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A NOTE ON FUNCTION SPACES GENERATED BY RADEMACHER SERIES

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Let X be a rearrangement invariant function space on [0, 1] in which the Rademacher functions (r_n) generate a subspace isomorphic to ℓ^2. We consider the space $\Lambda(\mathcal{R}, X)$ of measurable functions f such that $fg \in X$ for every function $g = \sum b_n r_n$ where $(b_n) \in \ell^2$. We show that if X satisfies certain conditions on the fundamental function and on certain interpolation indices then the space $\Lambda(\mathcal{R}, X)$ is not order isomorphic to a rearrangement invariant space. The result includes the spaces $L_{p,q}$ and certain classes of Orlicz and Lorentz spaces. We also study the cases $X = L_{\exp}$ and $X = L_{\psi_2}$ for $\psi_2(t) = \exp(t^2) - 1$.

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Let $\Lambda(\mathcal{R})$ be the space of measurable functions f such that $fg \in L^1[0, 1]$ for every $g \in \mathcal{R}$, where $\mathcal{R}$ is the subspace generated by the Rademacher functions (r_n) in $L^1[0, 1]$. Endowed with the norm $\|f\| = \sup\{\|fg\|_1 : g = \sum b_n r_n, \sum b_n^2 \le 1\}$ it is a Köthe function space. This space arises as the space of functions which are integrable in the sense of Bartle, Dunford and Schwartz with respect to the vector measure $v(A) = (\int_A r_n(t)\,dt) \in \ell^2$, see [3]. It can alternatively be described as the space of functions f such that for every measurable set A the Rademacher-Fourier coefficients of the function $f\chi_A$ are in ℓ^2. The question we investigate is the following. Can the functions in $\Lambda(\mathcal{R})$ be described by their distribution function? In other words, is $\Lambda(\mathcal{R})$ order isomorphic to a rearrangement invariant space?

We study the problem in a more general setting, replacing $L^1[0, 1]$ by a rearrangement invariant (r.i.) function space X on [0, 1] in which the Rademacher functions generate a subspace isomorphic to ℓ^2 (these spaces have been studied in [**6**] and [**5**, Theorem 2.b.4]). We will denote by $\Lambda(\mathcal{R}, X)$ the space of measurable functions $f : [0, 1] \to \mathbb{R}$ such that $fg \in X$ for every function $g = \sum b_n r_n$ with $(b_n) \in \ell^2$. It is a Köthe function space when endowed with the norm

$$\|f\|_\Lambda = \sup\left\{\|fg\|_X : g = \sum b_n r_n, \sum b_n^2 \le 1\right\}.$$

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This space can also be viewed as the space of multiplication operators from the subspace spanned by the Rademacher functions in X into the whole space.

In this paper we prove that if the fundamental function of the space X is nearly submultiplicative (see below) and the lower Boyd and fundamental indices coincide then the space $\Lambda(\mathcal{R}, X)$ is not order isomorphic to a rearrangement invariant space. The result includes the spaces $L_{p,q}[0,1]$, for $1 < p < +\infty$, $1 \le q \le +\infty$, or $p = q = 1$; the Orlicz spaces $L_\phi[0,1]$ where the function ϕ satisfies the Δ' condition globally (see [4, Definition I.5.1]), and the Lorentz spaces $L_{W,p}[0,1]$, for $1 \le p < +\infty$, where the function W is nearly submultiplicative.

We also show that if X is the Orlicz space L_{ψ_2} given by the function $\psi_2(t) = \exp(t^2) - 1$, then $\Lambda(\mathcal{R}, X)$ is order isomorphic to $L_\infty[0,1]$. For $X = L_{\exp}$ we show that $\Lambda(\mathcal{R}, X)$ is order isomorphic to L_{ψ_2}.

By $\varphi_X(t)$ we will denote the fundamental function of X defined by $\varphi_X(t) = \|\chi_{[0,t]}\|_X$. The dilation operators $E_t : X \to X$, for $0 < t < +\infty$, are defined by $E_t f(s) = f(st)$ for $s \le \min(1, 1/t)$ and $E_t f(s) = 0$ for $1/t < s \le 1$ (if $t > 1$). We will denote $h_X(t) = \|E_{1/t}\|$, where $\|E_{1/t}\|$ is the norm of $E_{1/t}$ as an operator on X. For any undefined notation we refer the reader to [1] or [5].

