.

**Lemma 1.7.** Let $N \trianglelefteq G$ and $x$ be an $\alpha_N$-regular element of $N$. Suppose that $\alpha_N$ is class-function cocycle of $N$. Then for each $g \in G$ and all $y \in N$,

$$f_\alpha(g, x) = f_\alpha(g, x^y).$$

**Proof.** By 1.2 and 1.3 we may let $\zeta \in \text{Proj}(N, \alpha_N)$ such that $\zeta(gxg^{-1}) \neq 0$. Now for $g \in G$ and all $y \in N$ we have that

$$\zeta^g(x) = f_\alpha(g, x)\zeta(gxg^{-1})$$
and

\[(\zeta^g)^{y^{-1}}(x) = f_\alpha(y^{-1}, x)\zeta^g(xy)\]

\[= \zeta^g(xy), \quad \text{since } \alpha_N \text{ is a class-function cocycle;}
\]

\[= f_\alpha(g, xy)\zeta(gxyg^{-1}).\]

But \(gxg^{-1}\) and \(gxyg^{-1}\) are conjugate in \(N\), since \(G\) permutes the classes of \(N\). Thus since \(\alpha_N\) is a class-function cocycle we have that both \(\zeta(gxyg^{-1}) = \zeta(gxg^{-1}) \neq 0\), and \(\zeta^g(x) = \zeta^g(xy)\), hence \(f_\alpha(g, x) = f_\alpha(g, xy)\).

**Theorem 1.8.** Let \(N \triangleleft G; \mathcal{C}_1, \ldots, \mathcal{C}_t\) be the \(\alpha_N\)-regular conjugacy classes of \(N\) fixed by \(g \in G\), and \(x_i \in \mathcal{C}_i\). Suppose that \(\alpha_N\) is a class-function cocycle of \(N\). Then \(\sum_{i=1}^t f_\alpha(g, x_i)\) is the number of \(g\)-invariant elements of \(\text{Proj}(N, \alpha_N)\).

**Proof.** (Brauer, Isaacs). Let \(\mathcal{G}_i\) for \(1 \leq i \leq t\) denote respectively the elements of \(\text{Proj}(N, \alpha_N)\) and the \(\alpha_N\)-regular classes of \(N\). Let \(x_i \in \mathcal{G}_i\), if \(\mathcal{G}_i^g = \mathcal{G}_j\) we shall write \(x_i^g = x_j\). For \(g \in G\), we define \(A(g) = (a_{ij})\), where \(a_{ij} = 1\), if \(x_i^g = x_j\) and is zero otherwise. We also define \(B(g) = (b_{ij})\), where \(b_{ij} = f_\alpha(g, x_j)\) if \(\mathcal{G}^g_i = \mathcal{G}_j\) and is zero otherwise. We note from 1.7 that \(f_\alpha(g, x_j)\) is independent of the choice of \(x_j \in \mathcal{G}_j\). Finally let \(P = (p_{ij})\), where \(p_{ij} = \mathcal{G}_i^g\). Then we have that the \((l, m)\)th entry of \(A(g)P\) is \(\sum_{j=1}^t a_{ij} \zeta^g_j(x_m) = \zeta^g_l(x_m)\); whereas the \((l, m)\)th entry of \(PB(g)\) is

\[\sum_{j=1}^t \zeta_l(x_j)b_{jm} = f_\alpha(g, x_m)\zeta_l(gxmg^{-1}) = \zeta^g_l(x_m).\]

Thus \(P^{-1}A(g)P = B(g)\) and so \(\text{trace}(A(g)) = \text{trace}(B(g))\). But \(\text{trace}(A(g))\) is the number of \(g\)-invariant elements of \(\text{Proj}(N, \alpha_N)\), whereas \(\text{trace}(B(g)) = \sum_{i \in I} f_\alpha(g, x_i)\) where \(I = \{i: \mathcal{G}_i^g = \mathcal{G}_i\}\).

As applications of the above theorem we have the following results.

**Corollary 1.9.** Let \(N \triangleleft G\), and suppose that every element of \(N\) is \(\alpha\)-regular. Then each \(g \in G\) fixes at least one element of \(\text{Proj}(N, \alpha_N)\).

**Proof.** By 1.4(ii) we may assume that \(\alpha\) is a class-function cocycle of \(G\). Let \(g \in G\), then by 1.8 the number of \(g\)-invariant elements of \(\text{Proj}(N, \alpha_N)\) equals the number of classes of \(N\) fixed by \(g\).

**Corollary 1.10.** Let \(N\) be a normal abelian subgroup of \(G\) such that \(G/C_G(N)\) is cyclic. Suppose that every element of \(N\) is \(\alpha\)-regular. Then there exists \(\delta \in \text{Proj}(N, \alpha_N)\) with \(\delta(1) = 1\) which is \(G\)-invariant.

