RELATIVITY IN ASTROMETRY

C A Murray
Royal Greenwich Observatory
Herstmonceux Castle
Hailsham, Sussex
UK

ABSTRACT. The coordinate dependence of the definition of light deflexion is discussed and the concepts of "natural" and "proper" reference frame are introduced in order to derive astrometric observables from coordinate quantities. Confusion in published formulations of VLBI data analysis is discussed in the context of the IAU definition of the coordinate time scale TDB. It is pointed out that with this definition, the unit of TDB differs from that of TDT and it is suggested that the speed of light in barycentric units be adopted as a defining constant in the system of astronomical constants.

1. INTRODUCTION

It is now more than 35 years since I started working in astrometry, and for most of that time I have not needed to consider relativity in connection with optical observations. Indeed, although "observational relativity" may be said to have started with an astrometric experiment at the famous total solar eclipse of 1919 May 29, it was only since last year, with the introduction of the IAU 1984 reference system, that relativity has become firmly established in optical astrometry. It is of course true that the excess perihelion motions of the planets have been incorporated in planetary ephemerides for many years, but until comparatively recently these have been on an empirical basis without any underlying theory.

My own interest in relativistic astrometry started at about the time of the centenary of the birth of Einstein, in 1979, which happened to coincide with the 60th anniversary of the 1919 eclipse, when my colleague G M Harvey (1979) remeasured those plates from the Greenwich expedition to Sobral in Brazil, which have survived in our plate archives. At about the same time it became clear that the data which will be obtained from the HIPPARCOS satellite, then only in the Phase A planning stage and as yet unnamed, would need full relativistic reduction. Therefore, as a comparative newcomer to this field, I hope the specialist relativists in the audience will forgive me if I address this short review to those who, like me, are primarily concerned with the

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 169–175.

https://doi.org/10.1017/S0074180900148132 Published online by Cambridge University Press

practical applications of relativity to the reduction and interpretation of astrometric observations.

I take as my starting point the formulation of General Relativity in Euclidean Terms by the late Robert Atkinson (1963). In that paper he wrote:-

"There is a very real gap between those who appreciate the beauty and symmetry of the formal mathematics so keenly that they may even deny any need for 'visualization' altogether, and those on the other hand whose natural interests and abilities lie in the field of specific observation and measurement, but who are, almost as a consequence, overwhelmed by the conceptual difficulties of four dimensions and curved space, and so cannot consider the fundamental aspects of relativity with the appropriate confidence."

Reading these lines now, more than 20 years after they were written, we may feel that they represent rather an extreme view. With the advent of the new techniques, such as space ranging, Doppler tracking and VLBI, relativity is now becoming part of everyday life in some branches of astrometry, but nevertheless I believe that there is still a gap to be bridged between specialists in relativity and practical astrometrists.

2. DEFLEXION OF LIGHT

I like to define astrometry as the measurement of space-time coordinates of photon events. It is therefore necessary to define precisely the frames of reference which are to be used and the units in which our coordinates are to be specified.

I hope I may be forgiven for mentioning once again the different representations of the track of a photon in a spherically symmetric gravitational field, which are obtained with the Schwarzschild standard and isotropic spatial coordinates.

If r, θ denote polar coordinates along the track, with $\theta = 0$ in the direction of perihelion, the slope in standard coordinates is

$$-\mu c^{-2} r^{-1} \tan\theta \; (2 + \cos^2\theta)$$

and, in isotropic coordinates, is

$$-2\mu c^{-2} r^{-1} \tan\theta$$

where $\mu = GM$, G being the gravitational constant, M the central mass and c is the speed of light.

Both these expressions appear in the literature (e.g. Brandt 1975, Mikhailov 1976) yet the difference between them can amount to about 1 mas at $r = 1$ au, $\theta = \frac{1}{2}\pi$, which is by no means negligible by modern standards. As I have pointed out elsewhere (Murray 1981) the resolution of this paradox lies in translating "coordinate" quantities into "observables". We postulate that the space-time reference frame actually used by an observer is locally flat; by a suitable coordinate transformation we obtain an "observed" direction which is independent of the coordinates used to describe the photon track (Murray 1983, p 32). I have called this locally flat reference frame, for a fixed observer, the "natural" frame, and a direction measured in this frame a "natural" direction; this I believe accords with the use of the word "natural", in this context, by Eddington (1923).

The unit vector $\underline{r}_n$ in the natural direction to a source can be expressed in the form

$$\underline{r}_n = \underline{r}_c + \underline{\varepsilon} \times \underline{r}_c \tag{1}$$

where $\underline{r}_c$ is the coordinate direction obtained by Euclidean geometry using the spatial coordinates of the source at the instant of emission of the observed photon and those of the observer at the instant of observation, $\underline{\varepsilon}$ is a small vector representing the deflexion and $|\underline{\varepsilon}|^2$ is neglected. Both $\underline{r}_c$ and $\underline{\varepsilon}$ depend formally on the coordinate system chosen but $\underline{r}_n$ does not. If isotropic coordinates are used for calculating $\underline{r}_c$, then*

$$\underline{\varepsilon} = 2\mu c^{-2} a_o^{-1} (1 + \underline{u}'\underline{u}_o)^{-1} \underline{u} \times \underline{u}_o \tag{2}$$

where $\underline{u}$, $\underline{u}_o$ are unit vectors in the heliocentric directions to the source and observer respectively, and a_o is the heliocentric distance to the observer.