Lemma 1. *There exists a constant $K > 0$ such that for each n of the form $n = 2^m$ there are measurable sets B_n, D_n of equal measure $n2^{-n}$ with*

$$K \cdot \|\chi_{B_n}\|_\Lambda \cdot \|\chi_{D_n}\|_\Lambda^{-1} \le \frac{h_X(n2^{-n})}{n^{1/2} \cdot \varphi_X(n2^{-n})} + \left(\frac{\varphi_X(2^{-n})}{\varphi_X(n2^{-n})}\right)^{1/2}.$$

Proof. Let n be a fixed positive integer such that $n = 2^m$, for $m \in \mathbb{N}$. Consider the dyadic intervals of order n, $A_j = ((j-1)2^{-n}, j2^{-n})$, for $1 \le j \le 2^n$. Consider the $n \times 2^n$ matrix (a_{ij}), where a_{ij} $(1 \le i \le n;\ 1 \le j \le 2^n)$ is the value of the function r_i on the interval A_j. Let J be a subset of $\{1, \dots 2^n\}$ with cardinality n. Associated with J we have the measurable set $A = \cup_{j \in J} A_j$ of measure $n2^{-n}$. Consider the characteristic function of the set A in $\Lambda(\mathcal{R}, X)$, then

$$\begin{aligned}\|\chi_A\|_\Lambda &= \sup\left\{\left\|\chi_A \sum_1^\infty b_i r_i\right\|_X : (b_i) \in B_{\ell^2}\right\}\\ &\le \sup\left\{\left\|\chi_A \sum_1^n b_i r_i\right\|_X : (b_i) \in B_{\ell_2^n}\right\} + \sup\left\{\left\|\chi_A \sum_{n+1}^\infty b_i r_i\right\|_X : (b_i) \in B_{\ell^2}\right\}.\end{aligned}$$

In order to estimate the second term in the right hand side take into account that

$$\chi_{[0,n2^{-n}]} \sum_{n+1}^\infty b_i r_i = E_{n^{-1}2^n}\left(\sum_{n+1}^\infty b_i r_{i+m-n}\right).$$

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Considering that X is r.i., we obtain:

$$\left\| \chi_A \sum_{n+1}^{\infty} b_i r_i \right\|_X = \left\| \chi_{[0,n2^{-n}]} \sum_{n+1}^{\infty} b_i r_i \right\|_X = \left\| E_{n^{-1}2^n} \left(\sum_{n+1}^{\infty} b_i r_{i+m-n} \right) \right\|_X$$
$$\leq \| E_{n^{-1}2^n} \| \cdot \left\| \sum_{n+1}^{\infty} b_i r_{i+m-n} \right\|_X \leq \| E_{n^{-1}2^n} \| \cdot M \cdot \left(\sum_{n+1}^{\infty} |b_i|^2 \right)^{1/2},$$

where M is a constant not depending on n. Hence

$$\sup \left\{ \left\| \chi_A \sum_{n+1}^{\infty} b_i r_i \right\|_X : (b_i) \in B_{\ell^2} \right\} \leq M \cdot h_X(n2^{-n}).$$

Select a set $J_1 \subset \{1, \ldots, 2^n\}$ with cardinal n, such that the $n \times n$ matrix $n^{-1/2}(a_{ij})_{j \in J_1}$ is orthogonal. Denote $B_n = \cup_{j \in J_1} A_j$, it follows that

$$\sup \left\{ \left\| \chi_{B_n} \sum_{1}^{n} b_i r_i \right\|_X : (b_i) \in B_{\ell_2^n} \right\} = \sup \left\{ \left\| \sum_{j \in J_1} \left(\sum_{i=1}^{n} b_i a_{ij} \right) \chi_{A_j} \right\|_X : (b_i) \in B_{\ell_2^n} \right\}$$
$$= \sup \left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (\beta_j) \in n^{1/2} B_{\ell_2^n} \right\}$$
$$= n^{1/2} \cdot \sup \left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (\beta_j) \in B_{\ell_2^n} \right\}.$$

