BREAKING THE BREXIT IMPASSE: ACHIEVING A FAIR, LEGITIMATE AND DEMOCRATIC OUTCOME

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Unanimity on the question of UK exit from the EU is not within reach, but this does not mean that the House of Commons or the population at large can not find a way out of the current Brexit impasse that is fair and legitimate. We discuss different voting procedures which satisfy some important principles of democracy and which can select the option that can win a majority against all other alternatives in a head-to-head majority vote. We argue that strategic considerations play an important role and we propose a procedure that works well and can help break the impasse when voters act strategically. The procedure requires (1) that all options with some minimum support are on the agenda, (2) that voting takes place in multiple rounds and (3) that in each round the alternative with the least support is eliminated until in the last round only two alternatives are left and the majority winner is selected. We discuss how this procedure can be modified to take into account that some voters may vote non-strategically and how it, in practice, could be used either in the House of Commons or in a new referendum.

Keywords: Brexit; Multi-stage voting; sequential elimination; Condorcet consistency; top cycle.

JEL Codes: P16; D71; C72

"A critical role of the constitution and other political institutions is to place restrictions on the state or the sovereign. These institutions in part determine whether the state produces rules and regulations that benefit a small elite and so provide little prospect for long-term growth, or whether it produces rules that foster long-term growth. Put simply, successful long-run economic performance requires appropriate incentives not only for economic actors but for political actors as well."

North and Weingast, 'Constitutions and Commitment', 1989.

I. Introduction

The UK is set to leave the European Union at the end of March 2019. The process of leaving was triggered by the result of the Advisory Referendum held on 23 June 2016, which led to invocation of Article 50 on 29 March 2017. The Withdrawal Agreement with the EU on 25 November 2018 is the government's concrete proposal for EU exit. Following the Commons vote on 15 January in which the Withdrawal Agreement was voted down by the House of Commons by 432 to 202 votes, a number of alternative plans have been suggested, including leaving without a deal, retaining a customs union with the EU, seeking a free trade deal *à la* Canada, and holding a new referendum.

Until the Brexit impasse is broken the United Kingdom faces a constitutional crisis. The political machinery is tangled in a series of questions on EU exit to which it cannot seem to give a clear answer and as a result many economic agents are in stasis and are crying out for clarity. The crisis threatens to undermine the constitutional order that since the Glorious Revolution has brought stability and economic prosperity (North and Weingast, 1989). Admittedly, the constitutional order has been under threat many times since 1688 but in each case revision to procedures and rules were made to preserve the order and to find a way forward.[1] We argue that this may also be needed now.

At the time of writing, it does not appear that a parliamentary resolution is in sight and it is unlikely that repeated votes for or against amended versions of the Withdrawal Agreement for or against the numerous

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other proposals that have been floated by groups both inside and outside parliament will resolve the situation. The problem with this form of procedure is not sufficiently appreciated and has given rise to the common perception that there is no majority in favour of any proposal in the House. The issue is this: when there are many possible alternatives to choose from, then each ballot is a vote on a given proposal against 'not that proposal'. For example, if there are three alternatives, A, B and C, then a ballot on A is in effect a ballot on A against either B or C. In the case of Brexit, 'not that proposal' is a bundle of many different options. The MPs clearly have strong preferences over different options and they may hold very different subjective views on how likely each option in the bundle 'not that proposal' is to become a reality. This opens up the real possibility that no proposal can command a majority against 'not that proposal' as the sequence of unsuccessful ballots held in 2003 in relation to reform of the House of Lords vividly illustrates. This, however, does not mean that there is no majority for any proposal.

We argue that a new procedure is needed, perhaps uniquely for the Brexit problem, to find a way forward. In this note, we outline some principles and methods, drawing on social choice and game theory, which may allow the House of Commons to break the impasse and to choose between the possible proposals systematically in a way that reaches a conclusion and preserves democratic legitimacy.[2]

In Section 2, we start by setting out the principles that we believe the procedure should respect in order to have legitimacy. In particular, we argue that the set of options to be considered must respect the principle of open agenda: any proposal that can muster some minimum level of support must be considered and voted on, and that each proposal must be treated in the same way and not given any particular status. The voting procedure should also treat all voters in the same way. This is the principle of anonymity. Finally, we argue that the voting procedure should be designed to identify the alternative that can command a majority against all the others in a pairwise ballot. This is the Condorcet consistency principle and the chosen alternative is called the Condorcet winner.

