REVIEWS

the value of a test statistic is just above or just below a percentage point. The likely random
error should always be borne in mind.

The printing and general appearance of the tables are still as excellent as in the old CEST
and no doubt the NCET will continue to serve their purpose as the standard tables at many
examinations and a useful set of tables for general reference.

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Algebra through practice, books 1, 2, 3, by T. S. Blyth and E. F. Robertson. Pp 100
(approx.) each. £3.50 each. 1984. ISBN 0-521-27285-8/6-6/8-2 (Cambridge University
Press)

These books are sub-titled ‘A collection of problems in algebra, with solutions’, and evidently
there are six books in all. Each book gives a considerable list of background reference
material with notes on its use. As notation in algebra can differ considerably from author to
author, the preface to each chapter of each book gives condensed notes on usage and main
concepts (about one page) followed by sets of problems. For example, Book 1 (Sets,
Relations and Mappings) has pages 1–23, questions; pages 24–89, solutions; pages 90–95,
test papers. Book 2 follows a similar plan for Matrices and Vector Spaces, and Book 3 for
Groups, Rings, and Fields.

Dipping into the problems, there is no particular grading of difficulty. The authors have the
average student in mind, and if average includes industrious, the method of “try the question,
then refer to the solution” should achieve considerable competence up to a non-specialist
level. A very good but acquiescent student might waste time working through a lot of detail,
without learning anything new, and his/her time would be better spent consulting the
reference texts. This is not to say that the questions are all easy; some solutions are long and
might be found difficult to follow.

As regards subject matter, the title of Book 1 is self-explanatory, Book 2 proceeds to inner
product spaces, but not to Jordan Normal Form or dual spaces, Book 3 includes questions
up to normal groups and an attractive diversity of questions on rings and fields.

In books such as these, it is clearly inappropriate to criticize content and method, as with
an ordinary text-book. The books are, as stated, supplementary to undergraduate courses,
and could be of much value for practice following a normal lecture course. They suffer from
the usual motivation problem of work divorced from the supporting text, and any problems
set by the lecturer should be carefully chosen for relevance, to preserve interest.

The books are attractively printed and bound, and easy to carry about. The price seems
reasonable until one realises that the three volumes together are equal to a 300 page book
selling at £10.50.

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Mathematical logic, by H. D. Ebbinghaus, J. Flum and W. Thomas, translated by A. S.

This undergraduate text “does not require special mathematical knowledge; however, it
presupposes an acquaintance with mathematical reasoning as acquired, for example, in the
first year of a mathematics or computer science curriculum”. It broadly follows the lines that
are taken by the majority of mathematical logic text books as regards choice of topics,
notation, definitions, theorems and proofs. Within that company of books it appears to come
out quite well as regards help to the student.
After an introduction, there are chapters on the syntax and semantics of first-order languages, and a formal calculus of deduction, and then the first major result is reached with Gödel’s completeness theorem, which is proved by Henkin’s method. Next there are the Löwenheim-Skolem and compactness theorems and chapters on set theory and languages more powerful than first order. Register machines are used, instead of Turing machines, as idealized computers, and we have the halting problem, the undecidability of first-order logic, self-referential statements (“mathematics through the looking-glass”) and Gödel’s incompleteness theorems. The final chapters include Fraissé’s and Lindström’s theorems.

Reading through the book, I was reminded how mathematical logic text books generally look more complicated than other mathematics text books. To see why this should be, compare the work in the two fields. In a general mathematical field, such as number theory, analysis or group theory, the worker studies mathematical objects such as numbers, functions or groups. Now we would expect such objects to have general properties which arise naturally and can be discovered and proved mathematically. Also the objects and their properties will often suggest a natural notation for the work. Thus mathematical objects lend themselves naturally to a mathematical theory.

On the other hand, in mathematical logic the worker studies the general mathematician and his work. This is not a mathematical object. Thus we should not expect a natural mathematical theory to develop as a matter of course.

Because the mathematician and his work are closely linked to mathematical objects, it is in fact possible to develop a theory of mathematical logic which is similar to general mathematical theories. However, the presence of the human element in that which is to be modelled means that the theory does not arise naturally and simply but has to be constructed artificially so as to capture the ways and idiosyncrasies of the human mathematician.

Thus the resulting theory of mathematical logic has an uncomfortable air to it, with symbols of a rather grotesque complexity, and definitions and theorems with a somewhat bizarre generality which grates against the examples in actual use. (Note how, because a proof is a human construction and a group is a mathematical object, a proof of $10^{100}$ lines produces a wry smile, whereas a group of $10^{100}$ elements does not.)

