

DETERMINATION OF ABSOLUTE X-RAY WAVELENGTH
WITH THE DOUBLE REFLECTION METHOD

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It has been shown^{1,2} that it is possible to determine absolute wavelength values of X-ray radiation emitted as a result of electron transitions in ions or atoms, without precise Spectrometers and without reference "lines". Knowledge of the lattice constants of the diffracting crystal is sufficient for the determination of the wavelengths. Here the first experiment describing the feasibility of the method is shown.

Usually spectra are obtained in form of "lines". In the X-ray case, however, it is possible to obtain spectra in form of points, ordered along lines. The lines represent the white radiation [Fig. 1].

A point spectrum along a line may intersect another point spectrum presenting the same transitions, which is reflected by other crystal planes. In the setting to be described the intersection point has the same wavelength in the two spectra. The intersection wavelength is a constant of the crystal², taking also into account the indices of the crystal planes involved. It turns out that such intersections may be found near any wavelength in the soft X-ray range. The intersections may be looked upon as absolute wavelength reference values, they appear simultaneously with the point spectra, and the intersection wavelength may be computed to the accuracy with which the crystal lattice constants are known.

Two kinds of crystal reflection planes are connected with the production of these point spectra, ordinary reflection planes and "forbidden" reflection planes. A forbidden reflection plane does not reflect X-rays because of extinction. However, certain forbidden reflection planes in a distinct group of crystals reflect along special directions on the forbidden reflection cone. Here the forbidden reflection is simulated through a pair of planes, the sum of the indices of them being equal to the indices of the forbidden reflection. For instance, the forbidden reflection [2,0,0] of a diamond lattice may be simulated by the pair $\{ [1,1,1], [1,-1,-1] \}$, in a very specified direction on the cone of reflection of [2,0,0]. This happens when the incoming radiation beam, for a given wavelength, will fulfill Bragg's law simultaneously for the [2,0,0] and [1,1,1] planes. The radiation reflected by the allowed reflection planes [1,1,1] will be in position to be re-reflected by the [1,-1,-1] plane into the "forbidden" direction. This is a simple description of a complicated, dynamical process.

Experimentally one uses an X-ray point source. The radiation impinges upon the crystal in form of an expanding cone [Fig. 1]. The surface of the crystal is ground and etched parallel to a suitable forbidden reflection plane. The crystal is set so that the central ray

of the beam hits, roughly, the central point of the crystal, with a Bragg angle for the central wavelength of the wavelength range of interest. The incoming and reflected central ray define a plane. In order that the double reflection of interest takes place, the azimuthal position of the crystal must also be set. This involves rotating the crystal in its own plane, so that a crystal axis in the forbidden reflection plane makes a predetermined angle with the plane of reflection, involving the indices of the double reflection and the wavelength.

No fine setting is involved. No collimator is used. The crystal reflects only the doubly reflected radiation.

The radiation which is not reflected is scattered into all directions. Each wavelength of the X-ray radiation coming from a point source will be reflected by an appropriate point on the surface of the crystal, in the double reflection process. For a strictly monochromatic wavelength the reflected radiation will be an infinitely thin ray, and intensity will not do down with distance from the crystal. However, as practically each electronic transition emits radiation of finite wavelength width, the point reflected turns into a thin streak, in the direction of dispersion.

The intensity of a "point" will be proportional to $1/r$ where r is the distance from the source, while the background intensity will be proportional to $1/r^2$. The farther away the film, the better the signal to noise ratio.

Figure 2 shows the results of the experiment.

Two wavelengths are involved, the $L_{\alpha_1, 2}$ Gold "lines" at 1.27639 and 1.28777 \AA . A fine focus X-ray tube is used as point source. It has practically the dimensions of 0.1mm. The crystal involved is Ge. The forbidden reflection is [6,0,0], involving a Bragg angle of about 43°. The distance of the film from the source, through the crystal, is 1.7m. Three double reflections are involved.

The two strong spots, on a nearly horizontal line of Fig. 2, belong to the double reflection $\{ [3, -1, 3] [3, 1, -3] \}$. This spectrum is intersected by the other spectra $\{ [1, -7, -1] [5, 7, 1] \}$ and $\{ [7, -3, 1] [-1, 3, -1] \}$.

The intersection wavelengths can be shown to be 1.287829 and 1.280169 \pm 0.00002 \AA . The accuracy is given by the accuracy with which the crystal lattice is known - the crystal constant being 5.6576 \AA . Refraction of the radiation in the crystal does not change the results in a significant way at this accuracy.

Measuring the distances involved, and interpolating for AuL_{α_1} , yields a wavelength consistent with the known wavelength value, within the accuracy of 2.10 \AA .

It should be noted that all points in Fig. 2 representing the L_{α_1} wavelength are ordered in a vertical direction, one above the other, as do also the L_{α_2} points. This is consistent with constant Bragg angle for each of the wavelengths. The inclination varies with the different double reflection, and their relative place above the $\{ [3, -1, 3] [3, 1, -3] \}$ double reflection varies according to the azimuthal angle of the various double reflections.

