PARAMETERS FOR DARK HALOS

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ABSTRACT. What are the characteristic scale lengths and densities for the dark halos of galaxies, and the typical ratios of dark to luminous mass? For elliptical galaxies, the best estimates come from X-ray data which will be discussed in a later session. For spirals, the best estimates come from rotation curves. I will concentrate on the halo parameters for disk galaxies. At the end, there will be a few comments on stellar dynamical data for ellipticals, and on the unique information available for the dark halo of our Galaxy.

1. THE HALOS OF DISK GALAXIES

The disks of disk galaxies are supported in the radial direction mainly by rotation. The equilibrium of disk matter is then given approximately by

$$GM(r)/r^2 = V^2(r)/r$$

where M(r) is the mass within radius r and V(r) is the rotational velocity at r. At large r, the integrated luminosity approaches the total luminosity. If M(r) also tends to a limit, then at large r we would expect V(r) to go like $r^{-1/2}$. This is not observed: usually V(r) is flat at large r, which suggests that M(r) increases like r, as for the isothermal sphere, and this is the most direct evidence for the presence of the massive dark corona component.

Does a flat rotation curve necessarily imply the presence of a dark corona? There has been some argument about this: we will see that the answer depends on how far out in radius the rotation data extend. Some work by Kalnajs (1983) illustrates this clearly.

1.1 Flat Rotation Curves

Kalnajs considered disk galaxies with measured rotation curves V(r) and surface brightness distributions I(r), and asked whether the gravitational field of the luminous components (bulge and disk) alone

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could produce the observed flat rotation curves. He assumed that the mass to light ratio M/L was constant, and devised an algorithm to calculate the expected rotation curve from the observed surface brightness distribution, assuming that the disk is in centrifugal equilibrium.

For several examples shown by Kalnajs, the rotation curves predicted from the surface brightness profiles were in excellent agreement with the observed rotation curves: no dark halo was needed. However, for these examples, as for most optically measured systems, the rotation data extends only to about 3 disk scale lengths. The important point that comes from this work is that, out to about 3 scale lengths, the disk and bulge together can produce a fairly flat rotation curve. This can also be seen from simple disk + bulge models. However, for a disk + bulge potential to produce a flat rotation curve requires a relationship between the scale lengths of the disk and the bulge, and the characteristic density of the bulge and surface density of the disk (see Freeman, 1985). This point is discussed further by Bahcall and Casertano (1985).

Rotation curves out to 3 scale lengths do not go far enough to show the gravitational effects of the dark halo unambiguously, so they tell us little about the presence of a dark halo. However, beyond about 3 scale lengths, the rotation curve predicted from the disk + bulge alone begins to fall, so a flat rotation curve extending beyond about 3 scale lengths provides strong evidence for the presence of a dark halo. If the rotation curve goes out far enough, into regions where the dark halo is dominating the gravitational field, then the shape of the curve can be used to estimate the scale length and density of the dark halo itself.

The procedure is to model the galactic rotation curve. The mass distributions for the disk and bulge components are given by surface photometry, and a simple model is used for the dark halo. The M/L ratios for the luminous components and the parameters for the dark halo are adjusted to fit the observed rotation curve. We will discuss this at some length. However, we should first make some preliminary comments on the shape and density distribution of the dark halo, and on the contribution that the luminous components make to the total gravitational field.

1.2 The shape of the dark halo

Here are some observations that are relevant to the question of the shape of the halo.

Many galactic disks show evidence for warping of the gaseous component. It is not yet understood how these warps survive. Some recent dynamical theories of warps (Dekel and Shlosman, 1983; Toomre, 1983; Sparke, 1985) require a somewhat flattened dark corona.

For several polar ring galaxies, rotation has been measured in both the disk of the parent galaxy and in the polar component, out to about $3R_{25}$ of the disk (see the paper by Whitmore et al. in this volume). The rotation curve in both components is flat, with V(polar)/V(disk) = 0.97 $\pm$ 0.08. If the dark component were in a disk, then this ratio would be in the range 0.6 to 0.8. Therefore, the dark matter is probably not disklike; the data suggests that it is more nearly spherical.

