Bull. Aust. Math. Soc. **104** (2021), 522–524
doi:10.1017/S0004972721000629

GEOMETRICALLY FLEXIBLE AND EFFICIENT NUMERICAL SOLUTION TECHNIQUE FOR BRAGG EDGE NEUTRON TRANSMISSION STRAIN TOMOGRAPHY

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(Received 6 July 2021; first published online 12 August 2021)

2020 *Mathematics subject classification*: primary 78M10; secondary 78A45.

Keywords and phrases: neutron transmission tomography, Bragg-edge transmission, elastic strain, longitudinal ray transform, finite element method, meshless method.

Neutron transmission spectroscopy on a pulsed source is a radiography technique that exploits characteristic cross-section energy-variations for element-specific imaging. The crystallographic details are acquired from transmission patterns of neutron time-of-flight (TOF) using neutrons which travel without interaction through a sample. A single axis is aligned with a neutron source, sample and detector. Neutron interaction when penetrating a sample depends on the nuclear properties of the sample atoms and the atom spatial distribution. For crystalline materials, neutron intensity is diffracted from the incident beam direction, leaving a characteristic pattern in the transmitted signal that can extract structural details.

In certain conditions, transmission geometry offers many advantages over a traditional diffraction set-up. Applying a pulsed neutron source provides Bragg-edge transmission patterns over a wide wavelength range with comprehensive statistics within seconds. The transmitted intensity is high compared with the neutron intensity diffracted in a particular solid angle. This allows kinetics to be studied in solids in times significantly less than an hour in structural phase transitions. As a result of the backscattering, the Bragg-edges do not add the scattering angle to the d-spacing error, making this technique an essential alternative for neutron applications requiring a high-resolution d distance, such as strain measurements. As no diffracted intensity beam-path is needed, the equipment can be designed more easily for particular environments, such as

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Thesis submitted to the University of Newcastle in February 2021; degree approved on 7 July 2021; supervisors Mike Meylan, Bishnu Lamichhane and Chris Wensrich.

temperature, atmosphere or pressure, than for diffraction experiments. For a transmission set-up, the precise sample location is not required and, furthermore, the testing is simplified.

Stress is a physical quantity that defines the distribution of force within a solid and is generally recognised as the key source of failure in engineering components. There is a strong interest in measuring and evaluating stress caused by development or subsequent use. Stress cannot be calculated explicitly but must be quantified by other associated quantities such as elastic strain. This thesis focuses on elastic strain from diffraction-based measurements and proposes a framework that unifies and models all elastic strain measurements. The strain tomography problem is reconstructed using lower-dimensional projected strain images from the full distribution of strain in two or three dimensions. It is similar to traditional computed tomography where a map of the attenuation within an object is reconstructed from lower-dimensional slices. Besides that, a strain is a tensor of the second order, while density is a scalar quantity, which makes strain tomography considerably more complicated. Recent technical advances in detector technology allow Bragg-edge neutron transmission to acquire high strain images. These provide strain field measurements modelled by the longitudinal ray transform (LRT). The mapping from the elastic strain field to the strain measurement via LRT has a null space if the strain components are deemed to be independent. For this purpose, additional prior knowledge is required to solve the inverse problem and reconstruct the strain field from the strain image collection. This prior understanding may be physical restrictions on the strain field, such as assuming that the strain field satisfies compatibility or equilibrium.

The primary focus of this thesis is to provide an alternative numerical approach to measuring strain, such as those offered by the finite element method and the meshless method. The benefit of the finite element method is that it provides a better approach to handling complicated sample shapes with ease; however, to remove noise we have to apply regularisation. In contrast, the meshless method offers a much smoother approach to reconstructing strain. This thesis represents a significant extension to the body of knowledge concerning Bragg-edge neutron strain tomography. As a result, numerical methods such as finite element and meshless methods are able to achieve high-resolution strain reconstruction.

We outline solutions and challenges associated with the finite element method, which lay the theoretical foundations for the algorithms we employ in this research and the results obtained using these algorithms. For the second theme, the meshless approach, the possibilities offered by an alternative numerical method using radial basis functions and a comparison with the traditional finite element method are discussed in the context of neutron transmission strain tomography. The descriptions of the algorithms, the comparison of approaches and the improvements in the resolution of the images are intended to serve as an entry-point guide for researchers wishing to continue this work further.

Some of this research has been published in [1–4].

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