Thoughts on Core-Collapse Supernova Theory†

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Abstract. An emerging conclusion of theoretical supernova research is that the breaking of spherical symmetry may be the key to the elusive mechanism of explosion. Such explorations require state-of-the-art multi-dimensional numerical tools and significant computational resources. Despite the thousands of man-years and thousands of CPU-years devoted to date to studying the supernova mystery, both require further evolution. There are many computationally-challenging instabilities in the core, before, during, and after the launch of the shock, and a variety of multi-dimensional mechanisms are now being actively explored. These include the neutrino heating mechanism, the MHD jet mechanism, and an acoustic mechanism. The latter is the most controversial, and, as with all the contenders, requires detailed testing and scrutiny. In this paper, we analyze recent attempts to do so, and suggests methods to improve them.

Keywords. supernovae: general – radiative transfer – stars: magnetic fields – stars: neutron – neutrinos – hydrodynamics – MHD

1. Introduction

After decades of theoretical exploration, it is now clear that the core collapse that leads to supernova explosions, and the supernovae themselves, are not spherical phenomena. During the violent dynamical sequence that gives birth to both a supernova and a neutron star (or black hole!), the core of a massive star runs a formidable gauntlet of hydrodynamic instabilities. First, the progenitor Chandrasekhar core experiences turbulent convection before collapse, ensuring density and entropy inhomogeneity at collapse. Then, the material behind the stalling bounce shock executes Rayleigh-Taylor overturn, aided later by neutrino heating from below that drives convection. The latter is boosted, if not overwhelmed, by the standing-accretion-shock instability (SASI; Blondin, Mezzacappa, & DeMarino 2003) that commences ∼100-250 milliseconds after bounce (Burrows et al. 2006), if there is no explosion before this, and has a frequency of from ∼20 to ∼80 Hz (Burrows et al. 2007a). The dominant SASI modes are ℓ = 1 and ℓ = 2 harmonics. Moreover, after bounce a shell below the neutrinospheres at ∼20-50 km in the inner core at a radius of ∼10-20 kilometers (km), executes convective overturn due to negative lepton (composition) gradients. It was hoped that such core convection, as well as doubly-diffusive instabilities (“neutron fingers”), could boost the neutrino luminosities that may ultimately be responsible for reenergizing the explosion. However, it has

† We acknowledge support for this work from the Scientific Discovery through Advanced Computing (SciDAC) program of the DOE, under grant numbers DE-FC02-01ER41184 and DE-FC02-06ER41452, and from the NSF under grant number AST-0504947.
been shown that the numbers are not encouraging (Dessart et al. 2006; Bruenn & Dineva 1996).

In addition, once the shock is launched, it progresses through a layer cake of outer zones, each bounded by density discontinuities and composition jumps. When the shock traverses these regions it trips further Rayleigh-Taylor-like and Richtmyer-Meshkov instabilities. Coupling all of the above mechanisms by which symmetry can be broken with rotation and magnetic stresses will only further enrich the multi-dimensional character of the hydrodynamics. Clearly, symmetry breaking is a key feature of core collapse supernova explosions.

Recently, Burrows et al. (2006) and Burrows et al. (2007abcd) have suggested that the violent turbulence around the inner core and the late-phase pounding of this core by accretion streams excite vigorous $\ell = 1$ and $\ell = 2$ core g-modes that damp by the emission of sound. If there had been no earlier explosion by other means, in their simulations Burrows et al. find that approximately $\sim 1$ second after bounce this sound can be sufficient to reenergize the shock. They suggest dumping acoustic power in the inner mantle at a rate of perhaps more than $10^{50}$ ergs s$^{-1}$ for many seconds can in principle lead to a supernova. The core g-mode is very aspherical and leads to an anisotropic, oftentimes unipolar, explosion. However, this g-mode/acoustic mechanism is quite controversial. Is the resolution (temporal as well as spatial) adequate? Can such simulations be trusted after $\sim 1,000,000$ timesteps? Is the result code-dependent? Do other mechanisms, such as neutrino heating or MHD jets (Burrows et al. 2007e), trump the acoustic mechanism before it has a chance to operate?