Then, for a certain constant $C > 0$, we have

$$\| \chi_{B_n} \|_\Lambda \leq C \cdot \left(h_X(n2^{-n}) + n^{1/2} \cdot \sup \left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (\beta_j) \in B_{\ell_2^n} \right\} \right). \tag{1}$$

Select next a set $J_2 \subset \{1, \ldots, 2^n\}$ with cardinality n, such that each column of the $n \times n$ matrix $(a_{ij})_{j \in J_2}$ has exactly one entry equal to -1 and the rest are equal to 1. Hence we have that $\sum_{i=1}^{n} a_{ij} = n - 2$, for each $j \in J_2$. Denote $D_n = \cup_{j \in J_2} A_j$. Then

$$\begin{aligned} \| \chi_{D_n} \|_\Lambda &\geq \left\| \sum_{j \in J_2} \left(\sum_{i=1}^{n} \frac{a_{ij}}{n^{1/2}} \right) \chi_{A_j} \right\|_X \\ &= (n^{1/2} - 2n^{-1/2}) \cdot \| \chi_{D_n} \|_X \\ &= (n^{1/2} - 2n^{-1/2}) \cdot \varphi_X(n2^{-n}). \end{aligned} \tag{2}$$

From (1) and (2) it follows that for a constant $K > 0$ not depending on n

$$K \cdot \frac{\| \chi_{B_n} \|_\Lambda}{\| \chi_{D_n} \|_\Lambda} \leq \frac{h_X(n2^{-n})}{n^{1/2} \cdot \varphi_X(n2^{-n})} + \varphi_X(n2^{-n})^{-1} \cdot \sup \left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (\beta_j) \in B_{\ell_2^n} \right\}.$$

Let X_n be the n-dimensional subspace generated in X by the functions χ_{A_j} for $j \in J_1$. Consider the operators $T_i : \ell_i^n \longrightarrow X_n$ given by $T_i(\beta_j) = \sum \beta_j \chi_{A_j}$, where $i = 1, 2, \infty$. Then

$$\begin{aligned} \| T_2 \| &= \sup\left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (B_j) \in B_{\ell_2^n} \right\} \\ \| T_\infty \| &= \left\| \sum \chi_{A_j} \right\|_X = \varphi_X(n2^{-n}) \\ \| T_1 \| &= \left\| \chi_{A_j} \right\|_X = \varphi_X(2^{-n}). \end{aligned}$$

Since $\sum_1^n x_i^2 \le \max |x_i| \cdot \sum_1^n |x_i|$, we deduce that $\| T_2^* \| \le \| T_1^* \|^{1/2} \cdot \| T_\infty^* \|^{1/2}$, where T_i^* are the adjoint operators, thus $\| T_2 \| \le \| T_1 \|^{1/2} \cdot \| T_\infty \|^{1/2}$. Hence

$$\begin{aligned} &\varphi_X(n2^{-n})^{-1} \cdot \sup\left\{ \left\| \sum \beta_j \chi_{A_j} \right\|_X : (\beta_j) \in B_{\ell_2^n} \right\} \\ &\quad = \| T_\infty \|^{-1} \cdot \| T_2 \| \le \| T_1 \|^{1/2} \cdot \| T_\infty \|^{-1/2} = \left(\frac{\varphi_X(2^{-n})}{\varphi_X(n2^{-n})} \right)^{1/2}, \end{aligned}$$

and the claim follows. □

We recall the following interpolation indices of a r.i. space X, the Boyd index $\beta(X)$ and the fundamental index $P_0(X)$, respectively

$$\beta(X) = \sup_{0<t<1} \log h_X(t)/\log t, \quad P_0(X) = \sup_{0<t<1} \log M_X(t)/\log t,$$

where $M_X(t) = \sup_s \varphi_X(st)/\varphi_X(s)$, see [**2**], and [**8**]. In general $0 \le \beta(X) \le P_0(X)$. We will say that a function f is nearly submultiplicative if $f(st) \le cf(s)f(t)$ for a constant $c > 0$. It is easily checked that the following holds.

