THEORETICAL CONSTRAINTS FROM ASTEROSEISMOLOGICAL HIGH S/N OBSERVATIONS

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ABSTRACT. The two firmly established cases of many-mode observations, the Sun and Kurtz's Ap stars, suggest that in stars extremely small pulsation amplitudes are to be expected. If spectroscopic Doppler techniques are used to measure the velocity pattern, then it is obvious that with better resolution and higher S/N ratio of the spectral lines the sensitivity to detect velocity amplitudes increases. Observed pulsation frequencies of any star will put new constraints on the theory of stellar evolution. Besides addressing the two most important issues, namely determination of age and chemical composition, observed periods will also help resolve open questions in the physics of stellar interiors.

1. INTRODUCTION

Stellar acoustic oscillation frequencies will likely be observed accurately in the near future, in analogy to the well-known solar five-minute oscillation frequencies. Of course we will never expect the wealth of the solar data, which is a result of the high spatial resolution of the Sun. Therefore we will not be able to solve the inverse problem, that is to probe physical quantities as functions of depth. Furthermore, in the case of the Sun, the large number of observed frequencies allows a unique mode identification (see, for instance, Deubner and Gough 1984). This will not be true for the small number of expected stellar frequencies. As has been emphasized by Däppen et al. (1988), asteroseismology should therefore not simply be understood as the stellar analogue of helioseismology (which it is not and cannot be), but rather as a method of testing stellar structure and evolution theory, using all available pulsation data, and not just oscillation frequencies.

Despite the lack of analogy with the solar case, prospects for asteroseismology are excellent. Given the fundamental importance of the theory of stellar evolution in astrophysics (for instance in the determination of age and chemical composition), it is clear that any observed quantity besides temperature and luminosity can be used to reduce existing uncertainties in the physics of stellar interiors

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(e.g. convection, opacity, equation of state, nuclear reaction, etc.). While this is intuitively clear, one is of course also interested to know how much observational errors propagate into the answers that asteroseismology promises to deliver, even if the theory were perfectly known. Only then one will be able to asses the gain from high S/N, high resolution spectroscopy.

2. SIMPLE (ASYMPTOTIC) THEORY OF OSCILLATION FREQUENCIES

Assuming a perfectly spherical star (that is with no distortion of the equilibrium state by rotation, magnetic fields or something else), the (linear) oscillation modes about the equilibrium can be classified by the angular degree ℓ of the spherical harmonic associated with the spatial variation of the surface velocity field, and by the number n of radial nodes of the velocity pattern inside the star. The radial number n can of course not be seen; it has to be deduced theoretically. The simplest theoretical analysis of oscillation frequencies is asymptotic theory (Tassoul 1980), which - to second order - yields the following expression for the frequencies $\nu_{n,\ell}$

$$\nu_{n,\ell} = (n + \ell/2 + \sigma)\ \nu_0 + \varepsilon_{n,\ell} \qquad (2.1)$$

Here, σ is a constant of order unity, and $\varepsilon_{n,\ell}$ is small compared with $\nu_{n,\ell}$.

At this point it is useful to introduce two definitions pertaining to the structure in the periodogram of high order pulsators.

2.1. Large and small frequency separations

(i) Large Separation:

$$D_{n,\ell} \equiv \nu_{n+1,\ell} - \nu_{n,\ell} . \qquad (2.2)$$

To first order asymptotic theory it is well known that

$$D_{n,\ell}^{-1} \approx \nu_0^{-1} \equiv 2 \int_0^R (1/c)\ dr . \qquad (2.3)$$

In simplified stellar models it is easy to show that

$$\nu_0 \propto (g/R)^{½} = (GM/R^3)^{½} . \qquad (2.4)$$

(ii) Small Separation:

$$d_{n,\ell} \equiv (\nu_{n,\ell+1} - \nu_{n,\ell}) - \tfrac{1}{2}(\nu_{n+1,\ell} - \nu_{n,\ell}) \; . \qquad (2.5)$$

The small separation serves to cancel the first-order term of (2.1), and thus reveals second-order details, which pertain to the central regions of the star (see below). The ratio between small and large separation is, to second-order asymptotic theory, given by (Tassoul 1980)

$$\frac{d_{n,\ell}}{D_{n,\ell}} \approx \frac{\ell+1}{2\pi^2 \nu_{n,\ell}} \int_0^R \frac{dc}{dr}\frac{dr}{r} \qquad (2.6)$$

Since sound speed increases steeply from the surface to the centre of a star, $D_{n,\ell}$ probes more the surface regions and $d_{n,\ell}$ more the central regions.