Whatever the ultimate solution, a central theme for modern supernova theory has emerged – whatever the mechanism, be it neutrino heating, MHD jets, or acoustic (and it might be a mix of all three), the breaking of spherical symmetry is an organizing principle of the theoretical debate. This puts a premium on the development and testing of multi-physics, multi-dimensional radiation hydrodynamic and magnetohydrodynamic computational approaches and codes. This imperative, and the limitations of current computers and algorithms, are important reasons the supernova puzzle is as yet unsolved.

Recently, Marek (2007) and Marek & Janka (2008) have looked into the generation of core g-modes and have challenged some of the findings in our recent series of papers (Burrows et al. 2006,2007abcd) on the core-oscillation/acoustic power mechanism. While it is vitally important to scrutinize such novel and provocative ideas as the acoustic mechanism in detail and directly, we believe that these two studies fall somewhat short of adequately addressing the issues raised by our series on this topic. Below we explain our conclusions and make various observations we think are germane to the case at hand. Since the Marek (2007) work has more details on what is to be found in both works, we focus on it, though refer to each work where appropriate. Though our remarks and observations are critical of these works, they are meant merely to improve the science return of any group seeking to critically explore our provocative acoustic findings and we very much appreciate the effort expended by these researchers to seriously address, check, confirm, or refute our controversial findings.

Therefore, in this short paper, we map out some of the numerical challenges and methodologies of testing the core-oscillation hypothesis. We don’t provide a systematic review of core-collapse supernova theory, such a task being beyond the scope of a short proceedings. For this, however, readers may profit from Burrows (2000) or Woosley & Janka (2005). Nevertheless, the enclosed discussion provides a snapshot of some of the current debate in supernova theory.
Thoughts on Core-Collapse Theory

2. Summary Comments on a Recent Investigation into Core G-mode Oscillations

Here, we summarize some of our technical reservations about the Marek & Janka work, and then address more specific points in §§3, §4, §5, and §6 below: 1) Their simulations do not extend to times after bounce when we say the core g-mode oscillations are large, but to times when we too say the excitation of such modes is minimal. Hence, they do not make the proper direct comparison, at the proper epoch; 2) They do their simulations in 1D in the central 1.6−1.7 km, which we believe artificially dampens the growth of the $\ell = 1$ core g-mode; 3) The measure they use for the presence of the mode is $\Delta P/P$, where $P$ is the pressure, at a given internal radius. In the convective region this is a measure of the strength of turbulence and of the potential for excitation of the interior g-mode by turbulence, but is not a good measure of the presence of the core g-mode itself; and 4) Marek & Janka (2008) claim that the onset of explosion might require a small nuclear incompressibility ($K$), and tout a value of $K = 180$ MeV. Their model with $K = 263$ MeV does not look as promising. However, the measured value of $K$ is $240 \pm 20$ MeV, much closer to the value employed in their non-exploding model, calling into question their central conclusion. We now proceed with a more specific discussion.

3. On $\Delta P/P$ as an Imperfect Measure of the G-mode

Marek (2007) and Marek & Janka (2008) use $\Delta P/P$ to discern the presence of the core oscillation itself, with Marek & Janka (2008) focussing only on the interior 10 and 20 kilometer (km) radii, and Marek (2008) including a discussion of the outer radius at 35 km as well. However, $\Delta P/P$ (as plotted at 35 km in Fig. 7 of Burrows et al. 2006) is a direct signature not of the core g-mode, but of the pressure fluctuations in the region between the shock and the inner core. The primary origins of these “outer” pressure fluctuations are the SASI and neutrino-driven convection, not the core oscillation. The restoring force for core g-modes is buoyancy in the inner $\sim 10$ km, not pressure, and by using $\Delta P/P$ as a g-mode index Marek & Janka focus unduly on a subdominant modal signature. A better measure of the presence and strength of an $\ell = 1$ core g-mode might be the overlap with the eigenfunction itself, particularly in the displacement or velocity, and the identification of the countervailing motion of the inner core with the shell around it; for the $\ell = 1$ core g-modes these regions oscillate out of phase.

