

TEERAWAT THEWMORAKOT, *Computability Theory on Polish Metric Spaces*, University of Connecticut, Storrs, CT, USA, 2023. Supervised by David Reed Solomon. MSC: 03D78. Keywords: computability theory, Polish metric spaces, index sets, classification problems.

### Abstract

Computability theoretic aspects of Polish metric spaces are studied by adapting notions and methods of computable structure theory. In this dissertation, we mainly investigate index sets and classification problems for computably presentable Polish metric spaces. We find the complexity of a number of index sets, isomorphism problems, and embedding problems for computably presentable metric spaces. We also provide several computable structure theory results related to some classical Polish metric spaces such as the Urysohn space  $\mathbb{U}$ , the Cantor space  $2^{\mathbb{N}}$ , the Baire space  $\mathbb{N}^{\mathbb{N}}$ , and spaces of continuous functions.

Abstract prepared by Teerawat Thewmorakot.

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WASEET KAZMI, *Ordered Groups, Computability and Cantor-Bendixson Rank*, University of Connecticut, Storrs, CT, USA, 2023. Supervised by David Reed Solomon. MSC: 03C57, 03D45, 06F15 Keywords: ordered groups, computability theory, computable groups, semidirect product, space of orders, Cantor–Bendixson rank.

### Abstract

We study ordered groups in the context of both algebra and computability. Ordered groups are groups that admit a linear order that is compatible with the group operation. We explore some properties of ordered groups and discuss some related topics. We prove results about the semidirect product in relation to orderability and computability. In particular, we give a criteria for when a semidirect product of orderable groups is orderable and for when a semidirect product is computably categorical. We also give an example of a semidirect product that has the halting set coded into its multiplication structure but it is possible to construct a computable presentation of this semidirect product.

We examine a family of orderable groups that admit exactly countably many orders and show that their space of orders has arbitrary finite Cantor–Bendixson rank. Furthermore, this family of groups is also shown to be computably categorical, which in particular will allow us to conclude that any computable presentation of the groups does not admit any noncomputable orders. Lastly, we construct an example of an orderable computable group with no computable Archimedean orders but at least one computable non-Archimedean order.

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DANIEL MOURAD, *Computability Theory: Constructive Applications of the Lefthanded Local Lemma and Characterizations of Some Classes of Cohesive Powers*. Department of Mathematics, University of Connecticut, Storrs, CT, USA. 2023. Supervised by David Reed Solomon. MSC: 03Dxx, 03C20. Keywords: local lemma, cohesive powers, Weihrauch reducibility.

### Abstract

The Lovász local lemma (LLL) is a technique from combinatorics for proving existential results. There are many different versions of the LLL. One of them, the lefthanded local