From the z-equilibrium of galactic disks, it is possible to make a direct estimate of the M/L ratio of the disk matter (Bahcall, 1984; van der Kruit and Shostak 1983; van der Kruit and Freeman, 1984). These M/L values are in the range 3 to 6, and include the contribution from the dark matter of the disk itself, but are not large enough to account for the amplitudes of the rotation curves in similar disk galaxies. It follows that about half of the galactic mass in typical spirals does not lie in the disk.

At this point, the shape of the dark halo is not well known, except that it is probably not disklike; it is usually taken to be spherical.

1.3 The density distribution of the dark halo

The main constraint is that the rotation curves are flat, in some cases to many disk scale lengths. This suggests that the density distribution $\rho(r) \propto r^{-2}$ at large r, although Bahcall et al. (1982) have shown that a steeper law may be appropriate. Some workers use an isothermal sphere to model the halo; others use a distribution of the form

$$\rho(r) = \rho_o[1 + (r/a)^{\gamma}]^{-1}$$

where $\gamma \approx 2$. Each model has a characteristic central density and scale length, and an associated velocity dispersion σ.

It would be better to use a selfconsistent bulge + disk + halo model, in which the halo is not simply imposed but is allowed to respond to the potential of the bulge + disk. Some work is already being done on such selfconsistent models, by Barnes and by van Albada.

1.4 Are the luminous components gravitationally significant ?

To estimate the halo parameters, the rotation curve for the disk + bulge + halo model is fitted to the observations by adjusting the model parameters. The shape of the rotation curve contribution from the luminous components (disk and bulge) is given by the surface photometry; the amplitude of this contribution is not known in advance, and depends on the adopted M/L ratios for the luminous components. The observed rotation curve puts an upper limit on these M/L ratios (ie when the luminous components provide the total potential gradient in the inner parts of the galaxy, as in Kalnajs's models).

Should the maximum values of M/L be used for the luminous components, or are smaller ones more appropriate ? Rubin and associates

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(see Rubin's review in this volume) point out the similarities in shape of rotation curves for different galaxies, and argue that the luminous components may not dominate the potential gradient anywhere. However, there are two further constraints on the M/L ratios for the luminous components:

(i) As mentioned above, it is possible to estimate M/L values for the disks of our Galaxy and other face-on systems, from the vertical velocity dispersions of gas and stars. This has been done now for several systems, and gives consistent values of M/L, between about 3 and 6. (Note that this includes the dark matter of the disk itself.)

(ii) Stellar population models give M/L ratios between about 1.2 and 7 for old disk and bulge populations, depending on their color (eg Larson and Tinsley 1978).

It seems unlikely that the M/L ratios of the luminous components (including the dark matter associated with the disk) could be much less than 2.

2. PURE DISK GALAXIES

These are disk galaxies with very small bulge components. The parameters of the dark halos can be measured fairly readily in these systems because:

(i) Their simple disk + dark halo structure makes it easier to identify the contribution of each component to the rotation curve.

(ii) They are usually late type galaxies with an extended HI distribution, so it is possible to measure their rotation curves out to many disk scale lengths. This is essential to tie down the halo parameters.

2.1 Carignan's work

Carignan estimated the halo parameters for three nearby pure disk galaxies: NGC 247, 300 and 3109 (see Carignan and Freeman 1985). From surface photometry he measured the I(r) distribution. Rotation curves V(r) out to large radii were obtained from HI and Fabry-Perot data. He used Kalnajs's procedure to calculate the rotation curve associated with the I(r) distribution, and fitted this calculated V(r) to the observed V(r) curve in the inner parts, where the disk probably dominates the radial potential gradient. This procedure determines the M/L value for the disk; it gives the maximum possible M/L for the disk (and therefore the minimum halo), as discussed in section 1.4.