Lemma 2. *Let X be a r.i. space, then φ_X is nearly submultiplicative and $\beta(X) = P_0(X)$ if and only if the functions φ_X and h_X are equivalent ($\varphi_X(t) \le h_X(t) \le a\varphi_X(t)$ for all $0 < t \le 1$ and a constant $a \ge 1$).*

Theorem. *Let X be a rearrangement invariant function space on $[0, 1]$ such that the Rademacher functions generate in X a subspace isomorphic to ℓ^2. Suppose that $\beta(X) = P_0(X)$ and φ_X is nearly submultiplicative. Then the space $\Lambda(\mathcal{R}, X)$ is not order isomorphic to a rearrangement invariant space.*

Proof. Since φ_X is submultiplicative, $\varphi_X(2^{-n})/\varphi_X(n2^{-n})$ is bounded above by a constant multiple of $\varphi_X(n^{-1})$. This converges to zero as n tends to infinity since $\varphi_X(0^+) = 0$. This last statement holds since the Rademacher functions generate in X a subspace isomorphic to ℓ^2, see [**6**, Lemma 3 and Theorem 7]. By Lemma 2

$h_X(n2^{-n})/\varphi_X(n2^{-n})$ is bounded from above, so $n^{-1/2}h_X(n2^{-n})/\varphi_X(n2^{-n})$ converges to zero. Hence we can apply Lemma 1 and deduce that there exists two sequences of measurable sets (B_n) and (D_n) of equal measure and such that the ratio $\|\chi_{B_n}\|_\Lambda \cdot \|\chi_{D_n}\|_\Lambda^{-1}$ converges to zero. This implies that the space $\Lambda(\mathcal{R}, X)$ is not order isomorphic to a r.i. space. □

The r.i. spaces X in which the Rademacher functions generate a subspace isomorphic to ℓ^2 are precisely those satisfying

$$\|f\|_X \le M\|f\|_{L_{\psi_2}} \quad \text{for every } f \in L_\infty[0, 1],$$

where $M > 0$ is a constant and L_{ψ_2} is the Orlicz space given by the function $\psi_2(t) = \exp(t^2) - 1$, [**6**, Theorem 6]. This condition is implied by $\beta(X) > 0$, or $P_0(X) > 0$ (which is equivalent to X being in the class $\mathcal{L}$ defined in [7]), or $L^p[0, 1] \subset X$ for some p, $1 < p < \infty$.

The conditions in the previous theorem are satisfied for the following classes of r.i. spaces:

(a) The spaces $L_{p,q}[0, 1]$ (see [**5**, Definition 2.b.8]) for $1 < p < +\infty$, $1 \le q \le +\infty$, or $p = q = 1$, since $h_X(t) = \varphi_X(t) = t^{1/p}$.

(b) The Lorentz spaces $L_{W,p}[0, 1]$ for $1 \le p < +\infty$ (see [**5**, p. 121]) such that the function W is nearly submultiplicative, since in this case $\Phi(t) = \int_0^t W(s)\,ds$ is nearly submultiplicative and $\varphi_X(t) = \Phi(t)^{1/p}$ and $h_X(t) = (\sup_{s>0} \Phi(s)/\Phi(s/t))^{1/p}$, [**2**, Theorem 4.1].

(c) The Orlicz spaces $L_\phi[0, 1]$ (see [**1**, Chap. 4.8]) such that the function ϕ satisfies the Δ' condition globally (which is precisely ϕ being nearly submultiplicative, see [**4**, Definition I.5.1]), since $\varphi_X(t) = 1/\phi^{-1}(1/t)$ where ϕ^{-1} is the right-continuous inverse of ϕ, and in the computation of $\beta(X)$ we can replace $h_X(t)$ by $g(t) = \limsup_{s\to\infty} \phi^{-1}(s)/\phi^{-1}(s/t)$, see [**1**, Theorem 4.8.18].

Remark. The Zygmund space $L \log L$ is included in case (c).