In Section 3, we discuss voting procedures that can satisfy these principles and help select a Condorcet winner. The appropriate choice of voting procedure depends critically on how strategic the relevant voters (MPs in the House of Commons or ordinary voters in a possible new referendum) are when they cast their votes. Voters are sincere (non-strategic) if they truthfully and myopically vote according to their preferences. They simply consult

their preferences over the options presented and vote for the one that they prefer the most. Voters, on the other hand, vote strategically if they deviate from voting for their true preference in order to influence the outcome of the voting procedure to their advantage. We argue that strategic voting is likely in the context of Brexit and show why standard voting procedures applicable when voters are sincere are problematic in this case.

Based on Bag, Sabourian and Winter (2009) and Sabourian (2019), we outline fair and legitimate voting procedures that can uncover the Condorcet winner and be adopted either in the House of Commons or in a referendum. Specifically, when there is strategic voting and a good level of understanding of what the alternatives are, we propose a sequential voting scheme called the Weakest Link (because it is rather like the TV show). The procedure involves multiple rounds of voting where in each round the 'weakest' alternative is eliminated until in the last round only two options are left and the one supported by the majority is chosen. Besides the Weakest Link show, a similar procedure is used in the selection of the host city for the Olympics, in leadership elections in the Conservative party and in Strictly Come Dancing (up till the final). In Section 4, we sketch the appropriate voting procedure if some voters are sincere and others are strategic. In Section 5, we discuss what happens if there is no Condorcet winner. In Section 6, we engage with various practical issues related to using the Weakest Link procedure to resolve the Brexit issue. In Section 7, we conclude.

2. The fundamental principles
We argue that the voting procedure needed to break the Brexit impasse should satisfy four principles of democracy.

First, the social choice literature on voting rules takes the set of options that voters can vote on as given (see, e.g., Myerson, 2013). In the case of Brexit, the options are, however, not given and must, therefore, be considered as an endogenous part of the voting procedure. We argue that any option that commands some support from some group of voters should be on the agenda. We call this the *open agenda principle*. Without this it is quite simple to argue against the chosen option on the grounds that an alternative had not been considered. The principle implies that any alternative that has some support cannot be excluded on the grounds that it is unfeasible, economically damaging, undemocratic, or for some other reason. If an alternative has some support, then that alternative is good enough for some people and should, therefore, be amongst the set of alternatives considered, irrespective of whether others think it is unacceptable. So, even if the claim that

No Deal is catastrophically costly were valid, this cannot be an argument for excluding the No Deal option from consideration. Similarly, even if the claim that in 2016 people knew Brexit meant 'Hard Brexit' and that any revisiting of the 2016 Brexit decision is undemocratic (as people have already voted) were valid, this cannot be an argument for excluding Soft Brexit, the Norway option, or Remain from the agenda as not everyone agrees. Hence, it is clear that the open agenda principle implies that all the various types of Brexit as well as Remain, and, for the House of Commons also a second referendum, should be amongst the alternatives that are considered. Also as we will discuss below, applied to the House of Commons, the open agenda principle removes the government's quasi-monopoly on setting the agenda and deciding on what is being voted.

Second, all options should be treated in the same way. This is the *neutrality principle* which ensures fairness. It means that the voting procedure, for example the order in which the choices are made, does not itself bias the final choice. Thus, voting procedures that treat different alternatives differently by say excluding some alternative at some stage of the procedure violate this principle. Violating neutrality would expose the procedure to the accusation that the process was rigged in some way.

Third, the procedure should also treat all voters the same way. This is the *anonymity principle*. The one-person-one-vote system ensures that all enfranchised voters are treated in the same way. However, it can be argued that what is really required is that all stake holders are treated in the same way and, in the context of a referendum, this raises questions about whether citizens below 18 years of age and all expatriates should be allowed to vote.[3]