From (2) we see that

$$|\underline{\varepsilon}| = 2\mu c^{-2} a_o^{-1} \tan \tfrac{1}{2}\alpha \tag{3}$$

where α is the angle between $\underline{u}$, $\underline{u}_o$. For the Sun,

$$\mu c^{-2} = 9.87063 \times 10^{-9} \text{ au} \tag{4}$$

and hence for an observer at a_o = 1 au

$$|\underline{\varepsilon}| = 0''.004072 \tan \tfrac{1}{2}\alpha \tag{5}$$

This of course gives the maximum deflexion of 1".748 for a Sun-grazing ray, with α = 179°.733, but clearly the deflexion is now observationally significant over much of the sky.

It is of interest to note that, for a fixed observer, the vector $\underline{\varepsilon}$ depends only on the angle α , and not on the distance to the source, provided that we use isotropic coordinates to calculate $\underline{r}_c$. Thus, in particular, the deflexions for a source situated at the limb of the Sun, and for a star 90° away from the Sun as seen by the observer, are both 0".004072.

3. ABERRATION

The natural reference frame is that for a hypothetical observer who is

* throughout this review, the prime symbol (') denotes scalar multiplication

https://doi.org/10.1017/S0074180900148132 Published online by Cambridge University Press

at rest in the coordinate reference frame. The "proper" direction $\underline{r}_p$ measured by a moving observer relative to his own local frame which we call his "proper" frame, is obtained by means of a Lorentz transformation between the two frames. Elsewhere (Murray 1983, p 46) I have derived a vectorial form for this transformation, which can be written as

$$\underline{r}_p = \frac{\{\beta^{-1}\underline{r}_n + (1+(1+\beta^{-1})^{-1}c^{-1}\underline{r}_n'\underline{V}_n)c^{-1}\underline{V}_n\}}{(1+c^{-1}\underline{r}_n'\underline{V}_n)} \qquad (6)$$

where $\underline{V}_n$ is the velocity of the observer relative to the natural frame and

$$\beta^{-2} = 1 - c^{-2}\,\underline{V}_n^{\,2}$$

I am indebted to Dr T Fukushima for pointing out that the form (6) is more suitable for precise computation than my original expression.

The jargon of classical astrometry includes words such as "mean", "true" and "apparent" to describe coordinates referred to different frames of reference. These words convey very little physical meaning, except to specialists. On the other hand, the words "coordinate" and "proper" have definite physical connotations in the context of relativity, and I would like to suggest that we adopt these in place of the old terminology. When the origin of a particular frame is important, then qualifiers such as barycentric, heliocentric, geocentric and topocentric can be used.

4. RADIO INTERFEROMETRY

The most accurate ground-based astrometric measurements of direction are now made with Very Long Baseline Interferometry, and the ultimate hope is that VLBI will provide a better approximation to an inertial reference frame than can be obtained at present from observations of stars. It is therefore very important to understand the principles of VLBI data analysis and the relationship between the reference systems of VLBI and optical astrometry. Already there has been some confusion over the origin of right ascension, elliptic aberration and the adoption of non-conventional values for some constants, but these matters are out of place at this Symposium. What does concern me however is the presentation of the fundamental principles of VLBI analysis.

Not working in this field myself, I cannot claim any degree of familiarity with the literature, but I am rather confused by what I have found. There seems to be general agreement on the modelling of coordinate time delay Δt_c between the arrival of a wavefront at two stations, when referred to the Solar System barycentric frame, but some confusion arises when this is converted to a terrestrial proper time delay, $\Delta\tau$, between the clock readings at the two stations. Thomas (1975) gives a derivation which can be expressed in the form

$$\Delta\tau = \Delta t_c(1 - c^{-2}\,p - c^{-2}\,\underline{v}'\underline{v}_2\,) - c^{-2}\,\underline{v}'\underline{b} \qquad (7)$$

https://doi.org/10.1017/S0074180900148132 Published online by Cambridge University Press

where $\underline{v}$, $\underline{v}_2$ are the barycentric coordinate velocities of the Earth and the second station, $\underline{b}$ is the baseline vector and $c^{-2}p$ represents periodic terms in the difference between coordinate time and proper time. The expression (7) agrees to the same degree of approximation with the formulation by Chopo Ma (1978) who quotes Robertson (1975).