After this fitting procedure was done, the observed V(r) curve lay well above the calculated V(r) in the outer parts of each galaxy. This shows the presence of a dark halo. The difference

$$\{V^2_{obs}(r) - V^2_{calc}(r)\}^{1/2}$$

was represented by the circular velocity curve for an isothermal sphere, which was chosen as a simple model for the dark halo. The isothermal sphere has a core radius r_c, central density ρ_o and velocity dispersion σ, where

$$4\pi G\rho_o r_c^2 = 9\sigma^2$$

At large radii, the circular velocity for the isothermal sphere tends to $\sqrt{2}\sigma$. This fitting procedure then gives estimates of r_c and σ for the dark halo. In some cases, it gives only the ratio σ/r_c, because the contribution of the isothermal sphere to the rotation curve is close to solid body rotation in the region where there is rotation data. However even this ratio is worth knowing, because it gives a direct estimate of the central density of the dark halo, from the above equation (subject to the lack of selfconsistency of the basic disk + halo model). Figure 1 shows how this procedure works for the galaxy NGC 3109.

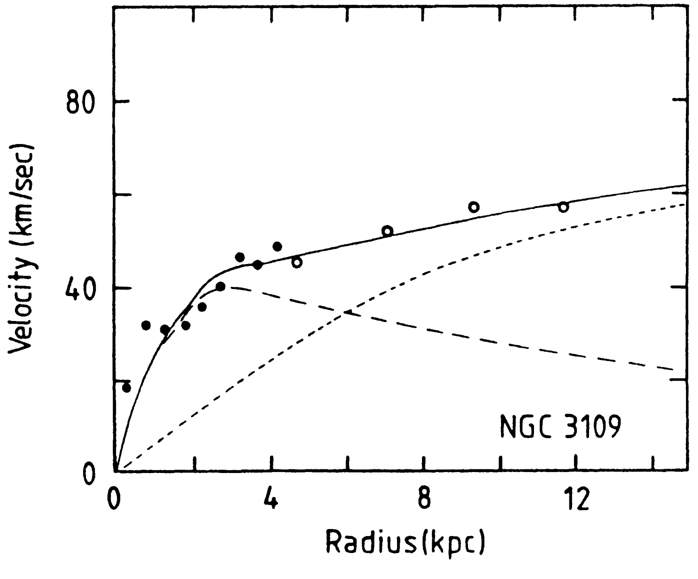

Figure 1. Disk + halo model for the rotation curve of NGC 3109, for the parameters given in Table I. The filled and open points represent the observed rotation curve. The long and short dashed curves are the contributions from the disk and the halo respectively, and the full curve is their sum.

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In a later study with the WSRT, Carignan et al. (1985) showed that the dwarf system UGC 2259 has a flat rotation curve and made a similar analysis to estimate the parameters of its dark halo. The results for these four pure disk galaxies are summarised in section 2.3.

2.2 NGC 3198: (van Albada et al. 1985)

The galaxies studied by Carignan are all fainter than $M_B = -18$. The Sc spiral NGC 3198 is brighter ($M_B = -19.4$), and is particularly interesting; for this galaxy, the WSRT rotation curve extends out to eleven disk scale lengths and the parameters for the dark halo can readily be estimated. van Albada et al. used a dark halo model of the form

$$\rho(r) = \rho_o[1 + (r/a)^{\gamma}]^{-1}$$

where $\gamma \approx 2$. The maximum disk/minimum halo solution has a M/L ratio of 3.6 for the disk, and the ratio of the halo mass to the disk mass is about 4, out to 30 kpc. However they point out that a model with less disk and more halo (say M/L = 1.2 for the disk) also gives an acceptable fit to the observed rotation curve. The fit is still adequate if the M/L ratio for the disk is taken as zero. (See however our arguments in section 1.4 against low values of M/L for the disk.)

2.3 Summary of results for pure disk galaxies

Table I gives parameters for the maximum disk/minimum halo solutions for the five pure disk galaxies described above. The columns give the name of the system, its absolute magnitude, the core radius (kpc), velocity dispersion (km s^{-1}) and central density ($M_\odot$ pc^{-3}) for the dark halo, the ratio of the halo mass to disk mass at the Holmberg radius and at the limit of the rotation data, and finally the maximum M/L ratio for the disk itself.