Nowhere in Burrows et al. (2006, 2007abcd) do we propose $\Delta P/P$ as a measure for the strength of the core g-modes pulsations. Rather, we use it as a measure of the pressure fluctuations at the surface of the protoneutron star near 25−35 km that can excite core g-mode oscillations in the first, weaker, phase of the excitation sequence we see (see also Goldreich & Keeley 1977; Goldreich & Kumar 1988,1990). The second phase is the more important, that of the “self-excited” oscillator during and after the onset of explosion, which in our calculations occurs more than $\sim 1$ second after bounce. The calculations of Marek & Janka (2008) do not extend to more than $\sim 610$ milliseconds after bounce, leaving a crucial gap of $\sim 400$ milliseconds. At this earlier time, we too see small amplitudes for the core g-mode oscillations and they aren’t yet having a significant dynamical influence (Burrows et al. 2007a).

Feedback (“extra”) pressure waves from the resultant g-mode core motion are not so manifest early on at radii of $\sim 35−50$ km, and certainly not at $10−20$ km, even when the amplitude of the $\ell = 1$ core g-mode oscillation (better measured with the Lagrangian displacement in the inner 10 km) is modest, not small. g-modes are predominantly “gravity modes” and the p-mode character they have in the outer region is because of their mixed
character and is sub-dominant. It is only when the core oscillation becomes non-linear that the p-mode character of the outer region of the core g-mode becomes interesting. Then, the outer pressure wave components of the complex g-mode can propagate out with vigor and steepen into shocks. All during this time the inner 10 km (with a node near 6–8 km) is oscillating in g-mode fashion, with gravitation/buoyancy as the restoring force. Note that this inner region is not convective and the inner g-mode is not, of course, in an evanescent region, though its outer tail is; it is only there in the evanescent region that the mode is predominantly p-mode in character. This is quite unexceptional (Unno et al. 1989).

In our simulations, supersonic accretion funnels that penetrate through the kinks in the outer shock structure created by the vigorous SASI are the ultimate agents of strong core g-mode excitation. It is the downflowing plumes and their ram pressure that excite the \( \ell = 1 \) core g-mode at very late times. Even when these accretion funnels are steadily impinging upon the inner core and do not have resonant frequency components, they can excite \( \sim 300 \)-Hz g-modes; witness the generation of gravity waves on a pond due to a steady jet of water. It is the width of the exciting stream, not its temporal fluctuation, that sets the “wavelength” of the g-modes that can be excited. The frequency spectrum of the excitation is a consequence of this wavelength and the dispersion relation of gravity waves on the inner core. This excitation frequency spectrum easily overlaps the core g-mode spectrum.

4. On the Amplitudes of \( \Delta P/P \)

\( \Delta P/P \) and Mach number in post-shock regions grow with time in models for which the shock is stalled and it bounds the inner turbulent region. This includes almost all models published to date. Mach numbers approach a few tenths to \( \sim 0.5 \) at late times (0.5-1.0 seconds). The Mach numbers provide a measure of the vigor of the motions in this region. In turbulent regions, high Mach numbers translate directly into high pressure fluctuations, with Mach numbers near one implying pressure fluctuations of order unity, i.e. large.