Fourth, the voting procedure should be *Condorcet consistent* and select the option among all the options (identified through the open agenda principle) that can command a strict majority of vote in head-to-head ballots against all the other options. Such an option is called the Condorcet winner (CW) after 18th century philosopher and mathematician the Marquis de Condorcet. A Condorcet winner reflects the majority view in the sense that there is no other option that can win a majority vote against it and it encapsulates what is commonly understood by the 'majority view'. The CW derives its legitimacy from the fact that it is stable, in the sense that once the CW is selected there is no other option that can win a majority vote against it. If there is no CW (something we return to below), then the voting procedure should not select an option which fairs badly against other alternatives in pair-wise votes. More specifically, as a minimum, the procedure

should not select options outside what is called the 'top cycle'. The top cycle is the smallest set of options such that the options in that set can beat every other option outside the set in pair-wise votes. If there is a Condorcet winner, then that option is the unique element in the top set. If the voting procedure selects an option outside the top cycle, then it selects an alternative that is a majority loser to all the options in the top cycle and that is clearly not desirable and may even be considered undemocratic.

3. Voting procedures

The next question is which voting procedures will satisfy these principles. Let us assume that the set of relevant options have been identified through the principle of open agenda. If it turns out that only two options emerge through this process, the majority rule will satisfy the other principles irrespectively or strategically, and can be used (May, 1952). However, in the case of Brexit, this is unlikely to be the outcome and the choice will be between more than two alternatives. For simplicity, suppose that there are three: A, B or C. The established procedure in the House of Commons would be to have a sequence of votes to see if there is a majority in favour of any of them. This would very likely be indecisive even if there is a Condorcet winner, i.e., even if one of the options can, in fact, win a majority vote against each of the other two. To see this, it is useful to consider an example with three (types of) voters indexed 1, 2 and 3. Table 1 illustrates their preference ranking over the three alternatives. For example, voter 1 prefers option A to option B to option C and so on for the other voters.

In this example, option A is a CW because it wins a majority against both option B and C in a pair-wise vote (voter 1 and 3 vote A in a ballot between A and B and voter 1 and 2 vote A in a ballot between A and C). Now, suppose that the voting procedure is to put each proposal to a vote in the hope of finding a majority in favour of one of them, and for the sake of argument also suppose that voters vote sincerely, i.e., according to their true preferences for the options. Option C would clearly fail as a majority consisting of voters 1 and 2 would vote against because both option A and B are preferred by them to option C.

Table 1. Three voters and rank preferences over three options

	Voter 1	Voter 2	Voter 3
1st	A	B	C
2nd	B	A	A
3rd	C	C	B

Note: The options are listed in each column in declining order of preference.

The situation with the other two options is more interesting. Suppose that there is a vote of 'option A' against 'not option A'. Voter 1 would vote for this because he prefers option A to the other two. However, for voter 2 and 3 the situation is more complicated. 'Not option A' is a bundle of options B and C and while each of them prefers one of these options to A, they prefer A to the others. They may, therefore, reasonably vote against option A if they either think that voting A down means that their preferred option (B and C, respectively) is likely to be approved next or if they really value their most-preferred option relative to the rest, or a combination of the two. In short, if the expected utility of the bundle 'not option A' is higher than getting option A, then they will vote against A. In a vote of 'option B' against 'not option B', voter 2 would vote yes and voter 3 would vote no; Voter 1 will be decisive but could well vote against option B if his preference for A is sufficiently strong or he thinks A is sufficiently likely to become a reality if B is voted down. As a consequence, the outcome may be that there is no 'majority for anything', yet option A is a Condorcet winner.

This simple example clearly demonstrates two things. First, it demonstrates that the argument, which has been repeated over and over again in past days, that there is "no majority for any alternative in the House of Commons or in the country" is misleading, as it is perfectly possible, perhaps even very likely, that a Condorcet winner exists and at the same time there is no majority for any of the options when compared to 'not that option'. Second, the example clearly demonstrates the need for thinking about a better procedure which can, in fact, identify the Condorcet winner and so reach a decision. To find a way forward, let us first consider sincere and strategic voting one at a time before discussing more complicated scenarios. For now, we assume that a CW exists; we return to the issue of what happens if it does not in Section 6.

a. Sincere voting

Voters are sincere if they truthfully and myopically vote according to their preferences. Sincere voters do not take the broader context in which a particular choice is embedded when they cast their vote on their choice; they do not think strategically about how their vote may interact with those of others and the outcome to which that might lead. They simply consult their preferences over the options presented and vote for the one that they prefer the most.