TABLE I

PARAMETERS FOR DARK HALOS OF PURE DISK GALAXIES

					M(halo)/M(disk)		
	M_B	r_c	σ	ρ_o	R_{Ho}	Limit	$(M/L)_d$
NGC 247	-18.0	9	80	0.003	0.8	4	7.0
NGC 300	-18.0	12	60	0.004	0.7	2	2.2
NGC 3109	-16.8	10	45	0.002	1.5	7	3.0
UGC 2259	-16.4	7	57	0.013	1.0	2	7.0
NGC 3198	-19.4	12	100	0.008	1.5	4	3.6

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For these pure disk systems, it is relatively straightforward to estimate the parameters of the dark halos: the disk and the halo are the only two components and, having the surface photometry, the M/L ratio is the only free parameter for the disk. Table I shows that the typical core radius for the dark halos is about 10 kpc and the typical central density is a few x 0.001 $M_{\odot}$ pc^{-3}. The mass of the halo out to the Holmberg radius is about the same as the disk mass. The last column shows that the maximum disk/minimum halo solutions give M/L values for the disk that are similar to those derived independently from the vertical equilibrium of the disks and from population models (section 1.4); there is no obvious reason at this stage to reject these solutions.

3. OTHER DISK GALAXIES

It is more difficult to study the dark halos of disk galaxies with significant bulges. The M/L ratio of the bulge comes in as a second free parameter for the luminous component, and in many cases the rotation curves do not extend much beyond the critical three disk scale lengths (see section 1.1). However there are many such galaxies with measured rotation curves, and recently Athanassoula, Bosma and Papaioannou (unpublished) have studied a sample of 60 systems with optical or 21-cm rotation curves.

Their procedure is similar in concept to that described earlier for the pure disk systems, with two extra features:

(i) for systems with bulges, the M/L value for the bulge appears as another free parameter;

(ii) swing amplifier theory (Toomre, 1981) was used provide another constraint on the M/L ratio of the disk. For example, if a particular galaxy shows no apparent m=1 mode but a clear m=2 mode, then its surface density was constrained to be low enough to inhibit the m=1 mode but high enough to allow the m=2 mode. In many cases, this constraint allowed the maximum disk permitted by the rotation curve, but in some cases it did not.

This is an interesting extra constraint. Is it an improvement on just assuming the maximum disk ? For systems in which the maximum disk is excluded by swing amplifier theory, the inferred dark halo will clearly have a core radius that is of the same order as the disk scale length, ie significantly shorter than the typical value of about 10 kpc found for the pure disk systems. The characteristic density for the dark halo will be correspondingly high. We would then need to know whether this high density and short core radius are just artifacts of the procedure, or are real features perhaps resulting from the dark halo responding to the potential of the disk and the bulge.

Preliminary results from this study include the following:

(i) For many of their systems, the parameters are fairly similar to those given in Table I for the pure disk galaxies. However for some systems, the halos do have significantly shorter core radii and characteristic densities that are higher by at least an order of magnitude, as discussed in the previous paragraph.

(ii) The ratio of halo mass to (disk + bulge) mass out to R_{25} is typically between 1 and 2, and shows a weak decreasing trend with increasing (B-V) color.

(iii) They also made minimum halo solutions for this sample. The M/L values for the disk derived in this way show an increasing trend with increasing (B-V) color. Quantitatively, this trend agrees well with the predictions from the Larson-Tinsley (1978) models (local IMF, monotonic decreasing star formation rates, ages 10 Gyr). Again, this favors the view that the minimum halo solutions are appropriate in most cases.

4. DARK MATTER IN ELLIPTICAL GALAXIES

The best estimates of dark matter parameters for elliptical galaxies at this time come from X-ray observations, which are discussed elswehere. Here I will just mention briefly some stellar dynamical items.