**Proof.** Let \(C = C_G(N)\) and \(\delta \in \text{Proj}(N, \alpha_N)\). Then \(\delta(1) = 1\) and \(C\) is a subgroup of the inertia subgroup, \(I_G(\delta)\), of \(\delta\) in \(G\), since \(N\) is abelian and every
element of $N$ is $\alpha$-regular. Let $g \in G$ such that $(gC) = G/C$, then by 1.9 $g$ fixes some $\delta' \in \text{Proj}(N, \alpha_N)$. Thus $G = \langle g, C \rangle \leq I_G(\delta')$.

It is interesting to note that Mangold in (5.1) of [7] claimed that every element of $G$ is $\alpha$-regular if and only if $[\alpha] = [1]$. The first of the following examples demonstrates that this is in fact false in general for a non-abelian group, and hence also shows that the condition of every element of $G$ being $\alpha$-regular is not even sufficient generally, to guarantee the existence of an element of $\text{Proj}(G, \alpha)$ of degree 1.

**EXAMPLES.** Let $p$ be a prime number, and $H$ be the “einfachste” representation group for $(C_p)^4$ as in (3.5.4) of [3], so that $|H| = p^{10}$ and $H = \langle x_1, x_2, x_3, x_4 : x_i^p = [x_i, x_j, x_k] = 1, \text{ for } 1 \leq i, j, k \leq 4 \rangle$.

Let $s = [x_1, x_2] [x_3, x_4]$ and $A = \langle s \rangle$, so that $A \leq Z(H) \cap H'$ and $|A| = p$. It is easy to show that no non-trivial element of $A$ is a commutator, see [5] for a generalization of this result. Now let $\lambda \in \text{Irr}(A)$ be defined by $\lambda(s^j) = \omega^j$ for $\omega = e^{2\pi i/p}$, and let $\alpha$ be the cocycle of $G_1 = H/A$ constructed in the normal way from $\lambda$, see pages 180–182 of [2] for example. Then by construction $o([\alpha]) = p$. Now with the definition and results of pages 195–197 of [2], we have that every element of $G_1$ is ‘$\lambda$-special’ trivially, and hence every element of $G_1$ is $\alpha$-regular.

For a different type of example let $B = \langle s, t \rangle$ where $t = [x_1, x_3]$, $M = \langle x_1, x_3, A \rangle$, and define $\mu \in \text{Irr}(B)$ by $\mu(s^j t^k) = \lambda(s^j)$. One can then check that every element of $N = M/B$ is $\mu$-special, but that not every element of $MZ(H)/Z(H)$ is $\nu$-special for any extension, $\nu$, of $\mu$ to $Z(H)$. So if $\beta$ is the cocycle of $G_2 = H/B$ constructed from $\mu$, we have shown that every element of the abelian group $N$ is $\beta$-regular, but that no element of $\text{Proj}(N, \beta)$ can be $G_2$-invariant.

### 2. The inflation-restriction sequence

Let $N \trianglelefteq G$. Then we have the Lyndon-Hochschild-Serre exact sequence of cohomology:

$$
\{1\} \to H^1(G/N, \mathbb{C}^*) \xrightarrow{\inf_1} H^1(G, \mathbb{C}^*) \xrightarrow{\text{res}_1} H^1(N, \mathbb{C}^*) \xrightarrow{\text{tra}} M(G/N) \xrightarrow{\inf_2} M(G)
$$

where the action of all groups on $\mathbb{C}^*$ is trivial, see [6, page 354].

It is clear that we may replace $M(G)$ in this exact sequence by $M(G)^\# = \{ [\alpha] \in M(G) : [\alpha_N] = [1] \}$. In this section we shall extend this new sequence one term to the right, and in doing so we shall give a practical test to see whether an element of $M(G)^\#$ is in the image of $\inf_2$. Thus it is 1.5 which connects the results of Section 1 to those of this section.
LEMMA 2.1. Let $N \leq G$, $\alpha$ be a cocycle of $G$ such that $[\alpha] \in M(G)^\#$, and $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$. Then

(i) the mapping $\alpha': G/N \to H^1(N, \mathbb{C}^*)$ defined by $\alpha'(gN) = \delta/\delta^g$ is a crossed homomorphism;

(ii) the mapping $\tau: M(G)^\# \to H^1(G/N, H^1(N, \mathbb{C}^*))$ defined by $\tau([\alpha]) = [\alpha']$ is a homomorphism.

PROOF. (i) Let $g \in G$. Then $\delta^g, \delta \in \text{Proj}(N, \alpha_N)$, and so since $\delta(1) = 1$ we have that $\delta/\delta^g \in H^1(N, \mathbb{C}^*)$. Now let $g_1, g_2 \in G$, and suppose that $g_1 x = g_2$ for $x \in N$. Then

$$\alpha'(g_2 N) = \frac{\delta}{\delta^g_1 x} = \frac{\delta}{(\delta^g_1)_x} = \frac{\delta}{\delta^g_1} = \alpha'(g_1 N),$$

since $N \leq I_G(\delta^g_1)$. Thus $\alpha'$ is well defined. Finally let $g_1, g_2 \in G$. Then

$$\alpha'(g_1 g_2 N) = \frac{\delta}{\delta^g_{1,2}} = \left(\frac{\delta}{\delta^g_1}\right)^{g_2} \frac{\delta}{\delta^g_2} = (\alpha'(g_1 N))^{g_2 N} \alpha'(g_2 N).$$