Under this assumption, it is well-known that a number of commonly used voting procedures may not select the CW when there are more than two options on the table (see,

e.g., Mueller, 2003, p. 150). This includes the plurality rule (the alternative that gets the most votes wins); majority run off (where if an option fails to gain a majority in the first round, then the two options with the highest vote count are voted against each other in the second round); the Borda rule (where each option gets a score according to the ranking in a voter's preference ordering and the option with the highest score wins) and many others.

However, there is a simple method that does work with sincere voters: the Condorcet method. The method is to ask the voters to write down their complete ranking of options and then mechanically count how many pair-wise votes each option can win; the alternative that can win K-1 pair-wise ballots with K options is the Condorcet winner. In the example in table 1, this is straightforward. Once each voter has (sincerely) submitted their ranking, we can see that option A wins against B (with two votes from voter 1 and 3) and against C (with two votes from voters 1 and 2) and option B wins against C (with two votes from voters 1 and 2). This establishes that A is the CW. An alternative way to do this is to simply ask the voters to vote on each pair of alternatives. In the example in table 1, this is also straight forward and it only requires three votes, but if there are many alternatives, the number of votes is large.

This provides a strong rationale for using the Condorcet method to decide amongst multiple options with sincere voters. In an important analysis, Dasgupta and Maskin (2008) provide another strong rationale. They show that under the assumption of sincere voters, the Condorcet method (which they call the majority rule) can satisfy a set of standard axioms (including neutrality and anonymity) for a broader class of preferences than any other voting rule, including plurality rule, super-majority rule, the Borda rule and other similar rank order rules.

What would happen if voters vote strategically rather than sincerely and the Condorcet method has been adopted? In this case, they may deviate from voting their true preference in order to influence the outcome of the voting procedure to their advantage. To understand what may happen in this case, we need to use Game Theory and think about the interaction between the voters as a game of strategy.

To be concrete, let us return to the example from table 1, but assume that there are two voters of type 1, three of type 2 and two of type 3, so that there are seven voters in total. It continues to be the case that option A is the CW; it can, if voters vote sincerely, win a head-to-head majority vote against B and C.

Table 2. Three voters and their voting strategies

	Type 1	Type 2	Type 2	Type 2		Type 3
	2	1	Ann	Peter		2
1st	A	B	B	*B*	**B**	C
2nd	B	A	C	*C*	**A**	A
3rd	C	C	A	*A*	**C**	B

Note: The columns specify the strategies of the different types of voters and the options are listed in declining order of preference according to each strategy. For Peter, we show two possible strategies: sincere voting is in bold and the non-sincere voting is in italic. The information in the second row refers to the number of voters of each type (two type 1 and 3 voters, respectively, and three type 2 voters). We list the strategy for the three type 2 voters in separate columns and indicate explicitly who is Ann and who is Peter.

What would happen if the voters started to think strategically? Suppose, for example, that one of the type 2 voters, call him Peter, thinks that all other voters will vote sincerely except that one of the other type 2 voters, call her Ann, will vote strategically and move A to the bottom of her ranking when she votes; that is, she uses the non-sincere ranking B>C>A where '>' means will vote for. The situation is described in table 2.

What would Peter do, assuming that all the others vote according to the rankings specified for them in the table? If he votes sincerely and chooses the ranking B>A>C then A wins a majority against B and against C.

Next, suppose that Peter deviates from sincere voting and does the same as Ann; that is, he votes according to the ranking in the table indicated in italics (B>C>A) rather than according to his true preference indicated in bold (B>A>C). What will happen in this case is that A wins a majority against B; B wins a majority against C, but critically, C can now win a majority against A because it gets support from Ann and Peter plus the two type 3 voters. This creates a Condorcet cycle. We need to say how this is resolved. One way to do this is to assume that the Borda rule is used and if there is a draw in the Borda count, then the winner is selected via a lottery between the tied options. The Borda rule requires the voters to submit their ranking of the options and then allocates three points to the preferred alternative, two points to the second and one to the lowest ranked option and adds up the points. The option with the most points wins. So, with the specified strategies and with Peter voting non-sincerely to create the cycle, the Borda count for option A is 14; the Borda count for option B is 15 and for option C it is 13. Consequently, option B is selected and that is strictly better for Peter than getting A which would result if he voted sincerely.

So, in plain English, Peter can benefit by voting against A and in favour of C despite the fact that he actually prefers A to C.