It is not clear why in the calculations of Marek (2007) \( \Delta P/P \) in the turbulent regions is generically small, even at the latest times achieved (for most of his models, around \( \sim 350 \) milliseconds after bounce). Both Burrows et al. (2006, 2007abc) and Yoshida, Ohnishi, & Yamada (2007) have published much larger values that seem more consistent with the character of the late post-shock turbulent flow. As noted, Marek (2007) generally does not simulate long enough after bounce. Also, his initial seed perturbations might be small. With small seeds it takes longer to erase the memory of initial conditions and to achieve a given amplitude. During \( \sim 0.05 \) to \( \sim 0.5 \) seconds after bounce, Yoshida, Ohnishi, & Yamada obtain values of \( < \Delta P/P > \) for the \( \ell = 1 \) component near 35 km of \( \sim 0.1 \), with excursions to 0.2. For this same quantity, Burrows et al. (2006) see a steady growth in its value from \( \sim 0.05 \) at 0.3 seconds after bounce to \( \sim 0.2 \) at \( \sim 0.55 \) seconds after bounce. However, Marek (2007) obtains values of \( \Delta P/P \) for the \( \ell = 1 \) component at \( r = 35 \) km and at \( \sim 350 \) ms after bounce of only \( \sim 0.005 \) to 0.02. Interestingly, the value Marek obtains depends upon the EOS employed (see Marek 2007, Figures 5.9 and 5.10), with ten times larger amplitudes for the softer EOS. Marek & Janka (2008) do calculate one model to \( \sim 610 \) ms after bounce, but this model is rotating modestly and rotation is expected to partially suppress the SASI implicated in the turbulence generated in this region (Burrows et al. 2007a). Nevertheless, even in this model \( < \Delta P/P > \) achieves a value of \( \sim 0.05 \) and is still rising when it is halted. In fact, \( < \Delta P/P > \) is rising at the end of all the Marek (2007) simulations. To directly compare the values of \( \Delta P/P \) in the
outer turbulent regions and the g-mode amplitudes in the inner core with those Burrows 
et al. (2007abc) obtain, it is important for Marek & Janka to continue their simulations 
for another \( \sim 500 \) ms. In Burrows et al. (2007a), we needed to evolve for more than 
0.9–1.0 seconds to see vigorous core g-mode oscillation. It takes a long time for the core 
oscillation to manifest itself and the simulation time needs to be commensurate.

5. On Simulating the G-mode

To show that the MPA hydro code can support and simulate \( \ell = 1 \) g-modes, Marek 
(2007) calculates a few test models in which he imposes such a mode in the inner core 
and follows it for 10 – 20 milliseconds. Ten to twenty milliseconds is only a few oscillation 
cycles. However, in all but one model, he constrains the inner 1.6 km to 1D motion. In 
Marek & Janka (2008), this constrained inner core has a radius of 1.7 km. The g-mode 
is, thereby, forced to flow around this inner 1.6/1.7 km. Since the node of this mode 
is near 6–8 km, we believe that 1.6 km is too large a region in which to inhibit the 
necessary multi-D flow. Importantly, the \( \ell = 1 \) g-mode has its greatest amplitude in 
this central region, where its eigenmode motion is straight through \( r = 0 \) (the modal 
velocities are the same for \(+x\) and \(-x\); spherical coordinates with a reflecting boundary 
at \( r = 0 \) unphysically flip the sign, by construction). Though in Marek (2007) and Marek 
& Janka’s (2008) 2D/1D calculations the pressures and velocities are fluctuating around 
this 1.6/1.7-km region, the implied constraint force that keeps the very inner core from 
naturally responding will by its nature mute the expression of the \( \ell = 1 \) g-mode. 