The run of velocity dispersion with radius in ellipticals is typically flat or rising: see for example Illingworth (1983), Efstathiou _et al._ (1982), Dressler (1979). However, this does not necessarily mean that these systems have a dark halo: Tonry (1983) and Richstone (unpublished) have pointed out that such dispersion profiles can be consistent with the observed light distribution and a radially uniform M/L ratio, if the velocity ellipsoid becomes more tangential with increasing radius. For the dark matter problem, it would therefore be very useful to find an observational way to estimate the anisotropy of the velocity ellipsoid in ellipticals, well outside their cores.

One (difficult) possibility comes from the _shape_ of the line of sight velocity distribution of stars in the outer parts of ellipticals; this shape will depend in some way on the distribution of the stellar orbital eccentricities and therefore on the degree of anisotropy of the velocity ellipsoid. As a simple example, take a spherical galaxy with the logarithmic potential $\Phi = V^2 \ln(r)$. Consider the extreme case in which the stellar orbits in the outer parts of this galaxy are circular and randomly orientated (ie the radial component of the velocity ellipsoid is zero). Then the distribution of observed line of sight velocities for these stars would be uniform between -V and +V, and zero elsewhere. On the other hand, for an isotropic velocity ellipsoid, the distribution of observed line of sight velocities would be peaked at zero, with wings going out beyond ±V. It would probably be difficult to measure the shape of the broadening function accurately enough, from integrated spectra and Fourier techniques, to estimate the anisotropy of the velocity dispersion in the outer parts of ellipticals. A more hopeful procedure would be to acquire a large sample of radial velocities for the globular clusters around M87, say; this would give the shape of the observed line of sight velocity distribution directly, for the outer parts of the galaxy.

An independent dynamical approach to dark matter in ellipticals comes from the shells observed around many elliptical galaxies. In Quinn's (1984) dynamical theory for the origin of the shells, the relationship between the radii of successive shells depends on the potential field of the galaxy. The radii of shells in some well-observed systems suggests the presence of dark matter, but not out to very large radii.

We should mention here the question of dark matter in globular clusters, which has been discussed by Peebles (1984). To test for dark matter, we would need to know the radial behaviour of the velocity ellipsoid in a globular cluster. This information is not yet available. Probably the best data for this purpose comes from ω Cen, for which Seitzer and Freeman (to be published) have measured the line of sight velocity dispersion over the entire range of radius, from the cluster center to the tidal radius. A simple isotropic King model, derived from star counts, reproduces this velocity dispersion profile very well. Its M/L ratio is about 2.5, which is well within the range of acceptable values for a globular cluster stellar population. If there is a significant amount of dark matter in the outer parts of this cluster, then the velocity ellipsoid in the outer parts must be highly anisotropic.

5. THE GALACTIC DARK HALO

From the galactic rotation curve, it is possible to estimate the parameters of the galactic dark halo, just as for other spirals. We can also use the kinematics of high velocity stars in the solar neighborhood, the M31/Galaxy timing arguments, and the properties of the outer spheroidal component to put additional constraints on the parameters of the galactic dark halo.

5.1 The Rotation Curve

The rotation curve of the Galaxy can be used to estimate the parameters of its dark halo, using procedures similar to those described in Sections 3 and 3. However, there are some extra difficulties. (i) The galactic rotation curve is not as well determined in the outer regions as it is for some other spirals. (ii) The structure of the luminous component (eg the scale length of the disk) is also not as well determined. Several groups have recently constructed models to estimate the dark matter content. Schmidt's (1985) model is simple and

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illustrative. He uses three components to model the galactic rotation curve: a bulge, an exponential disk with a scale length of 3.5 kpc, and a dark halo with density distribution $\rho = \rho_0(1 + r^2/a^2)^{-1}$. The density scale of the disk (which for other galaxies was given by the M/L ratio) comes in here through the local surface density $\Sigma_\odot$ at the sun. The model parameters are not tightly constrained. For example, for $\Sigma_\odot = 50\ M_\odot\ pc^{-2}$, the halo scale length a = 4.6 kpc and the local density of the halo is 0.010 $M_\odot\ pc^{-3}$; for $\Sigma_\odot = 65\ M_\odot\ pc^{-2}$, a = 6.5 kpc and the local density of the halo is 0.004 $M_\odot\ pc^{-3}$.