The previous result does not hold for every r.i. space in which the Rademacher functions generate a subspace isomorphic to ℓ^2. This is shown by the following examples.

Example 1. Consider the Orlicz space L_{ψ_2} mentioned above. This space is the smallest r.i. space in which the Rademacher functions generate a subspace isomorphic to ℓ^2, in the sense that every such space must contain the closure in L_{ψ_2} of bounded functions, [**6**, Theorem 6]. Let B be an interval in [0, 1] and A a dyadic interval of order a suitable n, so that A is included in B. For appropriate signs $a_i = \pm 1$ we have $\sum_1^n a_i r_i = n$ on A. Consider χ_B in $\Lambda(\mathcal{R}, L_{\psi_2})$, then for a constant $C > 0$ not depending on n we have

$$\|\chi_B\|_\Lambda \geq \|\chi_A\|_\Lambda \geq \left\|\chi_A \sum_1^n \frac{a_i}{n^{1/2}} r_i\right\|_{L_{\psi_2}} = n^{1/2}\|\chi_A\|_{L_{\psi_2}} = n^{1/2}\varphi_{L_{\psi_2}}(2^{-n}) \sim C,$$

since $\varphi_{L_{\psi_2}}(t) = \log(1+1/t)^{-1/2}$, up to some normalization constant. Let D be a measurable set of positive measure. By Lebesgue's Density Theorem we can find a dyadic interval A so that the Lebesgue measure of A and of $D \cap A$ are as close as desired. Let n be the order of A, then as before

$$\|\chi_D\|_\Lambda \geq \|\chi_{D\cap A}\|_\Lambda \geq \left\|\chi_{D\cap A} \sum_1^n \frac{a_i}{n^{1/2}} r_i\right\|_{L_{\psi_2}} = n^{1/2}\|\chi_{D\cap A}\|_{L_{\psi_2}},$$

but $\|\chi_{D\cap A}\|_{L_{\psi_2}} \sim \|\chi_A\|_{L_{\psi_2}}$, so $\|\chi_D\|_\Lambda \geq C$. Hence $\Lambda(\mathcal{R}, L_{\psi_2})$ contains no unbounded functions. From [**6**, Lemma 3] it follows that $\Lambda(\mathcal{R}, L_{\psi_2})$ is order isomorphic to $L_\infty[0,1]$.

Example 2. Consider the space L_{ψ_1} given by the Orlicz function $\psi_1(t) = \exp(t) - 1$, also known in the literature as $L_{\exp}$. We will see that $\Lambda(\mathcal{R}, L_{\exp})$ is order isomorphic to L_{ψ_2}. Let f be in L_{ψ_2} and consider $\sum a_n r_n$ for $(a_n) \in \ell^2$, which is also in L_{ψ_2}. Since in general $g \in L_{\psi_2}$ implies $g^2 \in L_{\exp}$ and $4xy = (x+y)^2 - (x-y)^2$, it follows that $f\sum a_n r_n$ is in $L_{\exp}$, so f is in $\Lambda(\mathcal{R}, L_{\exp})$. For the reverse inclusion let us establish the next claim.

Lemma 3. *For each $p\,(1 \leq p < \infty)$ and for each function f in $L^p[0,1]$ there exists a norm one sequence (a_n) in ℓ^2 such that*

$$\|f\|_p \leq 3\,p^{-1/2} \cdot \left\|f \sum a_n r_n\right\|_p.$$

Proof. Given $\varepsilon > 0$ let g be a simple function supported on dyadic intervals and such that $|g| \leq |f|$ and $\|f\|_p \leq (1+\varepsilon)\|g\|_p$. Let N be the highest dyadic order of the intervals of constancy of g. Consider $m \in \mathbb{N}$ such that $m - 1 \leq p < m$. Define $a_n = 0$ for $n \leq N$, $a_n = m^{-1/2}$ for $N+1 \leq n \leq N+m$, and $a_n = 0$ for $n > N+m$. Direct computation shows that $\|\sum a_n r_n\|_p \geq m^{1/2} 2^{-(m-1)/p} \geq p^{1/2} 2^{-1}$. Since g and $\sum a_n r_n = \sum_{n>N} a_n r_n$ are independent, we have

$$\int_0^1 \left|g \sum a_n r_n\right|^p = \int_0^1 |g|^p \cdot \int_0^1 \left|\sum a_n r_n\right|^p.$$