In the calculation reported by Marek & Janka (2008), they see the imposed g-mode 
oscillation decay within a few cycles, which is rather fast. They state in reference to their 
tests: “These demonstrate that our code is well able to track large-amplitude g-modes, 
also of dipole character, if such modes are excited in the neutron star core [our italics].” 
However, they do not in fact demonstrate that their code can track in a self-consistent 
fashion long-term excitation by anisotropic accretion and turbulence. This is the crucial 
question and they have not shown that their inner 1D region doesn’t inhibit excitation. 
The decay they witness could easily be due in part or in large measure to the fact that 
their inner core is anchored. One way to address this would be to explore the dependence 
of the decay time on the size of the region that one does in 1D. For this test, the radius 
of the 1D region could be varied from, say, 0.5 km to 4 km. 

Marek (2007) does indeed perform one test calculation all the way to the center \( (r = 0) \), 
and presumably imposes a reflecting boundary condition there. However, he calculates 
this model for a total of only \( \sim 10 \) ms and starts the calculation only \( \sim 30 \) ms after 
bounce. As seen in Burrows et al. (2006, 2007a), this is far too early to start and far too 
short a time to perform such a calculation if one wants to witness the excitation of core 
oscillations. Hence, by constraining the inner 1.6 km or employing a reflecting boundary 
at \( r = 0 \), the most important and largest amplitude region of the \( \ell = 1 \) mode is thereby 
neutered and its excitation inhibited. To clearly avoid this problem, we believe that 
simulations should be done with a quasi-Cartesian grid at the very center. Otherwise, 
the amplitude of the g-mode seen in a given hydrodynamic environment is artificially 
suppressed. This fact and our experience are the origins of our caveats in Burrows et al. 

6. Additional Observations

Marek (2007) claims that the acoustic-driven explosions seen in the simulations of 
Burrows et al. (2006, 2007ab) arise rather abruptly. However, in fact, these explosions
emerge in those simulations over a period of $\sim 100\text{-}200$ ms. For comparison, when a neutrino-driven explosion is witnessed in the calculations of Buras et al. (2006), Kitaura et al. (2006), or Burrows et al. 2007c, it emerges on a timescale of $\sim 50\text{-}150$ ms. We suggest that this is a characteristic timescale for any explosive instability in the “supernova” core and that the acoustic mechanism is not exceptional in this regard. Marek (2007) also claims that at the late times when Burrows et al. (2006, 2007ab) see vigorous core oscillations there is not sufficient accretion power to maintain it. As Figures 2 and 7 in Burrows et al. (2007a) clearly show, though the accretion rate has subsided by these times, there is still ample accretion power to maintain such oscillations. It is merely a matter of the efficiency of the conversion of accretion power into mechanical power, as opposed to neutrino luminosity. A $\sim 10\%$ efficiency would be adequate for this purpose.

Marek (2007) suggests that when the shock radius achieves a value larger than $\sim 300$ km this indicates the onset of explosion, that this is a “point of no return.” The calculations of Marek (2007) and Marek & Janka (2008) are stopped near the time when the average shock radius achieves this value, though why is not clear. Marek & Janka (2008) add in the discussion of their results the timescale $\tau_{\text{adv}}$ versus $\tau_{\text{heat}}$ condition advocated by Thompson, Quataert, & Burrows (2005), but such arguments are no substitute for actual calculation. During the vigorous SASI phase, it is often the case that the shock radius substantially exceeds 300 km, only to recede again and continue non-linear and bounded SASI pulsation (Burrows et al. 2006, 2007a). Hence, we caution against using simple criteria for the onset of explosion. In fact, none of the SASI- and neutrino-aided “explosions” seen by the MPA group, neither the 11.2 $M_\odot$ model of Buras et al. (2006) nor the 15 $M_\odot$ model of Marek & Janka (2008), is actually followed for more than a few tens of milliseconds after explosion seems to ensue. It is crucially important that any claim of explosion be buttressed by calculations in which the shock actually achieves a radius of thousands of kilometers (preferably larger), and not just 300-400 km. The calculations of Burrows et al. (2006, 2007abcd) were carried out until the shock reached a radius of 4000-5000 km, and even this should not be considered far enough.