The purpose of the method is to obtain accurate wavelengths for "lines" emitted in the X-ray range by highly ionized atoms in Tokamak or Laser produced plasma. A point source for Tokamak may be obtained by using a pin hole opening facing a Be window on the wall of the Tokamak. A system containing the pin hole, the crystal and the film is independent of the vibrations in the Tokamak, so that any number of discharges may be used to produce the spectrum without detriment to accuracy.

Intersection wavelength is obtained² by solving the equations:

$$r_x h_i + r_y k_i + r_z l_i = 1/2 [h_i^2 + k_i^2 + l_i^2]/a; \quad i = 1, 2, 3$$

where r_x , r_y , r_z are the coordinates of the center of the sphere of reflection in the reciprocal lattice, and h_i , k_i , l_i are the indices of the forbidden reflection and of the two first reflections of the two pairs of double reflections involved.

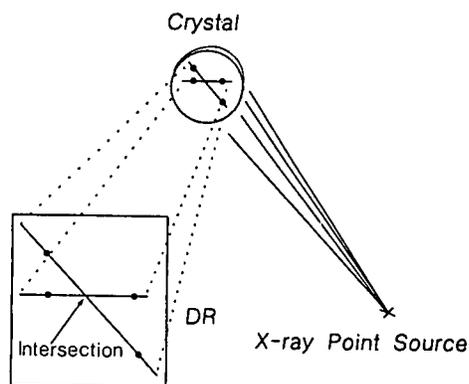
$$\text{Then } \lambda_{\text{intersection}} = [r_x^2 + r_y^2 + r_z^2]^{-1/2}.$$

Relative values of wavelength may be absolutely determined by this method. For the [2,0,0] of Ge this covers the range from 1Å to about 5Å. Also relative values of lattice constants of various appropriate crystals may be determined by this method, and probably change of crystal lattice values with temperature.

REFERENCES

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Fig. 1 - A beam of X-rays impinges on a crystal from a point source. The crystal reflects the image of the beam at two different wavelengths. The reflection takes place through two pairs of planes. The image of the source is obtained on a film DR by double reflection. The two images of each spectrum are joined by a line of white radiation. These lines intersect at a wavelength dependent only on the crystal structure and planes involved.



Intersections

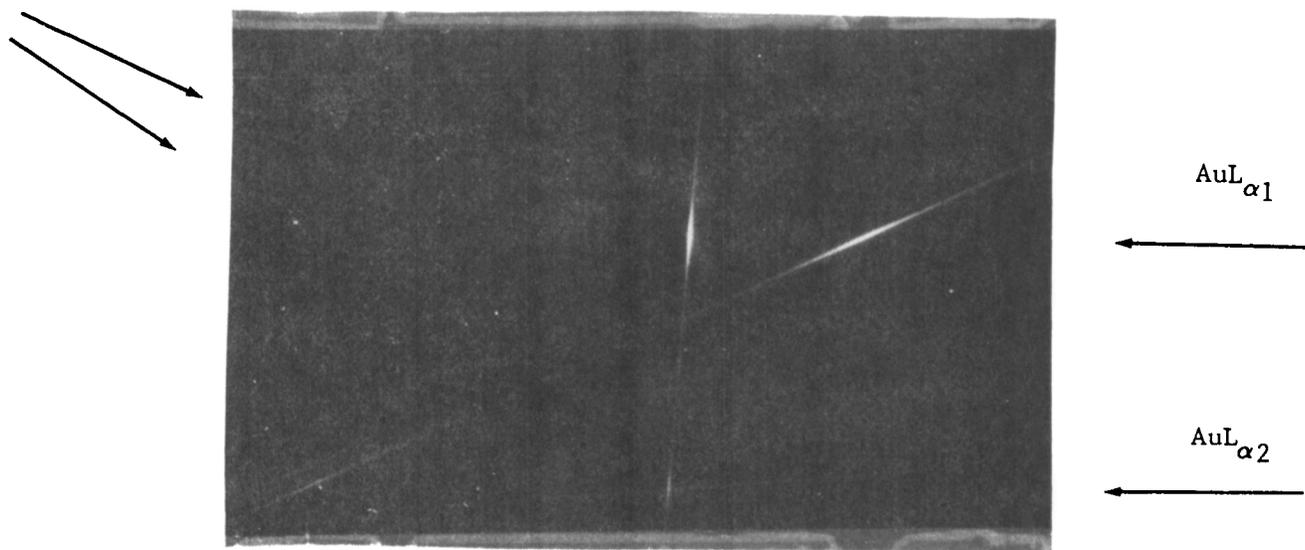


Fig. 2 - Double reflection spectra of Au $L_{\alpha 1}$ and $L_{\alpha 2}$. Two intersections are seen, the strong one between the two points, and the other one on the $L_{\alpha 2}$ point. The wavelengths of the two points are 1.27639Å and 1.28777Å. The wavelengths of the intersections are 1.287829Å, and 1.280269Å. The longish shape of the points is the result of the wavelength not being strictly monochromatic.