This shows that it can pay off for Peter to deviate from his true preferences if he thinks that at least one other voter thinks like him, but it does not tell us if this behaviour is consistent with what all the others are doing. Game theory uses the concept of Nash Equilibrium – a situation where the strategies of the voters are mutually consistent in the sense that nobody has an incentive to deviate and do something else given what the other voters are doing – to make a prediction about what will happen. Nash equilibrium, however, does not take us very far when it comes to voting because with many voters each one of them is rarely pivotal in any vote. As a consequence, there will, in general, be many different voting strategies that are a Nash equilibrium. A minimum refinement that is commonly used to analyse strategic voting is to require the strategies that underpin a particular Nash equilibrium to be (weakly) undominated.[4] In the context of voting, an undominated strategy is one that is preferred to any other alternative, for some situation.

Returning to the example, we showed above that for Peter B>C>A is better than sincere voting given the situation that all the others vote according to the rankings specified for them in table 2. In fact, B>C>A is better than any other strategy for Peter in this situation. This means that the non-sincere strategy of B>C>A is undominated for any type 2 voter. Furthermore, sincere voting is also undominated. Hence, the following profile of strategies are undominated: voters of type 1 and 3 vote sincerely while all three voters of type 2 vote non-sincerely according to the ranking B>C>A. In fact, it can be shown that this profile of strategies is also a Nash equilibrium as no voter can do better by deviating from these strategies.

There are many other examples that one can give, some which do not involve creating cycles, that can also sustain non-sincere behaviour in (undominated Nash) equilibrium.

b. Strategic voting

The example above shows that the Condorcet method does not necessarily select the CW because some voters might be better off misrepresenting their true preferences. Given the possibility of strategic voting, an alternative to the Condorcet method is required. Bag, Sabourian and Winter (2009) show that a large class of voting procedures based on repeated ballots and elimination of one option in each round will uniquely select the CW for certain when voters behave strategically.

An example of this procedure is what they call the Weakest Link. This is a multi-round election in which in each round voters would vote between all remaining alternatives and the one with the least votes would be eliminated. Voting continues until only one alternative is left. This procedure satisfies the principle of neutrality and anonymity and it can ensure that the CW is selected (if there is one and voting is strategic) and that a decision is made. The Weakest Link procedure can be interpreted as the natural extension of plurality voting with sequential voting in that the procedure eliminates the worst plurality loser in each round.[5]

The basic idea can be understood by considering the final round of the Weakest Link procedure. At that point there will be two options, for example, A and B in the example in table 1. At that point, the best thing for each voter to do is to vote for his or her preferred option, i.e., sincerely. Hence, whichever of the two is preferred by the majority will be elected in the showdown vote in the last round. In the example, option A would win against either B or C because it is the CW, and B would win against C.

If we then work backwards to the previous round when there are three alternatives and it is known to the voters that if a CW reaches the last round (with two options), then it will win against any other option, as shown above. So in this penultimate round the rational strategy for a majority of voters is to make sure that the CW is not eliminated and, therefore, to vote for it. In the example in table 1, voter 1 would vote A because this is his first preference. Voter 3 ideally would like option C, but knows that option cannot win in the final round and he, therefore, strategically votes for A to make sure it is not eliminated. So, by a process of backward induction, we can see that the Weakest Link procedure will get us to the CW.

Bag, Sabourian and Winter (2009, Proposition 3) also show that procedures designed with only one round of voting (including the single transferable vote and many other commonly used voting procedures) are insufficient to ensure that the CW is selected under strategic voting. This deficiency undermines the legitimacy of any standard one-round voting procedures and underscores the importance of multi-stage voting with sequential elimination of options.

The theoretical insight that the Weakest Link procedure selects the CW with strategic voters is predicated on the assumption of complete information about what the preferences of different voters are. In practice, information will be incomplete, but we stress that with just three alternatives, only a minimum amount of information and strategic thinking is required for the procedure to work: the voters need to know what the CW is and they need to think ahead and realise that if the CW gets to the last round it cannot lose to any other option. With more options than three, more information is needed and a more complex backwards induction reasoning is required of voters.