To reproduce the observed rotation curve with the potential field of the luminous components alone would require unrealistic parameters, such as a disk scale length of 8 kpc and $\Sigma_\odot = 200\ M_\odot\ pc^{-3}$. From the galactic rotation curve, it seems very likely that the Galaxy has a dark halo, but it is probably too early to expect accurate values of the parameters for the galactic dark halo from this approach.

5.2 The Escape Velocity

High velocity stars passing through the solar neighborhood have velocities of up to at least 500 km s^{-1} relative to a nonrotating frame. If these stars are bound to the Galaxy, they give some information about the properties of the dark halo. For example, assume the galactic rotation curve is flat (with velocity V) out to some radius R_m, and Keplerian beyond R_m. If $v_\odot$ is the velocity of a star in the solar neighborhood, relative to a nonrotating frame, and $R_\odot$ is the galactocentric distance of the sun, then a star that just escapes from the Galaxy has

$$v_\odot^2 = 2V^2[1 + \ln(R_m/R_\odot)].$$

Therefore R_m is at least 40 kpc and the total mass of the Galaxy is then at least $5.10^{11}\ M_\odot$. See Carney's paper in this volume for a more extensive discussion of this work.

Such estimates are probably lower limits: we do not yet fully understand the dynamics of the population of high velocity stars, and it may be that all the high velocity stars in the solar neighborhood are firmly bound to the Galaxy.

5.3 Timing

The M31/Galaxy timing arguments (eg. Gunn, 1974) give a galactic mass of about $10^{12}\ M_\odot$, if the M/L ratios of the Galaxy and of M31 are similar. If the mass is so large, then the flat rotation curve of the Galaxy must continue out to radii of 80 to 100 kpc.

5.4 The Isothermal Spheroidal Component

Velocity dispersions have now been measured for many classes of objects belonging to the spheroidal component (RR Lyrae stars, metal weak

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giants, globular clusters, M stars), with galactocentric distances ranging from near zero up to about 60 kpc. The velocity dispersion of the spheroidal component is remarkably constant over this entire range in radius, at about 120 km s^{-1} (see Freeman 1985). The density distribution for the spheroidal component is observed to follow an $r^{-3.5}$ law (see for example Zinn (1985). Then, if the velocity ellipsoid is isotropic, the galactic mass within 60 kpc is about 7.10^{11} $M_\odot$. This agrees well with the value that would be estimated from the flat rotation curve out to this distance (see section 5.3).

5.5 Conclusion

It seems very likely that the galactic dark halo extends out to about 50-100 kpc and that its mass is $(5-10).10^{11}$ $M_\odot$. We can estimate the ratio of dark to luminous mass out to the Holmberg radius, for comparison with the other systems given in Table I. If we take the disk scale length as 4 kpc and the local column density as 65 $M_\odot$ pc^{-2}, then the luminous mass (including the bulge and the dark matter associated with the disk) is about 7.10^{10} $M_\odot$. Out to the Holmberg radius, the (dark + luminous) mass is about 18.10^{10} $M_\odot$, so the ratio of dark to luminous mass is about 1.5, which is similar to the values given in Table I. The ratio of the total dark mass to luminous mass could be as large as 15. However this is only a factor of 2 greater than the largest value of dark to luminous mass, out to the limit of the rotation data, for the galaxies given in Table I.

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DISCUSSION

SCHECHTER: Could you elaborate on your assumption that the disk mass-to-light ratio is constant as a function of radius? Isn't it curious that disks are supposed to provide all of the observed mass over the radius range in which they have roughly constant angular velocity, while halos are needed only where disks cease to have constant angular velocity?