Let me illustrate what I mean by “uncomfortable”. Consider the following,

5.4 Definition. Let $\mathfrak{A}$ and $\mathfrak{B}$ be $\mathcal{S}$-structures.

(a) A map $\pi: A \to B$ is called an isomorphism of $\mathfrak{A}$ onto $\mathfrak{B}$ (written: $\mathfrak{A} \cong \mathfrak{B}$) iff

1. $\pi$ is a bijection of $A$ onto $B$;
2. for $n$-ary $R \in \mathcal{S}$ and $a_0, \ldots, a_{n-1} \in A$,
   \[ R^{\mathfrak{A}}(a_0, \ldots, a_{n-1}) \iff R^{\mathfrak{B}}(\pi(a_0), \ldots, \pi(a_{n-1})); \]
3. for $n$-ary $f \in \mathcal{S}$ and $a_0, \ldots, a_{n-1} \in A$,
   \[ \pi(f^{\mathfrak{A}}(a_0, \ldots, a_{n-1})) = f^{\mathfrak{B}}(\pi(a_0), \ldots, \pi(a_{n-1})); \]
4. for every $c \in S$, $\pi(c^\mathfrak{A}) = c^\mathfrak{B}$.

(b) $\mathfrak{A}$ and $\mathfrak{B}$ are said to be isomorphic (written: $\mathfrak{A} \cong \mathfrak{B}$) iff there is an isomorphism $\pi: \mathfrak{A} \cong \mathfrak{B}$.

Incidentally, I feel that the initial ‘iff’ in (a) and in (b) should be ‘if’. The fact that it is a definition surely gives the required interpretation.

This definition is a mathematical way of saying that $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic if one can be obtained from the other by simply changing the names of the objects and leaving their properties unchanged.

The definition is followed by,

5.5 Isomorphism Lemma. If $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic $\mathcal{S}$-structures, then for all $\mathcal{S}$-sentences $\varphi$

\[ \mathfrak{A} \models \varphi \iff \mathfrak{B} \models \varphi. \]
This is saying that if $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic then a sentence which involves only properties and not names of objects is true of $\mathfrak{A}$ if and only if it is true of $\mathfrak{B}$. Is this obvious or do we need a proof? The proof given is just over a page long and contains,

... we shall show:

(i) For every $S$-term $t$, $\pi(\mathfrak{A}(t)) = \mathfrak{B}(t)$.
(ii) For every $S$-formula $\varphi$, $\mathfrak{A} \models \varphi$ if and only if $\mathfrak{B} \models \varphi$.

This will complete the proof. (i) can easily be proved by induction on terms. (ii) is proved by induction on formulas $\varphi$ simultaneously for all assignments $\beta$ in $\mathfrak{A}$. We only treat the case of atomic formulas and the steps involving $\forall$ and $\exists$.

Although omissions of 'obvious' parts of proofs do occur in general mathematical theories, they do not seem as common or as disturbing as in mathematical logic, where their presence suggests that the subject and the treatment are not quite in step.

Mathematical logic does not fit easily into the mould of general mathematical theories. That research mathematicians should develop mathematical logic as a mathematical theory is, of course, right and proper and, in fact, the further the subject moves away from the influence of actual use, the easier it should be to fit into the mould. However, the beginning undergraduate might be better served by an alternative treatment. If a proof-based approach is not entirely appropriate then perhaps an approach by way of computation and algorithms might be more fruitful.

To summarize, this book contains an appropriate selection of topics from mathematical logic, but the teacher using the book should consider carefully how best to introduce the topics to his students.

One final thought. As mathematical logic contains a human element, it could be that, when it is suitably taught, it has a particular appeal for girl students.

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General topology has become a popular topic for authors in recent years, and there is a need for books in which the subject is treated as a justifiable study in its own right at undergraduate level, and not just as a tool of the trade for functional analysts and others. This book is one of the Springer series; 'Undergraduate Texts in Mathematics', and to quote from the preface; "This volume covers approximately the amount of point-set topology that a student who does not intend to specialize in the field should nevertheless know. This is not a whole lot, and in condensed form would occupy perhaps only a small booklet. Our aim, however, was not economy of words, but a lively presentation of the ideas involved, an appeal to intuition in both the immediate and the higher meanings".

The chapters are: Fundamental concepts, Topological vector spaces, The quotient topology, Completion of metric spaces, Homotopy, The two countability axioms, CW-complexes, Construction of continuous functions on topological spaces, Covering spaces, The theorem of Tychonoff and set theory. Especially in view of the quotation from the preface, this seems a lot to achieve in an average-sized book. If the target audience is really an undergraduate one then I don't think it is achieved because all too often the reader has to make do with being introduced to the language and will have to go elsewhere to learn the mathematics.

The relative beginner will not have much idea of how he is getting on because there are no explicit exercises. There are plenty of hidden ones, often in throw-away remarks which appear with no hint as to whether they are immediately useful and worth attempting a proof, or which are to be accepted as general interest. Another most annoying feature is that the examples chosen to motivate or illustrate the importance of a theorem or idea are often far