How would the Weakest Link procedure work if all voters happened to be sincere and not strategic? The answer is that it may not select the CW in that case. To see this, return to the example in table 1 but assume that there are 100 voters, with 25 voters of type 1, 35 type 2 voters and 40 type 3 voters. In the first round, there are three options and option C would get 40 votes from all the voters of type 3, option B would get 35 votes from voters of type 2 and option A only get 25 votes from the voters of type 1 and be eliminated. In the second round vote with B against C, option B will win a majority of 60 against 40. It is therefore quite possible that the CW is eliminated in the first round. So in the design of a procedure to locate the CW, the assumption about how strategic voters are – in the case with three options, whether they can think through what would happen in the second round – is critical. We discuss this issue in the next section.

4. Strategic or sincere?

The previous discussion makes it clear just how much it matters for the design of the voting procedure whether voters are sincere or strategic. Indeed, it may be even more complicated if there are both types in the voting constituency. In the context of the vote on the Brexit withdrawal agreement, we believe it is highly unlikely that all voters are going to vote sincerely in a ballot with more than two alternatives, in particular if the voters in question are MPs voting in the House of Commons. However, it is hard to rule out the possibility that some voters are sincere voters. The population of voters may, therefore, consist of a mixture of strategic and sincere voters.

In this case, an amended version of the Weakest Link procedure will work and select the CW. Sabourian (2019) calls this the Sequential Condorcet procedure and it is a hybrid of the Condorcet method and the Weakest Link procedure. As in the Weakest Link, decisions take place sequentially and in each round one option is eliminated. However, instead of eliminating the option with the least votes, the Condorcet method is applied in each round and the option that loses to most other options is eliminated. This method ensures that the CW is selected at the end of the process under the assumption that the voters' preferences over options are not systematically related to whether they vote strategically or sincerely.

5. What if there is no CW?

When there are many alternatives, it cannot be guaranteed that there is a CW and Condorcet cycles in which option A beats B which beats C, which in turn beats A can arise for some preference ordering of the options. What then? While this is logically possible, research shows that the problem is most acute for small committees and is least acute when many individuals vote on a few alternatives (the situation in a House of Commons vote or a second Referendum), see, for example, Jones *et al.* (1995). Even if there is no CW, then under the assumption of strategic voters, the Weakest Link procedure still has much to recommend it. This is because it ensures that an alternative that commands minority support in head-to-head pair-wise voting cannot be selected and that the principle of neutrality is preserved. In other words, the selected alternative will always belong to the so-called top cycle which is the smallest set of options with the property that each option in the set can beat all options not in the set in a pair-wise majority vote.

6. Application to Brexit

We believe that there is a strong case for adopting the Weakest Link procedure or its extension, the Sequential Condorcet method, to deal with the possibility of sincere voters, alongside the principle of an open agenda to resolve the critical Brexit impasse. While the procedure can, in principle, be applied either in the House of Commons or in a referendum, the starting point would be to apply it to unlock the situation in Parliament.

Let us turn to practical matters. How can the Commons adopt our proposed procedure when the voting system for a parliamentary division is already fixed?[6] One possibility is for the House to adopt our procedure in order to arrive at an indicative choice and, then, proceed to its usual voting procedure of either accepting or rejecting it. Doing so would be a significant deviation from normal parliamentary procedure and it would remove the government's quasi-monopoly on the EU Exit agenda. The open agenda principle would allow MPs to propose options (with some minimum requirement that a proposal needs to be endorsed by a minimum number of MPs which could be set low) and would presumably result in a range of different options being proposed, such as the Government's deal, No Deal, a Norway++, a customs union, a new referendum etc., each of which would have some minimum support amongst some MPs.

We acknowledge that the adoption of this procedure could set a precedent that may not be desirable. However, this could be dealt with by asking the House to vote on a motion that says "this procedure can be used for the Brexit question and cannot be used to decide future issues unless a majority [or even a so-called 'super-majority'] calls for it be used".

An objection to our proposed procedure is that it may require several ballots. For example, with four alternatives, under the Weakest Link procedure three ballots are required. Organising multiple ballots in the House of Commons is not difficult. In fact, the idea of multiple ballots is not new and parliament has used the method before (but not with the particular procedure that we propose). For example, in the vote on House of Lords Reform in 2003, eight options were considered simultaneously.[7] The House of Commons voted against each of the options for reform including outright abolition. Each of the eight votes defeated an option to amend the status quo and implied that the Commons, was in effect, supporting the status quo. And yet the binary vote that most closely represented the status quo, that is with 0 per cent elected and 100 per cent appointed, was voted down by 245 to 323 votes by the Commons. The result represents something of a paradox and is a good illustration of the problem with a procedure that effectively asks the MPs to vote against one option against 'not that option'.[8] Using statistical and interview evidence, McLean *et al.* (2003) suggest that the MPs voted strategically rather than sincerely and voted against some acceptable options.