Importantly, we note that the incompressibility ($K$) of nuclear matter has been measured to be $240\pm20$ MeV (Shlomo, Kolomietz, & Colò 2006; Lattimer & Prakash 2007). Marek & Janka (2008) calculate models using values of 180 MeV and 263 MeV, and conclude that only the 180-MeV model witnesses the onset of explosion. As a result, it is not clear, given the dependence on the EOS they identify in their calculations, that a more realistic value of $K$ would lead to the same explosive behavior they claim for their model with $K = 180$ MeV.

In conclusion, we do not challenge the possibility that the neutrino mechanism might act on shorter timescales than the acoustic mechanism, and thereby abort it. However, we do hope that the community will redouble its efforts to test in a cogent and clear fashion its particulars and viability. This will require calculations to at least 1.2 seconds after bounce and the demonstration that these calculations do not suppress $\ell = 1$ and $\ell = 2$ core g-mode oscillations due to numerical exigencies. Conversely, we are redoubling our efforts to address aresh all the issues that attend core-collapse theory, including the neutrino, acoustic, and MHD mechanisms in all their particulars (see, for example, Dessart et al. 2007,2008; Hubeny & Burrows 2007; and Burrows et al. 2007e).

References

Thoughts on Core-Collapse Theory


Discussion

STANEK: Theorists fail to make core-collapse SNe. Does nature ever fail to make a SN from a massive star? Wh are the observational constraints?

BURROWS: I think that SN rates, OB star birth rates, pulsar birth rates, and nucleosynthetic constraints suggest that most massive stars must ‘supernova’, though a factor of two difference is still possible. However, I know of no compelling reason ‘success’ and ‘failure’ should alternate or vary wildly along the mass function. I don’t think rotation is a key ingredient of explosion, except for hypernova and perhaps GRBs, rare subsets of the massive star family outcomes, so this parameter does not provide an acceptable out.

LIMONGI: One of the most important issues for massive stars is their final fate. Many people refer to the WHW or our picture, but in my opinion all the pictures must be taken very carefully because they have been obtained with simulated explosions that suffer many uncertainties. For how long can you perform your computations and can you say what is the real fate of a massive star at least in some cases?

BURROWS: We calculate for approximately 1-1.5 seconds of physical time after core bounce. While this is quite long by the standards of the field, it is not long enough to determine much of what we would like to know, such as final explosion energy, kick velocity, matter fallback, or the r-process. For the least-massive massive stars (8-9 M⊙) we think we can conclude they are underenergetic (a few 10^50 ergs), but much of what...
we want and need to know about the fate of a generic massive star is currently beyond credible theory, alas.

MODJAZ: Would an observational signature/prediction for your new acoustic SN mechanism be that the SN are unipolar, not bipolar? So, for example, polarization measurements of SN Ib/c could test that by constraining the geometry (bipolar vs. unipolar).

BURROWS: Yes, more often than not I would expect acoustic-driven explosions to be top-bottom asymmetric. However, late-time neutron-driven explosions, of all but the least-massive massive stars, should also be top-bottom asymmetric and slightly unipolar. So, ‘unipolar’ vs. ‘bipolar’ is a means of distinguishing MHD-driven jet models of explosion from both acoustic and neutron models. To distinguish acoustic and many neutrino driven models from one another via morphology or polarization requires more subtle discriminants.

DAVIDSON: This is a little off the track of your main concerns, but those lovely graphics make it irresistible. Of course you have shown some impeccable jets, but do cannonballs ever occur? The Crab Nebula includes a line of very well separated, dense, compact ovoidal things, which MacAlpine called ‘argoknots’ for reasons you can guess. Morphologically they are astounding.

BURROWS: We do not get sprays and clumps of material, but instabilities in the star and circumstellar material due to the passage and progress of the shock and later cooling are more likely culprits. However, the explosion asymmetry we see certainly set the stage for the variegated structures and condensations that emerge and are captured in SNR images.