(ii) Suppose $\beta \in [\alpha]$, and let $\mu: G \to \mathbb{C}^*$ be a mapping with $\mu(1) = 1$ such that

$$\beta(g,h) = \frac{\mu(g)\mu(h)}{\mu(gh)} \alpha(g,h) \quad \text{for all } g, h \in G.$$

Let $\nu \in \text{Proj}(N, \beta_N)$ with $\nu(1) = 1$. Then by 1.4 we have that $\nu = \mu_N \delta_1$ for some $\delta_1 \in \text{Proj}(N, \alpha_N)$. But $\delta_1 = \lambda \delta$ for $\lambda \in H^1(N, \mathbb{C}^*)$ as in the proof of 1.6. Thus

$$\frac{\nu}{\nu^g} = \frac{\lambda}{\lambda^g} \frac{\mu_N \delta}{(\mu_N \delta)^g} = \frac{\lambda}{\lambda^g} \frac{\delta}{\delta^g},$$

as in the proof of 1.4, and so $\tau$ is well defined. Clearly $\tau$ is a homomorphism.

THEOREM 2.2. Let $N \leq G$. Then the sequence

$$M(G/N) \xrightarrow{\inf} M(G)^\# \xrightarrow{\tau} H^1(G/N, H^1(N, \mathbb{C}^*))$$

is exact.

PROOF. By 1.5 we have that $\text{Im}(\inf) \leq \text{Ker}(\tau)$. Let $[\alpha] \in \text{Ker}(\tau)$. Then for $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$, we have that $\delta/\delta^g = \lambda/\lambda^g$ for some $\lambda \in H^1(N, \mathbb{C}^*)$. But then $\delta \lambda^{-1}$ is $G$-invariant, and so by 1.5 we obtain that $[\alpha] \in \text{Im}(\inf)$.

The above theorem can be regarded as a generalization of a result of Read, see (4.4.5) of [3], which deals with the special case when $N$ is a central subgroup of $G$. We now mention some applications of 2.2, the first being well known.
COROLLARY 2.3. Let $N$ be a perfect normal subgroup of $G$. Then the sequence
\[
\{1\} \to M(G/N) \xrightarrow{\text{inf}} M(G) \xrightarrow{\text{res}} M(N)
\]
is exact.

PROOF. We start by noting that $H^1(N, C^*) = \{1\}$, since $N' = N$. Thus by 2.2 we have that $\text{Ker}(\text{res}) = M(G)^\# = \text{Im}(\text{inf})$.

Our next result was used by Liebler and Yellen in (2.4) of [4] to help prove that groups of central type are solvable.

COROLLARY 2.4. Let $N \trianglelefteq G$, and suppose that $(|G/N|, |N/N'|) = 1$. Then $M(G)^\# = \text{Im}(\text{inf})$.

PROOF. By the Schur-Zassenhaus theorem we have that $H^1(G/N, H^1(N, C^*))$ is trivial, and so the desired result is immediate from 2.2.

COROLLARY 2.5. Suppose $G$ is metacyclic, and let $N \trianglelefteq G$ such that both $N$ and $G/N$ are cyclic. Then $M(G)$ is isomorphic to a subgroup of $H^1(G/N, N)$.

PROOF. From 2.2 we have that the sequence
\[
\{1\} \to M(G) \xrightarrow{\tau} H^1(G/N, \text{Irr}(N))
\]
is exact, since $M(G)^\# = M(G)$. Thus $\tau$ is a monomorphism.

For our last application we can now explain the result of 1.6.

COROLLARY 2.6. Let $N_1, N_2 \trianglelefteq G$ with $N_2 \leq N_1, T$ denote the image of $\text{res}: H^1(N_1, C^*) \to H^1(N_2, C^*)$, and $M(G)^{\#_1} = \{ [\alpha] \in M(G): [\alpha_{N_1}] = [1] \}$. Then the homomorphism $\tau : M(G)^{\#_2} \to H^1(G/N_2, H^1(N_2, C^*))$ defined in 2.1, induces by restriction to $M(G)^{\#_1}$ a homomorphism from $M(G)^{\#_1}$ into $H^1(G/N_2, T)$.

PROOF. Let $[\alpha] \in M(G)^{\#_1}$, and $\delta \in \text{Proj}(N_1, \alpha_{N_1})$ with $\delta(1) = 1$. Then by 2.1 we have that $\tau([\alpha]) = [\alpha']$, where $\alpha'(gN_2) = (g\delta)^{-1}N_2 \in T$.

In the situation of 1.6 we have that $N_2 = N_1'$ and so $T = \{1\}$, with the above notation. Thus by 2.2 we obtain that $M(G)^{\#_1}$ is a subgroup of $\text{inf}: M(G/N_1') \to M(G)$. 

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