It is of course, more complicated to apply the procedure to a public referendum and it may be argued that it is too costly to use it. If the cost of multiple ballots is considered too high, however, an (imperfect) compromise might be to eliminate more than one alternative in each round; in the extreme limiting the number of rounds to two. Two-round voting is common (e.g., in presidential elections in France and Brazil) and remains preferable to one round of voting.

7. Conclusion

Will selecting a CW via some form of Weakest Link procedure settle the question of the UK's relationship with the EU? As long as people's preferences do not change over time such a procedure can deliver the result favoured by the majority in a manner that is fair to all views; hence it should deliver an enduring outcome. However, if preferences change over time or new information comes to light, we cannot rule out another future vote on the terms of our EU relationship. As Keynes put it "when the facts change, I can change my mind."

Unanimous consensus on the Brexit question is not achievable, but that is true on any significant issue.

Our procedures can be designed to consider the cases of sincere or strategic voting or some combination of the two. We can also take a view nearly three years on from the announcement of an imminent referendum about the extent of knowledge or information about the different options faced by the UK. However, it is within reach to structure the democratic process in such a way that the procedure is fair to all views and the outcome is preferred by a majority to any other alternatives. Anything else may set the country on a fragile path with limited support. In recent days the Queen has even asked for a fair solution to be found:[9]

"Of course, every generation faces fresh challenges and opportunities. As we look for new answers in the modern age, I for one prefer the tried and tested recipes, like speaking well of each other and respecting different points of view; coming together to seek out the common ground; and never losing sight of the bigger picture. To me, these approaches are timeless, and I commend them to everyone."

Our suggestion points to a fair and democratic way forward.

NOTES

1   Examples of this include Aidt and Frank (2015) who demonstrate a close relationship between the threats of violence from the Swing Riots of 1830 and 1831 and the subsequent passing of the Great Reform Act and Chadha and Newby (2013) who examine the parliamentary response to the suspension of gold convertibility in 1797 to ensure the stability of monetary value.

2   It was Nigel Lawson who was quoted in 1991 as saying: "To govern is to choose. To appear to be unable to choose is to appear to be unable to govern."

3   In the 2016 referendum, British, Irish and Commonwealth citizens who were UK residents were allowed to vote as were British ex-pats who had lived abroad for less than 15 years. Citizens below the age of 18 were not allowed to vote and neither were EU citizens living in the UK, except those from Ireland, Malta or Cyprus.

4   A strategy x for a player is weakly undominated if for any alternative strategy y for the player, there exist strategies z for the other players such that the player prefers x to y given that the others play z.

5   However, the weakest alternative does not need to be eliminated by plurality voting. Many other methods can be used to find the weakest link. These include the Borda rule and approval voting. Another common example of multi-round voting in which at each round one alternative is eliminated is binary sequential voting. In such a voting scheme the alternatives are ordered. First, the voters choose between the first two alternatives in the ordering. The one that receives less than half the vote is eliminated and in the next round of voting, the voters choose between the winner and the next alternative in the ordering. The process continues until the procedure reaches the last alternative in the ordering. When there is a CW, this procedure also selects it when voters are strategic. However, sequential binary voting does not obey the neutrality principle when there is no CW. This happens because some alternatives are considered before others and it can be shown that changing the order in which different alternatives are presented can affect the outcome.

6   See Rasch (2000) for an overview of voting rules used in various Parliaments.

7   Eight binary options for the House of Commons to vote upon were offered simultaneously on 4 February 2003, allowing for seven permutations of elected and appointed members, ranging from 0% elected and 100% appointed through to 100% elected and 0% appointed, as well as an additional choice for outright abolition.

8   In the case of reform of the House of Lords, the status quo (keep things as they are) might have served as a reference point for what it would mean to defeat a given proposal, but it does not rule out that some MPs voted against a particular reform proposal, not because they wanted the status quo but because they thought that one of the other options might win the ballot.

9   See, for example, *Guardian*, 24 January 2019, "Queen's speech calling for 'common ground' seen as Brexit allusion".

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