FREEMAN: You know from the vertical equilibrium constraints that disk M/L ratios are unlikely to be much less than 1, even in very blue systems. Sometimes this forces you to make a maximum-disk model. Otherwise, I can't say more than I've already said. The rotation-curve decomposition procedure is not yet robust.

FABER: How would it affect the swing amplifier criterion if the spiral structure were driven by a bar?

ATHANASSOULA: The swing amplifier criterion tells us only whether the disk can respond to forcing, it doesn't distinguish between different kinds of forcing. The point we are making is that the properties of a galaxy (disk surface density, halo mass, velocity dispersion, etc.) must be such as to allow the spiral structure that we see. This is what the swing amplifier criterion tells us.

FREEMAN: In particular, you won't get spiral arms if Q is too large.

RICHSTONE: Your maximum-disk models show that Vera Rubin's statement that dark matter contributes to g at all radii <u>need</u> not be correct. They do not show that it <u>is</u> <u>not</u> correct. How can one do better?

FREEMAN: One can look at the stellar dynamics of bulges in early-type systems. Velocity dispersions and stellar rotation measurements give $M/L \approx 7$ for bulges, which suggests that they are self-gravitating. Also, recall that the <u>vertical</u> equilibrium of disks (Bahcall, van der Kruit and associates) implies that disk M/L values are in the range 3 - 6. This puts a lower limit on the contribution of the disk to g.

E. TURNER: It would appear easier to understand the "conspiracy" of the disk, bulge, and halo to produce flat and relatively featureless rotation curves if these three components formed more or less at once and from the same type of material (i.e., baryons).

FREEMAN: Maybe it would not make too much difference if everything happens adiabatically, in whatever order.

RUBIN: I would like to urge you to tabulate values not only for the model including the maximum disk, but also for the model including the maximum halo, whenever both can be determined. As long as there is a question as to which fit is most plausible, it will be valuable to know the range of permitted halo parameters.

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GUNN: I think that it is probably impossible to make a maximum halo model, because all of these models assume some ad hoc form for the halo density distribution. N-body simulations indicate that the central concentration you get in a halo is critically dependent on how much substructure there was initially. The amount of substructure was certainly stochastically variable. So I don't think one knows a priori what to expect for the form of the halo density distribution.

FABER: Yes. And if the dissipative infall of baryons further perturbs the halo, then until we have a good theory for that process, we don't know the shape of the halo density distribution today.

FREEMAN: If the halo is adiabatic, we can get a long way without needing to know exactly how the baryons fell in. If we then make self-consistent bulge/disk/halo models, I think we will be one step closer to reality than we are now.

GUNN: We can't really make such models until we know more about halos.

OSTRIKER: In your decompositions, you stress the galaxies with small bulges, and you find that things fit nicely with normal M/L ratios. Did you look at the larger-bulge systems? Can you still model them with normal M/L ratios? Or do you need very low values, maybe even halos with negative masses.

FREEMAN: The cases of large bulges that I have looked at are from the work of Athanassoula and Bosma. There the bulge M/L ratios had very normal values, $M/L \sim 4 - 5$. They certainly found no negative values.

SELLWOOD: Are the disks resulting from these decompositions everywhere locally gravitationally stable?

ATHANASSOULA: Yes.

SCHECHTER: Was it a matter of choice that your bulge-free spirals all had very low luminosities or did you have difficulty finding high-luminosity systems?

FREEMAN: The choice was to avoid systems with bulges. In the early work that Carignan and I did, we had to choose nearby galaxies so that we could make single-dish HI observations. That meant that we worked on galaxies in the Sculptor group, and these have low luminosities.

LAKE: Jacqueline van Gorkom, Bob Schommer and I have measured an HI rotation curve for an elliptical galaxy. The galaxy is about 0.5 mag fainter than L_*, has B-V = 0.9 and a de Vaucouleurs-law profile. In other words, it is a perfectly normal elliptical. The rotation curve is flat out to four or five times the effective radius. The value of M/L thus changes by a factor of four over the radius range of the observations.

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