It is the task of the machine to give all letters a common orientation so that the addresses can be easily read and the stamps cancelled.

We would expect such a machine to be able to perform four basic operations:

I: Pass the letter through unchanged;
H: Twist the letter through 180° about a horizontal axis in the plane of the letter;
V: Rotate the letter through 180° about a vertical axis;
R: Roll the letter end for end about an axis perpendicular to the plane of the letter.

Since these operations just describe the symmetries of a rectangle, I, H, V and R determine a Klein four-group.

Now in the construction of an automatic facing machine, it is easy to design a moving belt system which will perform H. It is rather more difficult to effect V and R by mechanical means for a fast moving stream of letters. Since the structure of the four-group tells us that $HV = R$, and $HR = V$, we see that in practice only one of the operations V, R needs to be built into the machine.

The diagram below shows the design of an early letter facing machine. At each 'scan', the lower leading edge of the envelope is scanned by a sensor. If the stamp appears at that corner, the letter is directed to the lower route and passes on through the machine unchanged. We observe that this design makes use of the operations H, $HV = R$ and $HVH = RH = V$.

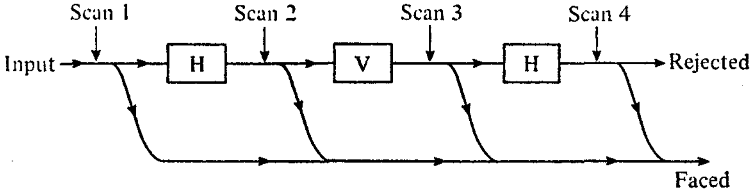

FIGURE 2.

It is an interesting and instructive classroom exercise to experiment with different possible designs for a letter facing machine, as any economical layout must make use of the four-group. Information on machines currently in use can be found in a paper on automatic letter facing by G. P. Copping in "British Postal Engineering", *Proceedings of the Institution of Mechanical Engineers* **184** (1969–70).

Yours sincerely,
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Minimum-MSE estimators

DEAR SIR,

In a recent article [1] Mr. B. J. R. Bailey draws attention to some of the problems which arise in point estimation. The article is very welcome, and one hopes it will be read by some of those who regard the Principle of Maximum Likelihood as an article of religious faith.

Mr. Bailey gives the impression, however, that the theory of minimum-MSE estimators is somewhat nebulous, and it would be a pity to leave this impression uncorrected. The classic paper by Pitman [2] deals with the subject in fiducial terms, and possibly for this reason his work is not as well known as it should be; but it is not difficult to obtain Pitman's results by purely frequentist arguments [3].

Consider the example quoted by Mr. Bailey, the estimation of the variance σ^2 of a normal distribution with zero mean. If S^2 is the sum of the squares of n observations, he rightly states that $S^2/(n+2)$ has uniformly smaller MSE than the unbiased estimator S^2/n, and points out that other estimators exist which, at least for some ranges of values of σ^2, have still smaller MSE. This is true if one places no restriction on the types of estimators to be used. But the observations in a problem such as this are usually physical quantities such as distances or speeds, and changes in the units of measurement in the observations should produce corresponding changes of units in the estimate. If we submit the observations to the transformation $x_i \to ax_i (a > 0)$, the parameter σ^2 undergoes the transformation $\sigma^2 \to a^2\sigma^2$. It is natural, therefore, to require that the estimator T should also undergo this transformation, $T \to a^2 T$, when the observations are transformed by $x_i \to ax_i$. If we so restrict T, then $S^2/(n+2)$ certainly has uniformly minimal MSE.

Formulae are given in [2] and [3] for minimum-MSE estimators for any scale parameter, and the ideas can be extended (less convincingly) to location parameters and to the simultaneous estimation of several parameters.

Yours sincerely,

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'T.A.A.B.'

DEAR SIR,

I wonder if I might take up some space over a small matter in which Dr. Maxwell may unwittingly have misled readers in his delightful obituary of Professor Broadbent. While it is true that Alan Broadbent succeeded Milne Thomson in the Chair at the Naval College and succeeded him as Gresham Professor of Geometry (and possibly these two events were simultaneous) the Gresham Chair was of course, as it still is, one of those delightful survivals from the past which can be held in plurality. Alan Broadbent filled this Chair, requiring as it does an exposition of mathematics to the layman, with rare distinction, and I know for a fact that members of the audience for the present Gresham Professor's lectures still remember him with affection and enthusiasm.

The casual observer might have associated the Gresham Chair with the Naval College because of the long period during which it was held by Milne Thomson and then Broadbent, but in the same way a casual observer now looking at the eight Gresham Professorships might assume they were in some way linked to another institution.

Yours sincerely,

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