3. DETERMINATION OF STELLAR AGES FROM SEISMOLOGY

The small separation carries an important signature of stellar age, because as hydrogen is converted into helium in the stellar core, the mean molecular weight μ increases, which causes a decrease of sound speed, thus reducing the small separation. An excellent diagnostic diagram that extracts the information contained in the small and large separation has been invented by Christensen-Dalsgaard (1984) (for a more detailed calculation see Christensen-Dalsgaard 1988). In this diagram, contours of constant stellar mass and age are plotted against the theoretically computed large and small separations. Since the mass and age contours are rather perpendicular than parallel to each other, they reveal the considerable diagnostic potential of these diagrams (hereinafter called JCD diagrams).

Going a step further, Gough (1987) discussed the accuracy of seismological mass and age determination, using JCD diagrams and calculations by Ulrich (1986, 1988). His discussion is purely *formal*: taking the theoretical model for granted, he computes the uncertainty in the mass and age determination, assuming given errors for the observed stellar parameters (effective temperature, luminosity, heavy-element abundance, large and small frequency separation). Gough's (1987) result is that mass and age determination are so sensitive to the *heavy-element abundance* that they cannot be carried out in this way, unless other stellar parameters are known by independent means. If, for instance, in the case of a binary system we can determine mass, or if we can estimate it from surface gravity (whose observation obviously profits from high

S/N and high-resolution spectroscopy), then the large separation can reveal the evolutionary information contained in the deviation from the simple relation (2.5) (otherwise the large separation mainly fixes M/R^3). Thus a more accurate age determination could become possible (see Gough 1987).

4. THE PROBLEM OF THE EQUATION OF STATE

An important physical issue to be addressed by solar and stellar oscillations is the equation of state. The principal open problem is the number of excited states of hydrogen and helium in the zones of partial ionization. While for many astrophysical applications simple equation-of-state recipes can be sufficient, it has been shown (Däppen 1987) that for finer helioseismological applications (e.g. helium abundance determination) such simple formalisms are not adequate. In contrast to various other improvements over the simple Saha equation, about which no disagreement exist, there are widely divergent opinions on the internal partition function of bound systems. There has been a recent controversy about the so-called Planck-Larkin partition function (Rouse 1983, Ebeling et al. 1985). The Planck-Larkin partition function essentially limits the number of excited states to those having a binding energy $\geq kT$. Optical hydrogen spectra, however, show more lines than predicted by the Planck-Larkin partition function (Däppen et al. 1987a). Rogers (1986) explains this fact by allowing resonances that are not counted in the partition function but could be seen in optical spectra. Thus the Planck-Larkin controversy cannot be resolved with optical experiments, but thermodynamical properties will have to be known. Stellar models with and without Planck-Larkin partition function will have to be compared. While thermodynamical quantities based on the Planck-Larkin partition function will soon become available (Rogers, private communication), an advanced and very smooth version of a more conventional equation of state has been developed in the framework of an ongoing opacity re-computation (Hummer and Mihalas 1987, Mihalas et al. 1987, Däppen et al. 1987b). If observational constraints on the partition functions can be obtained, helio- and asteroseismology could help answer this question from microphysics.

5. CONCLUSION

It is clear that high S/N and high-resolution spectroscopy will improve the quality of results from asteroseismology. Firstly, the determination of stellar parameters (age, mass, chemical composition), using given theoretical models of stellar evolution, will become more precise. Secondly, the observational data will enable us to develop better physical models for the theory of stellar evolution (equation of state, convection, opacity, nuclear reactions, etc.). Since first (rather crude) observations of stellar oscillation frequencies have already have led to theoretical problems [see, e.g. the case of ε Eridani (Noyes et al. 1984, Guenther and Demarque 1986, Soderblom and

Däppen 1987)], improvements of the observations will be most welcome.

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