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NUMERICAL AND ANALYTICAL SOLUTIONS OF THE KINEMATIC APPROXIMATION FOR FLOWS OVER SURFACE ELEMENTS

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The kinematic flow approximation is used to model overland flow on varying geometries. Converging and diverging surfaces are considered first. Solution of the equation for flow on a converging surface by the method of characteristics suggests an alternative method of solution in the form of a similarity variable. Whereas the method of characteristics reduces the partial differential equation to two ordinary differential equations, the similarity solution results in one ordinarv differential equation. However the similarity variable is only found if excess rainfall is described by any power of time, and its use is illustrated by considering flow on a diverging cone. The ordinary differential equation, easily solved by Runge-Kutta integration, is shown to be very accurate, as numerical calculations were checked for the case of constant excess rainfall, for which an exact analytic solution exists, see Campbell (Hairsine), Parlange and Rose [1].

It is shown that there are two parts to the solution of the ordinary differential equation, separated by a singularity. For the case of a constant excess rainfall, the asymptotic expansion for early times is given in order to obtain starting values for the Runge-Kutta integration.

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For longer times, that is, after the singularity, the exact analytic solution applies. A similar expansion to the one mentioned above, for early times, is obtained when excess rainfall is a power of time. For a non-constant rainfall, there is no analytic solution after the singularity, so an asymptotic expansion for later times is also needed for the integration. It is discovered that these expansions are accurate enough to be used as approximations to the solutions themselves. Thus the asymptotic expansions have two uses [2].

The exact analytic solution is used to check the accuracy of two numerical methods (finite-difference schemes) found in Singh and Woolhiser [7] and Singh and Agiralioglu [6]. Illustration is given of the improvement of these schemes by making use of the asymptotic expansions in a hybrid scheme. Here, the asymptotic expansion is used when excess rainfall is a power of time, and the finite-difference scheme is used otherwise. This hybrid method is shown to be a definite improvement in computatational time and accuracy over the other methods.

An analytic solution is presented for kinematic flow on a curved surface whose slope varies in any way with distance downstream. This solution is for a specific form of excess rainfall, that is, an initial, instantaneous dumping of a depth of water all over the surface at $t = 0$. An illustration of the use of the solution is given using a simple curved surface which allows algebraic manipulation of the solution. The formation of a shock wave on curves with varying degrees of curvature, and the determination of the correct position of the shock is demonstrated. The analytic solution is used to determine the errors involved if this curved surface were approximated by a two-plane kinematic cascade. Illustration is also given of how the analytic solution may be used to lessen these errors by making appropriate choices in the construction of the cascade, see [5].

Moving on to a different class of solutions, but still dealing with the kinematic cascade approximation, approximate analytic solutions to overland flow on a diverging conical surface and a converging inverted conical surface are developed, see [3]. The magnitude of error in these solutions is determined by undertaking a comparison between it and the solution given initially by an asymptotic expansion and then a numerical

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scheme.

Finally, a new method of obtaining a spatial average value for infiltration and runoff rates, is presented [*4*]. The theory is developed from the general analytic solution to the kinematic flow approximation of overland flow on a plane surface. Application of the method is illustrated both with analytic examples and field experimental data. The former indicates the good accuracy of the method, whilst the latter indicates how difficulties encountered when using experimental data can be overcome.

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