However a rather different formulation has been given by Fanselow and Sovers (1985). Their approach is to transform Δt_c to the terrestrial proper frame by means of a Lorentz transformation using the velocity $\underline{v}$. This gives, in our notation,

$$\Delta\tau = \beta \; \{ \Delta \; t_c(1 - c^{-2} \; \underline{v}'\underline{v}_2) - c^{-2} \; \underline{v}'\underline{b} \} \qquad (8)$$

where $\beta^{-2} = 1 - c^{-2} \; \underline{v}^2$. This expression is rigorous within the limits of the physical model adopted, namely neglect of general relativity in the time transformation and the assumption of rectilinear velocities. Within these limitations, the main practical difference between (8) and (7) is the Lorentz factor β, which differs from unity by about 5×10^{-9}. I have no doubt that this factor is absorbed elsewhere in the model for the delay, but my point is that there is an apparent inconsistency in the literature which can only confuse the non-specialist.

5. TIME SCALES AND CONSTANTS

This leads me to my final point. The apparent confusion in the VLBI formulation arises because of the conventional definition of "coordinate" time. For practical and understandable reasons, the coordinate time scale in the barycentric reference frame, TDB, has been arbitrarily defined to have the same average rate as the atomic time scale of terrestrial clocks, TDT. This is allowed for in (7) but not in (8), which is otherwise more elegant and readily understandable.

Astrometry has, in the past, been hindered by the use of conventions which are adopted for computational convenience at one epoch but lead to confusion later on. An excellent case in point is the omission of the elliptic aberration from the time of Bessel until 1984. Now with the definition of TDB we are in danger of making the same sort of mistake.

I have drawn attention elsewhere (Murray, 1983 p 27) to the consequence of this definition of coordinate time, namely that the unit of time used in the barycentric coordinate frame must be different from that used in the terrestrial frame. The implications of this on the definitions of some of the standard astronomical constants have been studied recently in some detail by Fukushima, Fujimoto, Kinoshita and Aoki (1986).

My own solution to the problem is to include as defining constants those which specifically refer to the barycentric reference frame. Thus in addition to the conventional Gaussian constant k, I would define the speed of light in barycentric units to be

$$c_* = 173.14463331 \text{ au d}^{-1} \qquad (9)$$

where the unit of time is the day (d) which consists of 86400 $(1 + \eta)$

https://doi.org/10.1017/S0074180900148132 Published online by Cambridge University Press

SI seconds, where

$$\eta = \frac{3}{2} c_*^{-2} k^2 \qquad (10)$$

In place of (10) it might be preferable to introduce a numerical constant to take full account of the mean rate of coordinate time relative to terrestrial time.

The primary constants would include only those constants pertaining to the Earth-Moon system, expressed in SI units. Apart from the substitution of c_* for the light time for unit distance, the only numerical change from the existing system would be the number of metres in 1 au and the heliocentric gravitational constant which are both currently quoted to nine significant figures.

6. CONCLUSION

In this review I have mentioned those aspects of relativity which impinge upon fundamental astrometric measurements. I have assumed as a basis that General Relativity is correct.

Optical astrometry provided the first direct test of the theory at the 1919 eclipse. In recent years the modern techniques of radar-echo and radio deflexion measurements have all converged to the prediction of General Relativity to much better than one per cent (Will 1980). The HIPPARCOS satellite, to be launched by the European Space Agency in 1988, will make of the order of 10^7 astrometric measurements of milli-arc second accuracy. Schutz (1982) has estimated that from these data it should be possible to make an independent check on General Relativity, again to much better than one per cent. Cowling (1983) has pointed out that all the HIPPARCOS measurements will be made at least 47° away from the Sun, so that, unlike the radio deflexion measurements, they cannot possibly be affected by the solar corona; furthermore he has proposed a generalized axially symmetric model for the Solar System metric whose parameters could be solved for. It is thus possible that, once again, optical astrometry will make a significant contribution to the study of relativity in the Solar System and thus play its part in bridging the gap between theorists and observers to which Atkinson drew attention more than 20 years ago.

REFERENCES

Atkinson, R. d'E., 1963 Proc. R. Soc. Lond. A 272, 60.

Brandt, V.E., 1975 Soviet. Astr. 18, 649.

Chopo Ma 1978 NASA Technical Memorandum 79582

Cowling S.A. 1983 Ph.D. Thesis University of Wales

Eddington A.S. 1923 The Mathematical Theory of Relativity p 80, Cambridge University Press.

Fanselow J.L. and Sovers O.J. 1985 JPL Publication 83-39 Rev. 1.

Fukushima T., Fujimoto M.K., Kinoshita H. and Aoki S. 1986 Celestial Mechanics, (in press).

Harvey G.M. 1979 Observatory 99, 195.

Mikhailov A.A. 1976 Soviet Astr. 20, 248.

Murray C.A. 1981 Mon. Not. R. Astron. Soc. 195, 639.
Murray C.A. 1983 Vectorial Astrometry. Adam Hilger, Bristol.
Thomas J.B. 1975 Astron. J. 80, 405.
Robertson D.S. 1975 Ph.D. Thesis M.I.T.
Schutz B. 1982 ESA SP-177, 181.
Will, C.M. 1980 Ann. New York Acad. Sci. 336, 307.