Hence

$$\begin{aligned}\|f\|_p &\leq (1+\varepsilon)\cdot\|g\|_p \leq (1+\varepsilon)2\,p^{-1/2}\cdot\|g\|_p \cdot \left\|\sum a_n r_n\right\|_p \\ &= (1+\varepsilon)2\,p^{-1/2}\cdot\left\|g\sum a_n r_n\right\|_p \leq 3\,p^{-1/2}\cdot\left\|f \sum a_n r_n\right\|_p.\end{aligned}$$

□

Assume now that f is in $\Lambda(\mathcal{R}, L_{\exp})$, that is $f\sum a_n r_n$ is in $L_{\exp}$ for every $(a_n) \in \ell^2$. In particular $f \in L^p[0, 1]$ for every p, $1 \le p < \infty$. Denote by h_p the function $\sum a_n r_n$ with (a_n) of norm one in ℓ^2 given by Lemma 3. Using the power series expansion of the functions $\psi_1(t) = \exp(t) - 1$ and $\psi_2(t) = \exp(t^2) - 1$ we have the following equivalent norms in the spaces $L_{\exp}$ and L_{ψ_2}:

$$\|f\|_{L_{\exp}} = \sup_{1 \le p < \infty} p^{-1} \cdot \|f\|_p \quad \text{and} \quad \|f\|_{L_{\psi_2}} = \sup_{1 \le p < \infty} p^{-1/2} \cdot \|f\|_p.$$

Then

$$\begin{aligned} \|f\|_{L_{\psi_2}} &= \sup\left\{p^{-1/2} \cdot \|f\|_p : 1 \le p < \infty\right\} \\ &\le 3 \sup\left\{p^{-1} \cdot \|f h_p\|_p : 1 \le p < \infty\right\} \\ &\le 3 \sup\left\{p^{-1} \cdot \left\|f \sum a_n r_n\right\|_p : 1 \le p < \infty, (a_n) \in B_{\ell^2}\right\} \\ &= 3 \sup\left\{\left\|f \sum a_n r_n\right\|_{L_{\exp}} : (a_n) \in B_{\ell^2}\right\} \\ &= 3\|f\|_\Lambda. \end{aligned}$$

So f is in L_{ψ_2} and $\|f\|_{L_{\psi_2}} \le 3\|f\|_\Lambda$. Hence both spaces are order isomorphic.

We can also consider the space $\Lambda(\mathcal{R}, X)$ for r.i. spaces where the subspace generated by the Rademacher functions is other than ℓ^2. If $X = L_\infty[0, 1]$ then the Rademacher functions generate a subspace isomorphic to ℓ^1, so $\Lambda(\mathcal{R}, X)$ is $L_\infty[0, 1]$.

Example 3. Let X be the Orlicz space L_{ψ_2} where $\psi_q(t) = \exp(t^q) - 1$ and $q > 2$. The Rademacher functions generate in X a subspace isomorphic to the sequence space $\ell_{p,\infty}$, where $1/p + 1/q = 1$, [**6**, Section 6]. Then, as in Example 1, for a dyadic set A of order n and for a constant $C > 0$ not depending on n, we have

$$\|\chi_A\|_\Lambda \ge \left\|\chi_A \sum_1^n \frac{a_i}{i^{1/p}} r_j\right\|_{L_{\psi_q}} \ge \left\|\chi_A \sum_1^n \frac{a_i}{n^{1/p}} r_i\right\|_{L_{\psi_q}} = n^{1/q} \varphi_{L_{\psi_q}}(2^{-n}) \sim C,$$

since $\varphi_{L_{\psi_q}}(t) = \log(1 + 1/t)^{-1/q}$, up to some normalization constant. Hence $\Lambda(\mathcal{R}, L_{\psi_q})$ and $L_\infty[0, 1]$ are order isomorphic.

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